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Black Holes and the Nature of Quantum Gravity

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Abstract

Hawking and Wald have recently argued that the process of quantum black hole evaporation requires the violation of the fundamental physical law which asserts that the time evolution of quantum states is governed by unitary operators. I show this violation can be avoided by a change in the global boundary conditions. It is remotely possible that astronomical observation could establish whether or not the universe has these boundary conditions in which quantum mechanical time evolution is governed by unitary operators.

In spite of 50 years of effort, no one has been able to construct a consistent theory of quantum gravity. The gravitational force has certain extraordinary properties which are not shared by the three (or one) other fundamental force(s). For example, the pure gravitational field is nonrenormalizable, in contrast to GUTs, and classical gravity is the geometrical structure of the space-time manifold itself, rather than geometrical structures on fiber bundles over the space-time manifold, as in GUTs. Since quantum gravity will fully manifest itself only at extremely high energies, one cannot use experiment as a guide to decide which features of classical gravity and standard quantum mechanics should remain in the fully quantized gravity theory. It has even been argued by a number of relativists, notably Hawking [1] and Wald [2, 3], that one of the basic laws of quantum mechanics, the axiom that temporal evolution of states is via a unitary S matrix, does not hold for quantum gravity! I shall show that this conclusion can be avoided by a change (admittedly radical) in the global boundary conditions imposed on the space-time manifold, and that it is remotely possible for this

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question of nonunitary quantum evolution vs. radical boundary conditions to be decided by astronomical observations. These observations would also give experimental constraints on the fully quantized gravity theory.

The arguments of Hawking and Wald are based on an analysis of the Hawking black hole evaporation process. These authors have shown that the evaporating black hole should terminate in a naked singularity. This singularity will annihilate information expressed in the quantum states which enter the event horizon [1, 2], and it is this destruction of information that gives the Hawking radiation its thermal character.

If S_1 is the spacelike partial Cauchy surface before a black hole has evaporated, and S_2 a partial Cauchy surface some time after the black hole has evaporated, then the existence of a naked singularity between S_1 and S_2 will cause a pure quantum state on S_1 to evolve into a mixture on S_2, provided the quantum states corresponding to the naked singularity—the states which went down the hole—do not collapse into a single unique state. (If it does collapse to a single unique state, then the evolution could still be given by a unitary S matrix [4].) As Page has shown [4], a single singular state for the end of a black hole cannot be ruled out in our present state of ignorance of quantum gravity, but Wald [2] and Hawking [1] have presented cogent reasons for believing it is unlikely. (In particular, if it were true, then information cannot be permanently lost inside the black hole, so the black hole is not really a black hole [6].) If this is so, then the Hawking process will sometimes cause pure states to evolve into mixtures, which is impossible if the evolution operator is unitary. Hawking and Wald therefore propose that the evolution operator is a "superscattering operator" rather than an S matrix. The superscattering operator would make the quantum evolution of one pure state into another nondeterministic (or impossible) in contrast to evolution via a unitary S matrix. Wald [2] and Page [4] point out that in addition the destruction of information in this way would also imply that quantum gravity violated CPT invariance. Page [5] also suggests that a naked singularity might even destroy the superscattering operator and make even the evolution of density matrices indeterminate. Thus if naked singularities indeed occur, then quantum gravity will not obey some of the most basic laws of quantum field theory.

On the other hand, we can save the laws of physics by simply insisting that naked singularities do not exist. This principle of "quantum cosmic censorship" says that if $H^+(S_1)$ or $H^-(S_2)$ are the Cauchy horizons for S_1 and S_2, respectively, then either $J^+(H^+(S_1))$ or $J^-(H^-(S_2))$—either the region of space-time to the future or to the past of the respective Cauchy horizon—simply does not exist. The physically existing spacetime manifold must either end or begin at a Cauchy horizon.[2] The principle of quantum cosmic censorship proposed here is different from the quantum cosmic censorship principle put forward by Page

[2] Definitions of H^+, J^+, etc. are given in [27].

[4], who required only that initial mixed states be mapped uniquely into final mixed states, but which assumed these initial and final states were given on asymptotically flat hypersurfaces. The version of quantum cosmic censorship proposed here does not require the existence of such hypersurfaces, and so can be applied in a cosmological context. I do assume however, that the total quantum universe can be pictured as a Feynman sum over many worlds (4-manifolds), and that there exists a set Σ dense in these 4-manifolds such that in each element of Σ there is an S_2 and an $H^-(S_2)$, with $J^-(H^-(S_2))$ nonexistent. (I shall assume in what follows that it is S_2 and not S_1 which really exists.) Another important difference between my version of quantum cosmic censorship and Page's is that mine requires pure states to be mapped into pure states. Page's version is concerned with mixed states, and may admit more indeterminism than standard quantum mechanics. In fact, if Page's mixed states are not merely pure states (pure states can be regarded as extremal mixed states), then Page's version *will* have more indeterminism.

Such a requirement of nonexistence does not contradict any physical laws, only the generally assumed global boundary condition which says that one should extend through nonsingular Cauchy horizons. On the contrary, such nonexistence is actually *required* by very general physical laws, as pointed out above. Experiments have to date confirmed the laws, but there is no experimental evidence whatsoever for supposing continuation through a nonsingular Cauchy horizon. The assumption that one can rests on nothing but a philosophical prejudice. Even if such a continuation were possible, there are no known general rules for continuation that would give a unique answer [7]. Even analytic continuation across a Cauchy horizon cannot always yield a unique result [8]. Furthermore, as Penrose has pointed out [25], it is quite possible that a Cauchy horizon arising from black hole evaporation would be singular, in which case no continuation could be made. Lake [26] has shown such singular horizons *do* arise for certain equations of state. Ellis [34, 35] has discussed the question of whether the universe has an extendible boundary, and the steady state model actually *has* such a boundary [27].

This principle of quantum cosmic censorship might actually be testable by astronomical observation of the primordial black hole (PBH) mass distribution. If PBH were formed in the Big Bang—and we would expect them to be formed unless the initial singularity were extremely regular—then according to the conventional theory, many of these PBH would be exploding and thus ending in naked singularities today. However, if quantum cosmic censorship holds, this is not possible. What is possible under quantum cosmic censorship is for the PBH mass distribution ρ to have the form of a step function in cosmic time:

$$\rho(M_{\mathrm{BH}}(t, t_0)) = \tilde{\rho}(M_{\mathrm{BH}}(t))\,\theta(t_0 - t) \tag{1}$$

where t measures cosmic time (or more precisely York time, which is the global absolute time defined by the constant mean curvature hypersurfaces [9-11]), t_0

is some definite time in our past; θ is the step function, and $\tilde{\rho}$ is some function of the primordial BH mass. With this form of ρ, the mass distribution will change with time, and will be identically zero for all $t > t_0$. Note that ρ is an implicit function not only of t, but also of t_0, which will be later than t for all values of t for which ρ is nonzero. At the initial Friedmann singularity, $t = 0$.

With this form of the mass distribution, the principle of quantum cosmic censorship would hold for $t > t_0$, but for the principle to hold at all times we must assume that the part of the space-time manifold before t_0 simply does not exist. With the mass distribution given in (1), the space-time manifold corresponding to cosmic time $t < t_0$, would of course *not* satisfy quantum cosmic censorship, but there is no reason to require it to do so, since by assumption, it does not exist. If, however, one did impose quantum cosmic censorship on both the physically existing manifold and the nonexistent manifold, then the mass distribution would have to have the form of a δ function. The consequences of such a distribution are discussed in [12]. Functional forms other than (1) are consistent with the principle of quantum cosmic censorship—a single BH explosion at a single event in the universe, with $J^-(H^-(S_2))$ nonexistent, for example—but only (1) is consistent in addition with the Copernican cosmological principle, which requires that a cosmological function cannot depend (in the large) on spatial coordinates. A small scale spatial variation could be present in (1).

Thus if BH explosions were observed to conform to a mass distribution like (1), this would confirm the principle of quantum cosmic censorship. A mass distribution like (1) is most unlikely to arise in the early universe for any other reason than forcing the universe to obey cosmic censorship. Even a PBH *initial* mass distribution which is a step function in the initial BH mass would be very unlikely to arise via purely astrophysical processes in the early universe, and a step function with cosmic time as the variable is even more unlikely. One might expect cosmic time and hole initial mass to be logarithmically proportional if the black holes were all formed at the same time, since the lifetime of a BH $\propto M^3_{\text{initial}}$, but the lifetime of a black hole is measured in the proper time of the BH, not cosmic time. Because of variable velocity relative to the cosmic time frame, and because of inhomogeneities of the gravitational fields through which the PBHs move in the time since the Big Bang, a step function in initial BH mass would result in a smeared out distribution of final explosion times, not the step function postulated by quantum cosmic censorship.[3] In other words, from the point of view of the nonexistent region $J^-(H^-(S_2))$, a distribution like (1) would appear to have teleological properties: the initial data in the early universe would be finely tuned so as to cause the BH explosions to occur simultaneously in the far future. We avoid teleology only by assuming $J^-(H^-(S_2))$ does not exist.

Such teleology appears to be an inevitable feature of *any* quantum gravity theory which is (1) based on the idea of a single space-time base manifold, and

[3] I am grateful to Professor D. M. Eardley for pointing out this difficulty.

(2) admits naked singularities in that manifold. The reason is that a manifold with an isolated singularity has a different topology from the same manifold without the singularity, and the global topology of the manifold is an *a priori* "given" in the initial data. As DeWitt put it: ". . . I believe that to talk of 'space-time foam' and 'changing topology' is nonsense. Topology is built in at the beginning, in the very definition of the base manifold of any theory. It is not a dynamical object, and to this date no one has come within twenty-five light years of making it so, despite a quarter of a century of handwaving about it [13]."

The principle of quantum cosmic censorship requires that this "prechosen" topology must contain no "isolated" or "timelike" singularities. The elimination of the latter type means that the boundary conditions on the wave function must be such that no space-time in the sum-over-histories contains a "timelike" singularity.

It should be emphasized that the sum-over-histories involves summing *only* over those metrics which can be placed on the "given" unique base manifold. The sum will not include other base topologies, because as DeWitt pointed out in the passage above, there are indications that this cannot be done in a consistent way so as to yield a complete, universal theory which is deterministic in the sense standard quantum mechanics is deterministic. Hawking [28] and his co-workers [29, 30] have made a valiant attempt to make rigorous the idea of a sum-over-histories of a space-time foam, but their effort founders over the non-classification theorem for 4-manifolds. It has been shown that there is no algorithm for deciding whether compact non-simply-connected 4-manifolds are homeomorphic [31], and one must have such an algorithm if one is to make sure *all* topologies are included exactly once in the Feynman sum. Hawking et al. are of course aware of this problem, and they try to avoid it by restricting the foam to simply connected compact spin 4-manifolds, which can probably be classified by the Euler number χ and Hirzebruch signature τ. Hawking justifies this restriction in two ways. First, he claims ". . . classifiability might not matter too much because one would probably not want to treat very complicated topologies exactly but only on some statistical basis [28]." Second, one can always pass to the simply connected universal covering manifold, and if this is noncompact it could be compactified "at some large volume with only a small change in the action per unit volume [28]."

I feel that both of these justifications are inadequate to overcome the non-classification problem, if all nonsimply connected topologies are indeed included in the space-time foam. In such a situation, a sum-over-histories restricted to the simply connected topologies could give the complete path integral only if such topologies were dense in the space of all topologies. The situation would be the same as with the standard quantum mechanical path integral, where the sum-over-histories is restricted to paths composed entirely of straight line segments. Such paths are dense in the space of continuous curves, so the exact path integral is obtained. But if there is no algorithm to show that two manifolds are homeomorphic, how can it be shown that there is a simply connected manifold which approximates a given manifold arbitrarily closely? Furthermore, one would want to give statistical weights to the alternative paths, and if one cannot classify the manifolds, it is impossible to be sure that one has not counted a given topology twice, or omitted it altogether.

In addition, passing to the compactified universal covering manifold, which in general contains several "copies" of the base manifold, would change the action at least by a factor equal to the number of "copies" in the covering manifold, unless the action integral were in some way restricted to a canonical single copy. But such a restriction is impossible, because it would amount to a classification of the non-simply-connected base manifolds. Requiring the non-simply-connected 4-manifolds to be spin manifolds does not help, because it is possible to construct a different spin manifold for each element of a finitely presented group, which implies that the compact spin manifolds themselves cannot be classified.

A final difficulty with the space-time foam picture is its inability to describe passage from one topology to another in a differentiable way [28]. This probably implies the presence of naked singularities in the space of all topologies, and I am requiring the base topology to be free of such isolated singularities.

If the global topology can contain no isolated singularities, then it must follow that it is impossible for protons or other elementary particles to decay via the tunneling of valence quarks to BH, which then decay to an instantaneous naked singularity and radiation[4] or else such a decay would destroy the universe. Such decays have been predicted [14-16] on the basis of semiclassical estimates of the likelihood that two subatomic particles would be within a Schwarzschild radius of each other. However, as the authors of these predictions are well aware, this involves dubious approximations. For instance, Carter has pointed out [17] that measured in geometrical units, the charge and spin angular momentum of an typical elementary particle is some 20 orders of magnitude larger than its mass. A proton, quark, or electron, regarded as a classical object, resembles a charged, rotating naked singularity more than a BH. Thus in order for two particles to come together to form a BH, the net charge and angular momentum would have to almost completely cancel. This is not impossible, but we would expect such a process to occur at an order higher than the semiclassical. Hawking [28] and his coworkers [29, 30] have also predicted the decay of proton by assuming that space-time has a foam of topologies on the scale of the Planck length. This assumption is contrary to the picture proposed here, which envisages the base manifold as having a single unique topology, and quantum gravity arises via metric fluctuations only on this base manifold. Thus my proposal would preclude the Hawking decay mechanism. Since proton decay via the gravitational interaction occurs in the Hawking space-time foam scheme, but not in my theory, it is

[4] I am grateful to Professor D. N. Page for pointing out that the decay of such particles would constitute a violation of quantum cosmic censorship.

possible, at least in principle, to decide between these two theories by looking for such a decay. In order for my theory to be consistent with observation, the proton must be either completely stable to decay via passage to a naked singularity, or else have a lifetime greater than

$$10^{80}\left(\frac{R}{R_{\mathrm{H}}}\right)^3(t - t_0)\text{ years} \tag{2}$$

where R_{H} is the Hubble distance, R is the radius of the universe, if the universe is closed, and $(t - t_0)$ is the actual age of the universe. If the universe is open, then the proton must be absolutely stable against decay via this process. The proton lifetime to gravitational decay predicted by Hawking [32] is 10^{122} years; no BH explosion has been seen closer that 10 light years [33], so $(t - t_0) > 10$ years, and $R \geqslant R_{\mathrm{H}}$. Thus I require a lifetime to decay via a gravitational process to be greater than 10^{81} years. Neither lifetime seems ever to be within range of experiments, which now can only measure lifetimes less than about 10^{32} years, which is comparable to lifetime predicted by GUTs. Thus there is no firm evidence for the decay of elementary particles to actual naked singularities, and it is unlikely there ever will be.

The boundary condition for quantum gravity which I am proposing in this paper has a number of advantages over the standard theory, which assumes the universe began at an infinite curvature singularity. As DeWitt has emphasized [13, 18], his new effective action method and indeed all other methods of quantizing gravity proposed to date have severe difficulties near infinite curvature singularities. On the other hand, if the universe is assumed to be infinite in proper time, this results in general in an infinite value for the universal action. One infinity is eliminated at the price of introducing another. With my boundary conditions, both infinities are eliminated if the standard picture of the naked singularity formed by BH evaporation is correct. On the one hand, by assumption the beginning of the universe occurred a finite proper time ago; on the other hand, the future side of a BH evaporation-produced naked singularity is considered to be flat space up to the singularity itself, which means a locally extendable singularity [19], with all curvature invariants and components finite. DeWitt [13, 18] is attempting to avoid both infinities by requiring the entire 4-manifold to be compact, but my boundary condition seems more plausible.

As a matter of fact, boundary conditions similar to mine are actually imposed in most papers on physical cosmology.[5] These papers assume that certain spectra of fluctuations are given at a certain time, say, the Planck time. From these data the evolution of the universe is then computed. No one has been able to deduce these initial data from boundary conditions on the singularity itself [36]. In both deterministic classical general relativity and standard unitary time evolution quantum mechanics, the actual time chosen does not matter in the sense that in either

[5] I am grateful to Dr. J. D. Barrow for pointing out this analogy to me.

case these initial data could never evolve anything new. Everything that happens must be implicit in the given initial data. Of course, my boundary condition is topological in nature, and cannot restrict the initial data totally. In order for quantum cosmic censorship to hold at all, however, it is necessary for the spectra of initial fluctuations to be close to the very regular Friedmann solution. Otherwise PBH would be very numerous and would have a wide range of masses, contradicting (1). Since quantum cosmic censorship must hold if the fundamental physical laws–such as the unitarity of the time evolution operator–are to hold, this simple requirement of physical law consistency provides a natural explanation for the extreme regularity of the early universe. More exotic explanations [20] are not necessary.

An observation of the PBH mass distribution, to see if it had the form (1), would do more than just test my theory of the universal boundary condition. The unitarity (or lack of it) of the time evolution operator is of crucial importance in two other fields of quantum mechanics: (1) the quantum theory of measurement, and (2) quantum statistical thermodynamics. In the Copenhagen interpretation the measurement process involves a nonunitary operation called "the reduction of the wave function," while the many-worlds interpretation [21, 22] is based fundamentally on the assumption that time evolution is always unitary, with the measurement process neither adding nor subtracting from the total information coded in the universal wave function. Standard quantum statistical thermodynamics also assumes unitary time evolution, and a consequence of this–Liouville's theorem–is what makes derivation of the second law of thermodynamics from quantum mechanics difficult, if not impossible [23]. For this reason Prigogine has suggested that standard statistical thermodynamics is inadequate and [23, 24] that there *must* be quantum systems in the universe for which the time evolution is *not* unitary, with pure states on occasion going to mixtures. The proposals of Hawking, Wald, and Prigogine actually undermine the basis of the many-worlds interpretation and standard statistical thermodynamics. Conversely, the observation that the PBH spectrum was of the form (1) would tend to confirm both, and to refute the theories of Hawking, Wald, and Prigogine. It would provide experimental support for the proposition that Nature would rather destroy the entire universe than allow a pure state to go to a mixture!

I do not propose that a major effort be made to confirm (1), because my proposal for the universal boundary condition is rather radical. I do think astronomers who are searching for PBH should be aware of the implications of (1), in case such a mass distribution is seen.

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