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Perry, Kelvin, and the age of the sun

Frank J. Tipler[a]

Department of Mathematics and Department of Physics, Tulane University, 70118 New Orleans, USA

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Abstract. Lord Kelvin argued that the Sun had to be between 20 and 100 million years old, based on the assumption that the Sun's energy source was gravitational contraction. As everyone now knows, the Sun's actual power source is the thermonuclear fusion of hydrogen into helium. But Kelvin's number is based on a physical assumption for which he could give no justification: the Sun's density is approximately constant. Had Kelvin assumed instead that the Sun had a small core near a black hole radius – an assumption allowed by the knowledge of physicists at the end of the nineteenth century – he would have obtained an age for the Sun as long as 10 trillion years, completely consistent with the long time scale required for evolution. Conversely, had Kelvin accepted the geologists' time scale, he would have been forced to acknowledge the existence of very dense objects, making it easier for twentieth century astronomers to accept the existence of black holes and neutron stars.

Three recent papers ([Stacey 2000], [England 2007a], [England 2007b]) have concluded that Lord Kelvin's calculation of the Age of the Earth should have been taken seriously by geologists, because, when combined with geologist's actual knowledge of the age of the Earth, it implied that convection currents were responsible for heat transfer deep in the Earth. Thus, had geologists taken Kelvin seriously, continental drift (which is driven by this heat convection) would have been accepted much earlier than it was. And it was not impossible that this earlier acceptance could have occurred. In 1895, Kelvin's own student John Perry published a series of three papers in *Nature* wherein he showed that that if Kelvin's heat flow calculation were modified by the assumption that main heat transfer mechanism was convection, the Earth could well be billions of years old. Kelvin replied to Perry [Kelvin 1895] that even if we grant that heat transfer inside the Earth is indeed due to convection, and indeed allow an Earth age of billions of years, the same calculation will not significantly increase the age of the Sun, which would still be less than 100 million years. Thus, concluded Kelvin, there is no reason to assume convective heat transfer. The geologists' and biologists' claim that life has existed for hundreds of millions of years, which is the only reason for assuming convection in the first place, would still necessarily be false.

[a] e-mail: `tipler@tulane.edu`

The three papers cited above agree with Kelvin that his estimate for the age of the Sun, 100 million years, was necessarily in error, given the physics known to Kelvin in the nineteenth century. I shall show that there was a possibility, known to Kelvin in the nineteenth century, that would allow gravitational energy to supply the Sun for gigayears. Thus, had the age of the Earth obtained by geology and biology been accepted as true – as it is – then not only would have the mechanism of continental drift been known prior to the publication of Wegener's book advancing the idea, but the possibility of black holes would have been generally recognized decades before they were.

The gravitation potential energy of a constant density sphere of total mass M and radius R, assuming zero potential energy at infinity is

$$-\frac{3}{5}\frac{GM^2}{R} \tag{1}$$

where $G = 6.672 \times 10^{11}\ \mathrm{N\,m^2/kg^2}$ is the gravitational constant. This formula was first obtained by Hermann von Helmholtz in 1854 [Helmholtz 1961], in a paper wherein he proposed that gravitational collapse energy was the Sun's energy source. Assuming that the Sun's luminosity comes entirely from gravitational collapse, and that it has had a constant luminosity $L_\odot = 3.827 \times 10^{26}$ W for its entire history gives a solar lifetime of

$$\frac{3}{5}\frac{GM_\odot^2}{R_\odot L_\odot} = 18.9 \times 10^6\ \text{years} \tag{2}$$

where $M_\odot = 1.99 \times 10^{30}$ kg is the mass of the Sun, and $R_\odot = 6.96 \times 10^8$ m is the Sun's radius. This number of around 20 million years is Kelvin's lower estimate for the age of the Sun.

However, there was no physical reason at the time forcing Kelvin to assume a constant density for the Sun. In fact, Kelvin realized that a non-constant density was a real possibility, and he also realized that a greater density in the Sun's core would imply a greater age to the Sun. As a consequence of a possible greater central density, Kelvin was willing in 1862 to extend the upper bound to the Sun's age to 100 million yeas [Kelvin 1862].

But suppose he had assumed instead that the Sun had a core with a super high density, much greater than he was willing to assume in 1862. For example, suppose he had assumed a fraction of the Sun's mass was such that this fraction was inside what we now call the "black hole radius". A black hole had been proposed in the eighteenth century independently by Laplace [Laplace 1799] and the English clergyman John Michell [Michell 1784], and it is well known that the radius of the black hole in Newtonian gravity is the same as the black hole radius in general relativity. Although in general relativity it is not possible for a star to be stable inside its black hole radius, there is no such stability limit in Newtonian gravity.

To compute the black hole radius in Newtonian gravity, we only have to assume that a light particle of mass m having the speed of light c at the surface of the black hole, makes it to infinity with speed zero. Then conservation of energy gives

$$-\frac{GMm}{R_{BH}} + \frac{1}{2}mc^2 = 0. \tag{3}$$

Solving this equation for the black hole radius R_{BH} gives

$$R_{BH} = \frac{2GM}{c^2}. \tag{4}$$

Putting this radius into equation 1 gives a total gravitational potential energy of

$$-\frac{3}{10}Mc^2 \tag{5}$$

which means that 30 per cent of the total rest mass of the core would be available to power the Sun. If the entire Sun were to emit at constant luminosity for its entire lifetime until it reached this radius, its total lifetime would be

$$\frac{3}{10}\frac{M_\odot c^2}{L_\odot} = 4.44 \times 10^{12}\,\text{years.} \tag{6}$$

Thus, had Kelvin been willing to assume that only one thousandth of the Sun's mass had collapsed to its black hole radius – once again, I emphasize that this assumption is consistent with the physics known to Kelvin – he would have obtained a current age of the Sun of 4.4 billion years, completely consistent with its known actual age of 4.6 billion years.

The above calculation is only concerned with whether gravitational energy is sufficient to provide the Sun's current luminosity for several billion years, which is far as Kelvin himself went. A more realistic calculation would have to address the question of whether gravitational collapse could provide an approximate constant luminosity for this length of time, a question which was not addressed, even for the much shorter interval considered by Kelvin.

If Kelvin had attempted to avoid the conclusion that gravitational collapse could provide sufficient energy by arguing that it could not do so at a constant luminosity, it is likely that he would have tried to support this argument by assuming the solar interior satisfied a polytropic equation of state. A polytrope relates pressure p to density ρ via the equation $p = K\rho^{(n+1)/n}$, where n is the polytropic index, and K is a constant. Chandrasekhar [Chandrasekhar 1957] has an excellent history of the development of the idea of the polytrope in his 1939 book on stellar structure. Kelvin himself introduced the idea of a polytrope in 1862, but only as applied to the Earth's atmosphere. The American Homer Lane first applied the idea to the Sun in 1869. Lane argued that one could assume the interior of the Sun was a monatomic gas (for which $n = 3/2$), because the high temperature in the solar interior would be destructive to molecules: "... their component atoms might be torn asunder." In 1880, A. Ritter considered the pulsation of a polytrope gas sphere, and showed that only if $n < 3$ would the polytrope be stable. In 1887, Kelvin applied the polytrope theory to the Sun himself. However, in none of these papers was there any discussion of the long-term evolution of the Sun, much less the crucial problem of how to ensure an approximately constant solar luminosity on any time scale.

Had the question been asked, a constant solar luminosity over several billion years could have still been defended if the physicists of the day were willing to abandon a polytrope equation of state for the deep solar interior, retaining the polytrope only for the outer layers of the solar atmosphere. With a polytrope equation of state, then a hyper-condensed center (required to get sufficient energy for billions of years) would generally require a very large outer radius, much larger than the actual solar radius.

What a defender of gravity as the energy source could have done is simply set the energy intensity at a given radius r_0 to be a constant independent of time, and adjust the equation of state for all lesser r so that this intensity and this radius are both constants over several billion years, under the assumption that gravity is the only energy source. This would be permissible as physics, because in the late nineteenth century, no other source of energy could be imagined, and it was understood that absolutely nothing was known about the equation of state of materials at extremely high temperatures, pressures, and densities. In fact, Lane, in his 1870 paper, admitted that all of his assumptions about the solar equation of state could be in error for precisely this reason.

In other words, all a defender of gravity as the energy source would have to do for the Sun is *exactly* what cosmologists do today in their theories of the very early universe: use standard physics for densities and temperature below certain values, and new physics for higher values. The only difference is that we are confident we know the physics at higher densities and temperatures than the nineteenth century physicists did.

The above discussion of what the nineteenth century physicists would have done if they had accepted the geologists' timescale is an example of *counterfactual history* which is now recognized as an important approach to the history of science. In 2008, *Isis* had a special issue devoted to this approach [Radick 2008]. In counterfactual history, one must image what some actors would have done in response to a change in the behavior of at least one other actor. I have included the above discussion of polytropes to investigate how, given a proof that gravitational energy could provide the necessary luminosity for billions of years, some opponents of the idea of a billion year old Sun could have reacted.

England, Molnar, and Richter point out ([England 2007a], [England 2007b]) that a substantial fraction of the total heat from the Earth's interior is indeed gravitational collapse energy, so Kelvin was partially correct about the source of the Earth's internal heat. Nor was Kelvin entirely wrong about the ultimate source of the Sun's energy. In the case of a massive star, the core begins as a mixture of hydrogen and helium – mainly protons with some neutrons – and ends as a neutron star, composed of particles more massive on average than the beginning particles. Where does the energy come from? Not from thermonuclear fusion, since the helium nuclei are torn apart during the formation of the neutron star. Ultimately, the energy is gravitational collapse energy, released when the core collapses to form the neutron star. Averaged over the entire star's lifetime, all the total radiation emitted comes from gravitational collapse, as astrophysicists have known for decades [Zeldovich 1971]. Over our Sun's lifetime, a substantial fraction of the Sun's energy will, in the end, come from gravitation, as our Sun's core collapses to form a white dwarf.

England, Molnar, and Richter rightly emphasize that had geologists combined their knowledge that the Earth was much older than Kelvin's 100 million years with Kelvin's proof that a greater age was impossible if the Earth was a solid body on all time scales, they would have been led to the conclusion that convection currents existed in the mantle. Geologists would have realized that these convection currents could have carried the continents to different locations on the Earth's surface. England, Molnar, and Richter believe, and I agree, that taking both geology and physics equally seriously would have led to the acceptance of continental drift theory decades earlier than it actually required.

Conversely, had Kelvin taken the geologists' age of the Earth seriously, and applied the physics known to him, he would have been forced to conclude that the Sun had a very dense core. Kelvin did not know about nuclear fusion; he believed that the ultimate source of energy was gravitational collapse energy. The only way to achieve a billion year Solar lifetime was to have a very dense core. If Kelvin had advanced this idea, physicists would have been prepared to accept the idea of black holes and neutron stars long before they actually did. I have argued elsewhere ([Tipler 1994], [Tipler 2005]) that even today physicists have not truly appreciated the enormous power of gravitation collapse energy: I have claimed that such energy is indeed the ultimate energy source (carbon dioxide free!) just as Kelvin believed.

In summary, physicists and geologists should always take the results of their colleagues in the other department very seriously. Physical reality does not respect departmental boundaries.

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