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# FDTD-Based Statistical Analysis of Crosstalk on Planar Data buses 

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#### Abstract

Crosstalk is a major limiting factor of signal quality and important EMC problem in high speed communication systems especially when fast data buses involved such a data bus is considered as a planar multiconductor transmission line. This paper will demonstrate how the finite difference time domain (FDTD) method provides an exact solution of the transmission-line equations to analyze the near end and the far end crosstalk. In addition, this study makes it possible to analyze pulse sources connection at the input according to random occurrences in order to closely approach real cases accurately. The paper also discusses a statistical analysis, based upon a set of several simulations. Such analysis leads to a better understanding of the phenomenon and yields useful information.


Keywords- Planar data bus, Multiconductor transmission line, Crosstalk, Finite difference time domain (FDTD), Printed-circuit board (PCB), Statistical analysis.

## I. Introduction

In the field of high-speed data communications, crosstalk becomes an increasingly important issue as data rates increase. Crosstalk can occur anywhere in electronic systems, such as the silicon die, package or printed circuit board (PCB). Crosstalk affects mainly and in a straight forward way the voltage of the victim signal, depending on the nature of the coupling between the aggressors and the victim. In this paper, we deal with data buses that are widely used in electronic systems to transfer data from one device to another or to multiple devices with a good level of precision. Such buses consist of a large number of coupled individual conductor traces implemented on a dielectric substrate within a printed circuit board (PCB). Therefore, unintentional electromagnetic coupling between closely spaced signal traces affect strongly the system performance and may contribute to radiated and conducted electromagnetic emissions[i]. In industrial applications, such as embedded systems and motherboard of sophistical devices, the bus conductors can be excited with different voltage sources and also have different loads. In most cases, binary sequences propagate along the coupled traces by means of polar NRZ (Non Return to Zero) coding that enables a whole pulse for full duration of a bit. Logic " 1 " represents the signal at positive voltage and logic " 0 " represents the signal at negative voltage [ii]. As far as many practical issues are concerned, we assume that the coupled conductors are excited randomly with synchronized or unsynchronized pulses depending on the targeted performances. In order to handle the issue of source random occurrences, we carry out a time-domain statistical method that turns out very efficient in terms of determining crosstalk voltage variation at both the near and far ends of the victim trace.
In this paper, the circuit model based on transmission line theory is introduced to analyze crosstalk problems and some factors affecting crosstalk. The analysis is based on numerical
simulations using the finite difference time domain (FDTD). The FDTD is nowadays an established method in computational electromagnetic and is well suited for parasitic mode simulation. Using a time-domain method enables the propagation of a pulse to be seen clearly without the need for any frequency-time transformations [iii]. The paper is organized as follows. In section II we describe the FDTD algorithm for the calculation of voltages and currents on the line especially at the input and output. The issue of transmission line excited with multiple sources is handled in section III. The aim is to determine the crosstalk variation in the case of more than one aggressor. The study is carried out in a deterministic context. In section IV we propose a statistical analysis of crosstalk under random circumstances depending on the timing and probabilistic law of occurrences of the pulses. Finally in section V the paper is concluded.

## II. The FDTD Formulation

The transverse electromagnetic or TEM-mode model of a uniform ( $\mathrm{n}+1$ )-conductor, lossless transmission line is embodied in the equations [iv].

$$
\begin{align*}
& \frac{\partial}{\partial z} \boldsymbol{V}(z, t)+\boldsymbol{L} \frac{\partial}{\partial t} \boldsymbol{I}(z, t)=0  \tag{1}\\
& \frac{\partial}{\partial z} \boldsymbol{I}(z, t)+\boldsymbol{C} \frac{\partial}{\partial t} \boldsymbol{V}(z, t)=0 \tag{2}
\end{align*}
$$

Where $\mathbf{V}$ and $\mathbf{I}$ are $\mathrm{n} \times 1$ vectors of the line voltages ( with respect to the reference conductor) and line currents respectively. The line cross-sectional dimensions are contained in the $\mathrm{n} \times \mathrm{n}$ per-unit-length parameter matrices of $\mathbf{L}$ (inductance) and $\mathbf{C}$ (capacitance). The position along the line is denoted as z and time is denoted as $t$. One of the important approximate solution techniques is the Finite Difference Time-Domain method or FDTD [iii]. In that method, the line axis z is discretized in $\Delta \mathrm{z}$ increments or spatial cells, the time variable $t$ is discretized in $\Delta t$ increments or temporal cells, and the derivatives in the multiconductor transmission lines (MTL) equations are approximated by finite differences. The solution voltages and currents are obtained at these discrete points and represent an approximate solution of the MTL equations. In general, the accuracy of the solution depends on having sufficiently small spatial and temporal cell sizes [v].
In the case of MTL's, the boundary conditions are lumped loads at the two ends of the line, $\mathrm{z}=0$ and $\mathrm{z}=\angle$ for a line of length $\angle$. Linear, resistive such terminations can be characterized by [vi].

$$
\begin{align*}
& \mathbf{V}(0, \mathrm{t})=\mathrm{V} \mathrm{~s}-R_{S} \mathbf{I}(0, \mathrm{t})  \tag{3}\\
& \mathbf{V}(L, \mathrm{t})=R_{L} \mathbf{I}(L, \mathrm{t}) \tag{4}
\end{align*}
$$

Where Vs is the voltage sources
In order to insure stability in the FDTD solution, the discrete voltage and current solution points are not physically located at the same point but is staggered one-half cell apart [iii].

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Fig.1. Spatial discretization of the line showing location of the interlaced voltage and current solution points.
To apply this method, we divide the line into NDZ sections each of length $\Delta \mathrm{z}$ as shown in Fig.1. Similarly, we divide the total solution time into segments of length $\Delta \mathrm{t}$. In order to insure stability of the discretization and to insure second-order accuracy we interlace the NDZ+1 voltage points, V1, V2, ... , VNDZ, VNDZ +1 , and the NDZ current points, $\mathrm{I}, 12, \ldots$, InDZ, as shown in Fig.1. Each voltage and adjacent current solution points are separated by $\frac{\Delta \mathrm{z}}{2}$. In addition, the time points are also interlaced, and each voltage time point and adjacent current time point are separated by $\frac{\Delta t}{2}$.
Discretizing the derivatives in the MTL equations using the FDTD method we obtain:

$$
\begin{align*}
& \frac{V_{k+1}^{n+1}-V_{k}^{n+1}}{\Delta z}+L \frac{I_{k}^{n+\frac{3}{2}}-I_{k}^{n+\frac{1}{2}}}{\Delta t}=0  \tag{5}\\
& \frac{I_{k}^{n+1 / 2}-I_{k-1}^{n+1 / 2}}{\Delta z}+C \frac{V_{k}^{n+1}-V_{k}^{n}}{\Delta t}=0 \tag{6}
\end{align*}
$$

Where we denote:

$$
\begin{align*}
& \boldsymbol{V}_{k}^{n}=V[(K-1) \Delta z, n \Delta t]  \tag{7}\\
& \boldsymbol{I}_{k}^{n}=V\left[\left(K-\frac{1}{2}\right) \Delta z, n \Delta t\right] \tag{8}
\end{align*}
$$

We will denote the vector of currents at the source as Is and the vector of currents at the load as $\mathbf{I}$. The equations (5) and (6) are discretized at the source and load:

$$
\begin{align*}
& \frac{1}{\frac{\Delta \mathrm{z}}{2}}\left[\boldsymbol{I}_{1}^{n+1 / 2}-\frac{\boldsymbol{I}_{S}^{n+1}-\boldsymbol{I}_{S}^{n}}{2}\right]+\frac{1}{\Delta t} \boldsymbol{C}\left[\boldsymbol{V}_{1}^{n+1}-\boldsymbol{V}_{1}^{n}\right]=0  \tag{9}\\
& \frac{1}{\frac{\Delta \mathrm{z}}{2}}\left[\frac{\boldsymbol{I}_{L}^{n+1}-\boldsymbol{I}_{L}^{n}}{2}-\boldsymbol{I}_{N D Z}^{n+1 / 2}\right]+\frac{1}{\Delta t} \boldsymbol{C}\left[\boldsymbol{V}_{N D Z+1}^{n+1}-\boldsymbol{V}_{N D Z+1}^{n}\right]=0 \tag{10}
\end{align*}
$$

Solving (9) and (10) gives the recursion relations at the source and the load:

$$
\begin{align*}
& \boldsymbol{V}_{1}^{n+1}=\boldsymbol{V}_{1}^{n}-\frac{2 \Delta t}{\Delta z} \boldsymbol{C}^{-1} \boldsymbol{I}_{1}^{n+\frac{1}{2}}+\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left[\boldsymbol{I}_{s}^{n+1}+\boldsymbol{I}_{s}^{n}\right]  \tag{11}\\
& \boldsymbol{V}_{N D Z+1}^{n+1}=\boldsymbol{V}_{N D Z+1}^{n}+\frac{2 \Delta t}{\Delta z} \boldsymbol{C}^{-1} \boldsymbol{I}_{N D Z}^{n+1 / 2}-\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left[\boldsymbol{I}_{L}^{n+1}+\boldsymbol{I}_{L}^{n}\right] \tag{12}
\end{align*}
$$

Using (3) and (4) we obtain:

$$
\begin{align*}
\boldsymbol{V}_{1}^{n+1}= & \left(\frac{\Delta z}{\Delta t} R_{S} \boldsymbol{C}+1\right)^{-1} \times\left[\left(\frac{\Delta z}{\Delta t} R_{S} \boldsymbol{C}-1\right) \boldsymbol{V}_{1}^{n}-2 R_{S} \boldsymbol{I}_{1}^{n+\frac{1}{2}}\right. \\
& \left.\left(\boldsymbol{V}_{s}^{n+1}+\boldsymbol{V}_{s}^{n}\right)\right]  \tag{13}\\
\boldsymbol{V}_{N D Z+1}^{n+1}= & \left(\frac{\Delta z}{\Delta t} R_{L} \boldsymbol{C}+1\right)^{-1} \times\left[\left(\frac{\Delta z}{\Delta t} R_{L} \boldsymbol{C}-1\right) \boldsymbol{V}_{N D Z+1}^{n}-\right. \\
& \left.2 R_{L} \boldsymbol{I}_{N D Z}^{n+\frac{1}{2}}+\left(\boldsymbol{V}_{L}^{n+1}+\boldsymbol{V}_{L}^{n}\right)\right] \tag{14}
\end{align*}
$$

The voltages and currents are solved by iterating k for a fixed time and then iterating time. The initial conditions of zero voltage and current are used to start the iteration.

$$
\begin{align*}
\boldsymbol{V}_{k}^{n+1}= & \boldsymbol{V}_{k}^{n}-\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left(\boldsymbol{I}_{K}^{n+\frac{1}{2}}-\boldsymbol{I}_{K-1}^{n+\frac{1}{2}}\right)  \tag{15}\\
& \mathrm{k}=2, \ldots, \mathrm{NDZ} \\
\boldsymbol{I}_{K}^{n+3 / 2}= & \boldsymbol{I}_{K}^{n+1 / 2}-\frac{\Delta t}{\Delta z} \boldsymbol{L}^{-1}\left(\boldsymbol{V}_{k+1}^{n+1}-\boldsymbol{V}_{k}^{n+1}\right)  \tag{16}\\
& \mathrm{k}=1 \ldots \mathrm{NDZ}
\end{align*}
$$

In the application of the conventional FDTD Yee algorithm for 1-D problems a necessary condition to insure stability of the solution is the Courant limit (CFL) [vii]:

$$
\begin{equation*}
\Delta t \leq \frac{\Delta z}{V p} \tag{17}
\end{equation*}
$$

Where $V p$ is the phase velocity of propagation of the waves.

## III. Transmission Line With Multiple Sources

In a deterministic context, we deal with a planar multiconductor transmission line having multiple voltage sources in order to determine the crosstalk level variation at both the near and far ends of the victim conductor. Let us consider a 4-conductor coplanar transmission line with two aggressors and one victim as shown in Fig.2. The nominal parameters are $w=2.159 \mathrm{~mm}$, $\mathrm{s}=1.139 \mathrm{~mm}, \mathrm{~h}=1.20 \mathrm{~mm}, \varepsilon \mathrm{r}=4.7$ and the total line length is 10 cm and is terminated at the near and far ends in $50 \Omega$ resistors. Two lines are excited by two pulses whose rise time and fall times are identical ( $\tau_{1}=200 \mathrm{ps}, \tau=1 \mathrm{~ns}$ ) rising to a level of 1 V .


Fig.2. Line under study. (a) The line dimensions and terminations. (b) The crosssectional dimensions. (c) The source voltage wave shape.

The objective of the first step of performed simulations, by means of the FDTD algorithm, is to evaluate the crosstalk voltage at the far and near ends of the victim trace by connecting only one voltage source at a time. The following simulations are aimed to examine the complete circuit in terms of the victim trace response as both sources are connected to the line and work in a synchronized way. Fig. 3 illustrates the time-domain NEXT and FEXT crosstalk voltages for the cases previously described.

(b)

(c)

Fig. 3 (a) NEXT and FEXT voltages with the two excitations enabled. (b) NEXT and FEXT voltages with Vs2 disabled (Vs2=0). (c) NEXT and FEXT voltages with Vs1 disabled (Vs1=0)
Fig. 3 shows the comparison of the near end (NEXT) and far end (FEXT) crosstalk waveforms of the victim trace between one excitation source (Vs2=0 or Vs1=0) and two excitation sources cases. We clearly notice, from the computed results, that there is a close relationship between the two cases under study. Indeed,
the overall crosstalk voltage at the near and far ends when considering two pulses simultaneously is increased, representing the sum of the crosstalk voltage levels. Therefore, the near end and far end crosstalk is decreased when we eliminate the pulse applied to the trace1 (Vs1=0) or trace2 (Vs2=0).Consequently, by analyzing the results in Fig.3, we demonstrate that the reduction of the near end and far end crosstalk is mainly caused by the variation of the distance between receptor and generator conductors.

## IV. Transmission Line Excited Randomly By Multiple Pulses

As well known, the transfer of data along a transmission line causes inevitably electromagnetic coupling between conductors leading to crosstalk voltages. The transfer is generally performed by a code according to a random law.
In order to address this issue, for better understand, a simple configuration is proposed and analyzed in this section. We are limited to the case of transmitting two bits, each on different trace. We choose the pulses presented in Fig. 4 standing for logic " 1 " and logic " 0 ".


Fig.4. Representation of logic"1" and logic"0".
The circuit under study is the same as that in section III with two aggressors and one victim (Fig.2).The coupling effect of these two active traces on the victim one surely takes place whatever the signal waveform. The crosstalk voltage level depends strongly on the random occurrence and the amplitude of the two pulses.
As an assumption, the amplitude is taken constant and deterministic. However, the two bits have random occurrences governed by the uniform law whether the pulses are synchronized or unsynchronized.
In most situations, unsynchronized pulses can have either a small time shift or large time shift between them. Both situations are analyzed by means of statistical study taking into account equiprobable and non equiprobable cases.

## 1. Small time shift

## a. Equiprobable occurrences

We first consider the combinations of the synchronized and small time shifted pulses possibilities representing logic " 1 " and logic "0" as illustrated by Fig.5. Excitation couples $\left(\mathrm{V}_{\mathrm{S} 1}, \mathrm{~V}_{\mathrm{S} 2}\right)$ are chosen randomly and independently with equiprobable occurrences according to the uniform law. Fig. 6 shows the computed results through the FDTD method, of the near end (NEXT) and far end (FEXT) crosstalk voltage running 3000 simulations.

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Fig.5. Possibilities of small time shifted pulses.

(b)

Fig.6. (a) FEXT crosstalk voltage for 3000 simulations. (b) NEXT crosstalk voltage for 3000 simulations.
As well expected, there are several responses depending on the injection of excitation sources. It is interesting to note that crosstalk voltage reaches a maximum level that can be described as the worst-case, at a precise instant. Thus, FEXT and NEXT voltages present worst-case values at 400 ps and 200 ps , respectively. Fig. 7 and Fig. 8 show the histograms illustrating the number of occurrences of the worst-case values of FEXT and NEXT voltages.


Fig.7. Histogram of NEXT crosstalk for 3000 simulations at $\mathrm{t}=200 \mathrm{ps}$


Fig.8. Histogram of the FEXT crosstalk for 3000 simulations at $\mathrm{t}=400 \mathrm{ps}$.
From Fig. 7 and Fig.8, we notice that both NEXT and FEXT voltages present almost the same number of occurrences of the worst-case with different values. Ranging from -0.2 V to 0.2 V .

## b. Non equiprobable occurrences

In order to account for more uncertainties affecting practical issues, we consider the same transmission line excited with time shifted pulses according to non equiprobable occurrences. The time domain responses with respect to near and far end crosstalk voltages are the same as those presented in Fig.6. We carried out 3000 time domain simulations to achieve another statistical study based on FEXT and NEXT histograms at 400 ps and 200 ps respectively.


Fig.9. Histogram of NEXT crosstalk for 3000 simulations at $t=200 \mathrm{ps}$

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Fig.10. Histogram of the FEXT crosstalk for 3000 simulations at $\mathrm{t}=400 \mathrm{ps}$.
It is obvious that there is an increase of the number of occurrences of the maximum NEXT voltage level. Contrary to the previous case, the greatest magnitude value $(0.18 \mathrm{~V})$ is likely to take place more repeatedly than the others. On the other hand, the FEXT crosstalk voltage occurrences have decreased.

## 2. Large time shift

a. Equiprobable occurrences

In this section, we propose a further study to collect more quantities information on the statistical behavior of the crosstalk response. This fact can be achieved by considering large time shifted pulses combined with synchronized ones as shown in Fig.11. The time domain responses are identical to those obtained for the small time shift case. The excitation couples $\left(V_{S 1}, \quad V_{S 2}\right)$ are randomly and independently chosen with equiprobable occurrences according to the uniform law. The histograms, corresponding to the computed results at 200 ps and 400 ps , are displayed in Fig. 12 and Fig. 13.


Fig.11. Possibilities of large time shifted pulses.


Fig.12. Histogram of NEXT crosstalk for 3000 simulations at $\mathrm{t}=200 \mathrm{ps}$.


Fig.13. Histogram of FEXT crosstalk for 3000 simulations at $t=400 \mathrm{ps}$.

According to the two histograms shown in Fig. 12 and Fig.13, we can see that the maximum magnitude values of NEXT and FEXT crosstalk voltage do not have the greatest number of occurrences.

## b. Non equiprobable occurrences

Finally, the proposed study takes into account the fact that the large time shifted pulses combined with synchronized ones can be injected into the line with non equiprobable occurrences.


Fig.14. Histogram of NEXT crosstalk for 3000 simulations at $\mathrm{t}=200 \mathrm{ps}$.

## V. CONCLUSION

A time domain method, based on the FDTD algorithm to solve multiconductor transmission line equations, has been proposed to perform a statistical evaluation of the crosstalk on a data bus. A circuit that consists of two aggressor traces and one victim trace has been analyzed by dealing with several cases related to the occurrences of injected sources in order to simulate real behavior. The transmission line circuit under study has been excited randomly with two pulses according to the uniform law. The excitations, synchronized or unsynchronized, can be injected with either equiprobable or non equiprobable occurrences. The statistical analysis has led to several histograms representing the number of occurrences of the crosstalk worst case level at both near and far ends of the victim trace. The crosstalk variation in the case of more than one aggressor has been handled producing great deal of useful information.

## References

i. Qin Yin, Bin Chen, Bo Yang, Zhixue Shao, Bihua Zhou " Analysis of Crosstalk in PCB Design". Lab of Electromagnetics, Nanjing Engineering Institute No. 1 Haifuxiang, Nanjing 210007, China.
ii. Christian Schuster, Wolfgang Fichtner, "Parasitic Modes on Printed Circuit Boards and Their Effects on EMC and Signal Integrity" IEEE Transactions on Electromagnetic Compatibility, vol. 43, no. 4, November 2001.
iii. K. Li, M. A. Tassoudi, R,T. Shin, and J. A. Kong, "Simulation of electromagnetic radiation and scattering using a finite differencetime domain technique," Comput. Appl. in Eng. Education, vol. 1, no, 1, pp. 45-62, Sept./Oct. 1992.
iv. C. R. Paul, Introduction to Electromugnetic Compatibility, New York Wiley-Interscience, 1992.
v. F. M. Tesche, "On the inclusion of losses in time-domain solutions of electromagnetic interaction problems,"IEEE Trans. Elecfromugn.
vi. J. Alan Roden,Clayton R.Paul,William T.Smith and Stephen D.Gedney, "Finite-Difference, Time Domain Analysis of Lossy Transmission Lines," Phil. IEEE Trans.Electromagnetic Compatibility,vol. 38,no,1, February 1996.
vii. C.R.Paul,Analysis of Multiconductor Transmission Lines.New York:Wiley Interscience, 1994.

