

Comparison on Two Harmonic Current Determinate Methods of p-q and i_p-i_q

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Abstract: This paper implements a comparison on p-q and i_p-i_q methods. The simulations demonstrated that it gives same accurate results in case source voltage is symmetric and undistorted. In vice versa case the harmonic current can be detected accurately by i_p-i_q method, while the p-q method gives results are not accurately.

Keywords— p-q method, i_p-i_q method, instantaneous reactive power, harmonics.

I. Introduction

As we all know, the presence of the harmonic elements in the system would make the power quality to deteriorate. One of the methods to improve the power quality is the use of active filtering circuit forms [i-vi]. The efficiency of these filters is very good and has been applied in practice. There are many factors contributing to an active power filter work effectively as: selection the parameters correctly, the method of determining the harmonic current, methods of control, styles of model... In which, the harmonic current determinate method is so important, it should contribute to the correctness of the filter, if the harmonic current determinate method doesn't exactly means the compensate signal into the grid will not properly, leading to compensate a false signal into the grid, power quality of the system more badly than at uncompensated.

Currently, there are many methods of determining the harmonic component as the Fourier expansion method [vii] instantaneous power theory, p-q, i_p-i_q [viii-x] and the application methods of fuzzy-neural to further enhance the accuracy of determining the harmonic component [xi-xii]. Of the above methods, the method of instantaneous power theory is more widely used because of its simplicity and ease of use. Whereas the methods using neural and fuzzy, though for more accurate results, but too complex and difficult to use in practice. However, do not have a paper would analyze the effectiveness, the scope of application of the method p-q and i_p-i_q . Therefore, this paper lays out a mathematical model analyzing of the two methods on which to compare their effectiveness in different cases. From the results of this comparison and the analysis will brings out the scope of its application, when to use the method of p-q when i_p-i_q method is should be used. This research has practical significance, contributing to further enhance work efficiency of active filter circuits.

II. Comparison on the two harmonic current determinate methods p-q and i_p-i_q

A. Three-phase instantaneous voltage is symmetric, no distortion + p-q method

The p-q method is proposed by Akagi et al [viii]. The principle scheme of this method is shown as in Fig 1.

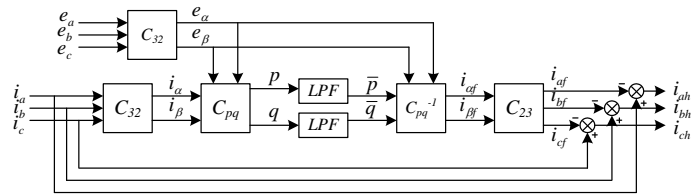


Fig1. p-q method

Suppose, three-phase instantaneous current of load can be presented as following

$$\begin{cases} i_a = \sum_{n=1}^{\infty} \sqrt{2} I_n \sin(n\omega t + \varphi_n) \\ i_b = \sum_{n=1}^{\infty} \sqrt{2} I_n \sin\left[n\left(\omega t - \frac{2\pi}{3}\right) + \varphi_n\right] \\ i_c = \sum_{n=1}^{\infty} \sqrt{2} I_n \sin\left[n\left(\omega t + \frac{2\pi}{3}\right) + \varphi_n\right] \end{cases} \quad (1)$$

and three-phase instantaneous voltage is symmetric, no distortion

$$\begin{cases} e_a = \sqrt{2} E_1 \sin(\omega t) \\ e_b = \sqrt{2} E_1 \sin\left(\omega t - \frac{2\pi}{3}\right) \\ e_c = \sqrt{2} E_1 \sin\left(\omega t + \frac{2\pi}{3}\right) \end{cases} \quad (2)$$

where ω is source angular frequency

I_n, φ_n are RMS and initial phase angle

According to transformation from a-b-c coordinate system to $\alpha-\beta$ coordinate, we have:

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = C_{32} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \sqrt{3} E_1 \begin{bmatrix} \sin \omega t \\ -\cos \omega t \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C_{32} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{3} \begin{bmatrix} \sum_{n=1}^{\infty} I_n \sin(n\omega t + \varphi_n) \\ \sum_{n=1}^{\infty} \mp I_n \cos(n\omega t + \varphi_n) \end{bmatrix} \quad (4)$$

$$\text{Where } C_{32} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Three-phase instantaneous active power p and instantaneous reactive power q can be calculated following as

$$\begin{aligned} \begin{bmatrix} p \\ q \end{bmatrix} &= \sqrt{3}E_1 \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \\ &= 3E_1 \begin{bmatrix} \sum_{n=1}^{\infty} I_n \cos[(1 \mp n)\omega t \mp \varphi_n] \\ \sum_{n=1}^{\infty} \pm I_n \sin[(1-n)\omega t - \varphi_n] \end{bmatrix} \end{aligned} \quad (5)$$

Use low-pass filter to filtering take the DC components \bar{i}_p and \bar{i}_q

$$\begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix} = 3 \begin{bmatrix} E_1 I_1 \cos(-\varphi_1) \\ E_1 I_1 \sin(-\varphi_1) \end{bmatrix} \quad (6)$$

Where $e^2 = 3E_1^2$

From Fig.1, we calculated out:

$$\begin{aligned} \begin{bmatrix} i_{af} \\ i_{bf} \\ i_{cf} \end{bmatrix} &= \frac{1}{3E_1^2} C_{23} C_{pq}^{-1} \begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix} = C_{23} \begin{bmatrix} \sqrt{3}I_1 \sin(\omega t + \phi_1) \\ -\sqrt{3}I_1 \cos(\omega t + \phi_1) \end{bmatrix} = \\ &= \begin{bmatrix} \sqrt{2}I_1 \sin(\omega t + \phi_1) \\ \sqrt{2}I_1 \sin(\omega t - \frac{2\pi}{3} + \phi_1) \\ \sqrt{2}I_1 \sin(\omega t + \frac{2\pi}{3} + \phi_1) \end{bmatrix} \end{aligned} \quad (7)$$

$$\text{Where: } C_{23} = C_{32}^T; C_{pq} = \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix}$$

From here, the harmonic current components determined as

$$\begin{cases} i_{ah} = i_a - i_{af} \\ i_{bh} = i_b - i_{bf} \\ i_{ch} = i_c - i_{cf} \end{cases} \quad (8)$$

+ i_p - i_q method

The i_p - i_q method scheme [viii-x] is shown as in Fig 2.

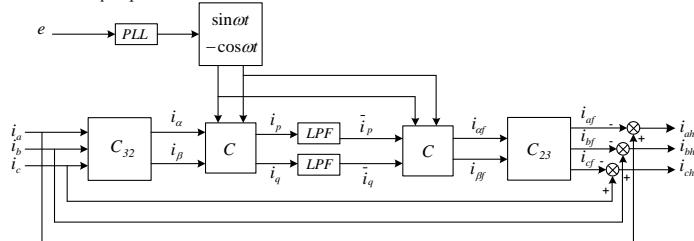


Figure 2. i_p - i_q method

The use of PLL is to generate the reference sinewave ($\sin(\omega t)$ and $-\cos(\omega t)$) with unity amplitude and synchronized with the source voltage.

Similarly, from Fig 2 we can calculate out:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C_{pq} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (9)$$

$$\begin{aligned} \begin{bmatrix} i_p \\ i_q \end{bmatrix} &= \begin{bmatrix} \frac{e_\alpha}{e} & \frac{e_\beta}{e} \\ \frac{e_\beta}{e} & -\frac{e_\alpha}{e} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \\ &= \sqrt{3} \begin{bmatrix} \sum_{n=1}^{\infty} I_n \cos[(1 \mp n)\omega t \mp \varphi_n] \\ \sum_{n=1}^{\infty} \pm I_n \sin[(1-n)\omega t - \varphi_n] \end{bmatrix} \end{aligned} \quad (10)$$

$$\begin{bmatrix} \bar{i}_p \\ \bar{i}_q \end{bmatrix} = \sqrt{3} \begin{bmatrix} I_1 \cos(-\varphi_1) \\ I_1 \sin(-\varphi_1) \end{bmatrix} \quad (11)$$

The fundamental harmonics current components of the i_a, i_b, i_c following formula as

$$\begin{bmatrix} i_{af} \\ i_{bf} \\ i_{cf} \end{bmatrix} = C_{23} C \begin{bmatrix} \bar{i}_p \\ \bar{i}_q \end{bmatrix} = \begin{bmatrix} \sqrt{2}I_1 \sin(\omega t + \varphi_1) \\ \sqrt{2}I_1 \sin(\omega t - \frac{2\pi}{3} + \varphi_1) \\ \sqrt{2}I_1 \sin(\omega t + \frac{2\pi}{3} + \varphi_1) \end{bmatrix} \quad (12)$$

$$\text{Where } C = \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix}$$

From here, the harmonic current components can be determined as

$$\begin{cases} i_{ha} = i_a - i_{af} \\ i_{hb} = i_b - i_{bf} \\ i_{hc} = i_c - i_{cf} \end{cases} \quad (13)$$

From (7) and (12) we can see that in case supply voltage is symmetric and not distortion, then the p-q method and i_p - i_q as the same.

B. Three-phase instantaneous voltage is symmetric and distortion

Suppose three-phase instantaneous voltage is symmetric and distortion

$$\begin{cases} e_a = \sum_{n=1}^{\infty} \sqrt{2}E_n \sin(n\omega t + \theta_n) \\ e_b = \sum_{n=1}^{\infty} \sqrt{2}E_n \sin \left[n \left(\omega t - \frac{2\pi}{3} \right) + \theta_n \right] \\ e_c = \sum_{n=1}^{\infty} \sqrt{2}E_n \sin \left[n \left(\omega t + \frac{2\pi}{3} \right) + \theta_n \right] \end{cases} \quad (14)$$

+ p - q method:

Transformation from a-b-c coordinate system to α - β coordinate, we have:

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = C_{32} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \sqrt{3} \begin{bmatrix} \sum_{n=1}^{\infty} E_n \sin(n\omega t + \theta_n) \\ \sum_{n=1}^{\infty} \mp E_n \cos(n\omega t + \theta_n) \end{bmatrix} \quad (15)$$

The instantaneous power components can be determined as

$$\begin{bmatrix} p \\ q \end{bmatrix} = 3 \begin{bmatrix} \sum_{n=1}^{\infty} E_n I_n \cos(\theta_n - \varphi_n) + \sum_{n=1}^{\infty} \sum_{m(m \neq n)=1}^{\infty} E_n I_m \cos[(n \mp m)\omega t + (\theta_n \mp \varphi_n)] \\ \sum_{n=1}^{\infty} \pm E_n I_n \sin(\theta_n - \varphi_n) + \sum_{n=1}^{\infty} \sum_{m(m \neq n)=1}^{\infty} \mp E_n I_m \sin[(n - m)\omega t + (\theta_n - \varphi_n)] \end{bmatrix} \quad (16)$$

The DC components of p, q is

$$\begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix} = 3 \begin{bmatrix} \sum_{n=1}^{\infty} E_n I_n \cos(\theta_n - \varphi_n) \\ \sum_{n=1}^{\infty} \pm E_n I_n \sin(\theta_n - \varphi_n) \end{bmatrix} \quad (17)$$

With $e^2 = 3 \sum_{n=1}^{\infty} E_n^2$

From here we can calculate the fundamental harmonics current components of the i_α, i_β, i_c following formula as

$$\begin{bmatrix} i_{\alpha f} \\ i_{\beta f} \\ i_{c f} \end{bmatrix} = \frac{1}{3 \sum_{n=1}^{\infty} E_n^2} C_{23} \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix} \begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix} \quad (18)$$

+ *ip-iq method:*

Because the i_p-i_q theory uses a Phase-locked loop (PLL). PLL is responsible for generating the reference sine-wave ($\sin\omega t$ and $-\cos\omega t$) with unity amplitude and synchronized with the source voltage. So, the fundamental harmonics current components of the i_α, i_β, i_c are calculated as in formula (12).

Comparison between (18) and (7) we can see that between these two equations have an error is

$$\begin{bmatrix} \Delta i_{\alpha f} \\ \Delta i_{\beta f} \\ \Delta i_{c f} \end{bmatrix} = C_{23} \begin{bmatrix} e_{\alpha h} \frac{\bar{p}}{e^2} + e_{\beta h} \frac{\bar{q}}{e^2} \\ e_{\beta h} \frac{\bar{p}}{e^2} - e_{\alpha h} \frac{\bar{q}}{e^2} \end{bmatrix} + C_{23} \begin{bmatrix} e_{\alpha f} \left(\frac{\bar{p}}{e^2} - \frac{I_1 \cos\varphi_1}{E_1} \right) + e_{\beta f} \left(\frac{\bar{q}}{e^2} - \frac{I_1 \sin(-\varphi_1)}{E_1} \right) \\ e_{\beta f} \left(\frac{\bar{p}}{e^2} - \frac{I_1 \cos\varphi_1}{E_1} \right) + e_{\alpha f} \left(\frac{\bar{q}}{e^2} - \frac{I_1 \sin(-\varphi_1)}{E_1} \right) \end{bmatrix} \quad (19)$$

Where: Δ shows the error value.

From (18) and (19) we can demonstrate that how have error as follows

* e_α, e_β in (19) have harmonic components, so the fundamental components $i_{\alpha f}, i_{\beta f}, i_{c f}$ also have harmonic

* \bar{p}, \bar{q} in (18) including the fundamental voltage and current, while in (19) consists of harmonic voltage and current components.

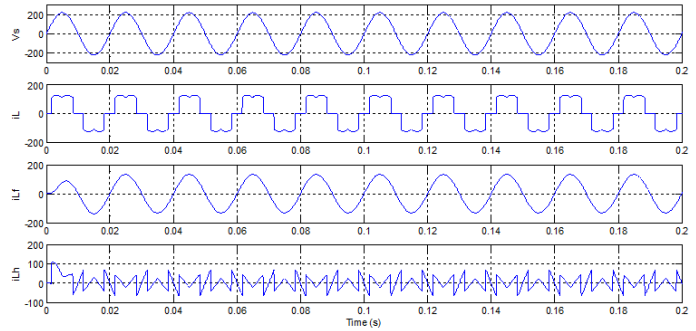
* $e^2 = 3 \sum_{n=1}^{\infty} E_n^2$ in (19) is larger than $e^2 = 3E_1^2$ in (6).

Therefore, according to the previous analysis, then we can see that: in case the source voltage is symmetric and no distortion, then we can use the $p-q$ or i_p-i_q theory method. In case the source voltage is symmetric and distortion, then the $p-q$ theory, method gives result have an error, while the i_p-i_q theory method give result very well, because the i_p-i_q theory method uses a Phase-locked loop (PLL)..

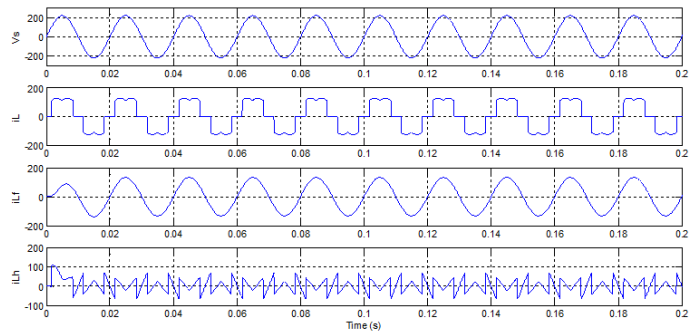
III. Simulation Results

To demonstrate the advantage and disadvantage of the above harmonic current detection methods. Simulation results have been carried out with MATLAB software for a system has voltage 380V-50Hz connected to a nonlinear load. The nonlinear load is an uncontrolled three-phase rectifier with a load $R=3\Omega$.

where: $V_s(V)$ is the voltage of supply source, $I_L(A)$ is load current, $I_{L1}(A)$ is load current at fundamental frequency and $I_{Lh}(A)$ is harmonic current of phase A.

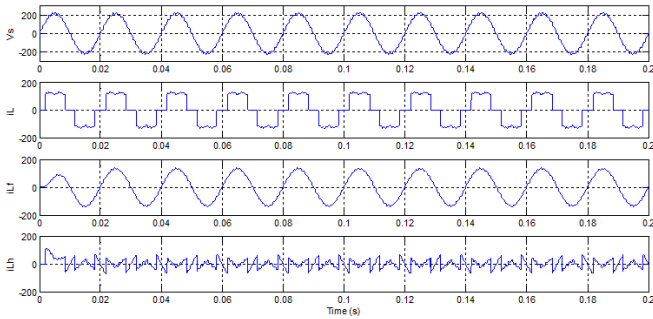


a) Harmonic current detection with the $p-q$ method

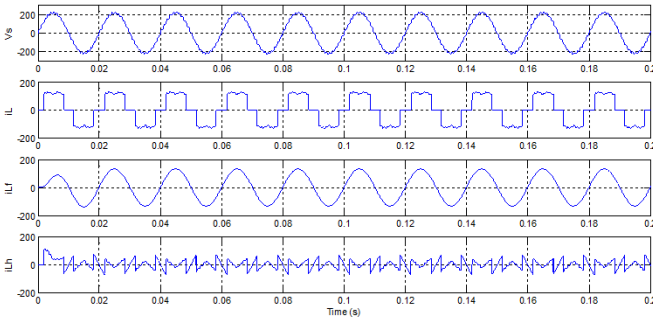


b) Harmonic current detection with the i_p-i_q method

Figure 3. Simulation results of harmonic detection methods in case the voltage is symmetric and not distortion.



(a) Harmonic current detection with the p-q method



(b) Harmonic current detection with the i_p-i_q method

Figure 4. Simulation results of harmonic detection methods in case the voltage is symmetric and distortion.

Performance comparison between the methods in cases the voltage is not distortion and distortion is shown as in table 1 and table 2. The amplitude values are represented in % of fundamental.

Table 1. Performance comparison between the methods in case the voltage is symmetric and not distortion

Harmonic order	i_L (%)	i_{Lf} (%)		i_{Lh} (%)	
		p,q method	i_p-i_q method	p,q method	i_p-i_q method
1	100	100	100	0.00	0.00
5	22.65	0.63	0.63	22.02	22.02
7	11.3	0.31	0.31	10.99	10.99
11	9.07	0.06	0.06	9.01	9.01
13	6.45	0.04	0.04	6.41	6.41
17	5.67	0.02	0.02	5.65	5.65
19	4.51	0.01	0.01	4.50	4.50
23	3.29	0.00	0.00	3.29	3.29

Table 2 Performance comparison between the methods in case the voltage is symmetric and distortion

Harmonic order	i_{La} (%)	i_{Laf} (%)		i_{Lah} (%)	
		p,q method	i_p-i_q method	p,q method	i_p-i_q method
1	100	99.9	100	0.01	0.00
5	23.15	0.34	0.12	22.81	23.03
7	10.59	0.22	0.11	10.37	10.48
11	9.26	0.12	0.01	9.14	9.15
13	5.53	0.02	0.00	5.51	5.53
17	5.61	0.01	0.00	5.60	5.61
19	5.98	0.00	0.00	5.98	5.98
23	1.01	0.00	0.00	1.01	1.01

From Fig 3, Fig 4, table 1 and table 2 we can see that: In the case of the voltage source is symmetrical and not distorted form, we may use the method of $p-q$ or i_p-i_q . In the case of voltage source is symmetrical and distorted forms, the methods of $p-q$ produces the error, while the i_p-i_q method that gives very good results. This comparison will be basis to decide to choose harmonic current determine the method in researches on the active power filter.

IV. Conclusion

Analysis of the mathematical model of the $p-q$ and i_p-i_q methods shown that in cases the source is symmetrical and not distorted, the $p-q$ and i_p-i_q methods give results as the same. However, in the case the source is distorted, the i_p-i_q method gives results more accurate the $p-q$ method.

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