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Passive Control of a Vibrating Bar with a Dynamic Vibration Absorber

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***Abstract* — The passive control of a vibrating bar is analyzed in this paper. It has been controlled passively by a Dynamic Vibration Absorber (DVA) which consists of a mass and a spring. A bar, which is fixed at one end and free at the other end is subjected to a harmonically excited force being positioned at the free end. By analyzing the motion of a bar, the equation of motion for a bar was derived by using a method of variation of parameters. To define the optimal conditions of a DVA, the cost function, which denotes the reduction of a force being transmitted to the fixed end of a bar was evaluated and discussed. The possibility of reduction of the force transmissibility was found to depend on the location and the spring stiffness of a DVA.**

***Keywords*— Passive control, a method of variation of parameters, the force transmissibility, a Dynamic Vibration Absorber (DVA), spring stiffness**

I. INTRODUCTION

In the industrial fields, vibrations of machines and structures had been a long standing problem to be solved. So far a lot of studies have been processed to have good results from the vibration control skills in machining operations in the past decades. Currently as of the optimal design condition, the fast manufacturing time and the accurate level of the machining center and mother machine are known to be the important factors. In the high speed operation, the vibration which may occurs at each parts of machine should be controlled actively or passively for the stability of the machine structure.

For the simplification of the complex structures, each part of the machines is considered as a continuous system such as bar, beam, plate or shell. Hence, the vibration problems of a continuous system had been studied by lots of researchers. Recently the vibration energy flow and the dynamic response of a beam, plate, shell or some compound system have been analyzed. By using a model of an elastic beam, the vibration energy and control technologies had been presented [1-3]. Furthermore, the complex frame which is the assembly of several beams had been used to analyze and control the vibration characteristics [4-5]. The Active control method had been proven to become the convenient control method for the known forcing frequency zone [6-8]. The passive control of the vibrations of the machining center and machine tools had been introduced [9-12].

A working arm or part in machinery is known to experience the longitudinal vibration being induced by the constant operation speed. The Dynamic Vibration Absorber (DVA) is known to be effective over narrow band of frequencies and is designed to make the natural frequencies of the main system be away from the forcing frequency. Hence in this study a theoretical system which consists of a Dynamic Vibration Absorber and a bar with an external force at the free end is employed to evaluate and reduce the force transmissibility of a bar. The edges condition of a bar is fixed at one end and free at the other end. Based on the wave equation, the vibration of a bar will be discussed. To define the optimal conditions for the passive control of the bar, the cost function which is the ratio of the force transmissibility of a bar without DVA to that of a bar with DVA will be discussed. Fig. 1 shows the theoretical model of a uniform bar which is subjected to a harmonically excited force F(t) at the free end and has a Dynamic Vibration Absorber (DVA) which is located at a distance of x_a from the left end of the bar.

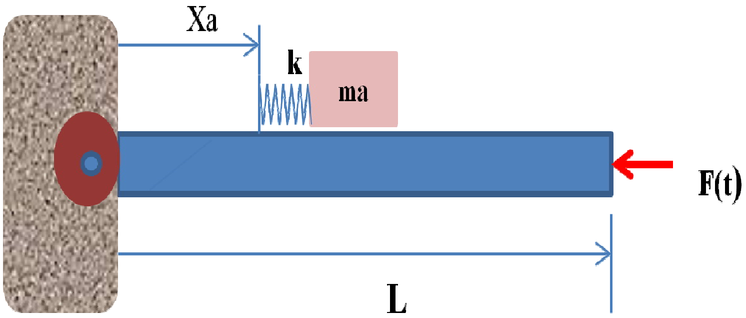


Fig. 1: Theoretical model of a bar with fixed and free ends

F(t) (the external force), Dynamic Vibration Absorber (DVA)[mass (m_a), spring stiffness (k), location (X_a)], BAR PROPERTIES [LENGTH (L), DENSITY (ρ), CONSTANT SECTION AREA (A), YOUNG'S MODULUS (E)]

II. THE GOVERNING EQUATIONS

A uniform bar of a cross section area A, mass density ρ and length L, which is fixed at the left end and free at the right end is subjected to a harmonically excited force F(t) at the free end and has a Dynamic Vibration Absorber (DVA) which is located at a distance of x_a from the left end of the bar. The governing equations of a vibrating bar and a Dynamic Vibration Absorber (DVA) can be written as:

$$EA\frac{\partial^2 u(x,t)}{\partial x^2} = \rho A\frac{\partial^2 u(x,t)}{\partial t^2} + k\Big(y(t) - u(x,t)\delta\big(x - x_a\big)\Big) \quad (1)$$

$$m_a\frac{\partial^2 y(t)}{\partial t^2} + ky(t) = ku(x,t)\delta\big(x - x_a\big) \quad (2)$$

Where *u(x,t) and y(t)* represent the displacement of a bar and a DVA, respectively. In the above equations, the displacements of a bar and a DVA are given as the time harmonic function. Hence the displacements (*u(x,t) and y(t)*) are expressed into the terms of the time harmonic motion as,

$$u(x,t) = \mathrm{u}(x)e^{i\omega t} \quad \text{and} \quad y(t) = \mathrm{Y}e^{i\omega t} \quad (3)$$

Where u(x) and Y mean the deflection of a bar and a DVA, respectively and ω is the circular frequency. By inserting Eq. (3) into Eq. (1) and Eq. (2), then suppressing the time term, Eq.(1) and Eq. (2) can be rearranged as,

$$EA\frac{\partial^2 \mathrm{u}(x)}{\partial x^2} + \rho A\omega^2\, \mathrm{u}(x) = kY - k\mathrm{u}(x)\delta(x - x_a) \quad (4)$$

$$Y = \frac{k\mathrm{u}(x_a)\delta\big(x - x_a\big)}{k - m_a\omega^2} \quad (5)$$

Then by using Eq. (5), Eq. (4) can be rewritten as

$$\frac{\partial^2 \mathrm{u}(x)}{\partial x^2} + \alpha^2\, \mathrm{u}(x) = \frac{m_a k\omega^2}{EA\big(k - m_a\omega^2\big)}\mathrm{u}(x_a)\delta(x - x_a) \quad (6)$$

Where the wave number $\alpha^2 = \rho\omega^2/E$. The boundary conditions of a bar with fixed and free ends are given as

$$u\big|_{x=0} = 0 \quad \text{and} \quad EA\frac{\partial \mathrm{u}}{\partial x}\bigg|_{x=L} = -f_o e^{i\omega t} \quad (7)$$

Where f_o is the magnitude of the external force. The total solution of Eq. (6) can be expressed into the sum of a homogeneous solution (u_h) and a particular solution (u_p) as

$$\mathrm{u}(x) = \mathrm{u}_h(x) + \mathrm{u}_p(x) \quad (8)$$

The homogeneous solution is determined by letting the right side of Eq. (6) be zero and then becomes as

$$\mathrm{u}_h(x) = A_1 \sin\alpha x + A_2 \cos\alpha x \quad (9)$$

Here the particular solution can be solved by means of a method of variation of parameters and then be assumed as

$$u_p(x) = V_1(x)\sin \alpha x + V_2(x)\cos \alpha x \tag{10}$$

Where the coefficients V_1 and V_2 are can be determined by means of the method of variation of parameters and defined as follows

$$V_1(x) = \frac{1}{k}\int^x \cos(\alpha\xi) f(\xi) d\xi \quad \text{and} \quad V_2(x) = -\frac{1}{k}\int^x \sin(\alpha\xi) f(\xi) d\xi \tag{11}$$

where the forcing function, $f(\xi) = \dfrac{m_a k\omega^2}{EA(k - m_a\omega^2)} u_p(x_a)\delta(\xi - x_a)$.

The final form of the particular solution can be obtained by using Eqs. (10) and (11). The complete solution of Eq. (8) can be written as

$$u(x) = A_1\left\{\sin \alpha x - \frac{m_a k\omega^2}{\alpha EA(k - m_a\omega^2)}\sin(\alpha x_a)\sin(\alpha(x - x_a))H(x - x_a)\right\} + A_2\left\{\cos \alpha x - \frac{m_a k\omega^2}{\alpha EA(k - m_a\omega^2)}\cos(\alpha x_a)\sin(\alpha(x - x_a))H(x - x_a)\right\} \tag{12}$$

Where H(x) represents the Heaviside unit step. By using the boundary conditions given in Eq. (7), the final solution of the dynamic response of a bar becomes as,

$$u(x) = A_1\left\{\sin \alpha x - \frac{m_a k\omega^2}{\alpha EA(k - m_a\omega^2)}\sin(\alpha x_a)\sin(\alpha(x - x_a))H(x - x_a)\right\} \tag{13}$$

Where A_2 becomes zero and the constant function A_1 is as follows,

$$A_1 = \frac{-f_o}{EA\left[\alpha\cos\alpha L - \dfrac{m_a k\omega^2}{EA(k - m_a\omega^2)}\sin(\alpha x_a)\cos(\alpha(L - x_a))\right]} \tag{14}$$

Here Eq. (13) introduces the deflection of a bar with a DVA, which is caused by the harmonically excited force applied to the free end.

III. NUMERICAL RESULTS AND DISCUSSION

All values obtained in this study have been expressed into the nondimensional forms. In Table 1, the nondimensional properties and the given values are introduced as

TABLE I - NONDIMENSIONAL PROPERTIES AND THE GIVEN VALUES

	EXPRESSION FORM	VALUE
A MASS RATIO	$m = m_A/\rho AL$	0 ~ 0.3
EXTERNAL FORCE RATIO	$F_O = F_O/EA$	1
A STRUCTURAL DAMPING		0.001
A DVA LOCATION	$X_A = x_A/L$	0.1 ~ 0.9
A STIFFNESS RATIO	$K_S = k/(EA/L)$	0.001 ~ 10

A. Resonance Frequency Coefficients(β)

In Eq. (14), the denominator part vanishes for certain specific values of the wave number α. The roots of denominator which are expressed in ω_r by $\alpha^2 = \rho\omega^2/E$ are called the resonance frequencies for the bar with a DVA. So the resonance frequency equation becomes as,

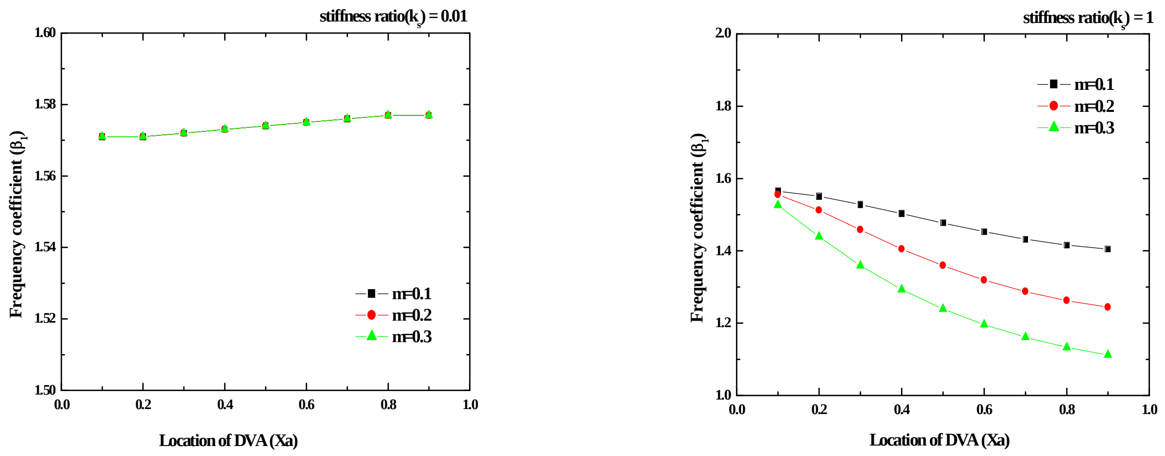
$$F(\alpha) = \cos\alpha L - \frac{m_a k\omega^2}{\alpha EA(k - m_a\omega^2)}\sin\alpha x_o \cos\alpha(L - x_o) = 0 \tag{15}$$

Here Eq. (15) can be rewritten in terms of the nondimensional form as,

$$F(\beta) = \sin\beta - \frac{m\beta}{1 - \dfrac{m\beta^2}{K_s}}\sin\beta X_a \cos\beta(1 - X_a) = 0 \tag{16}$$

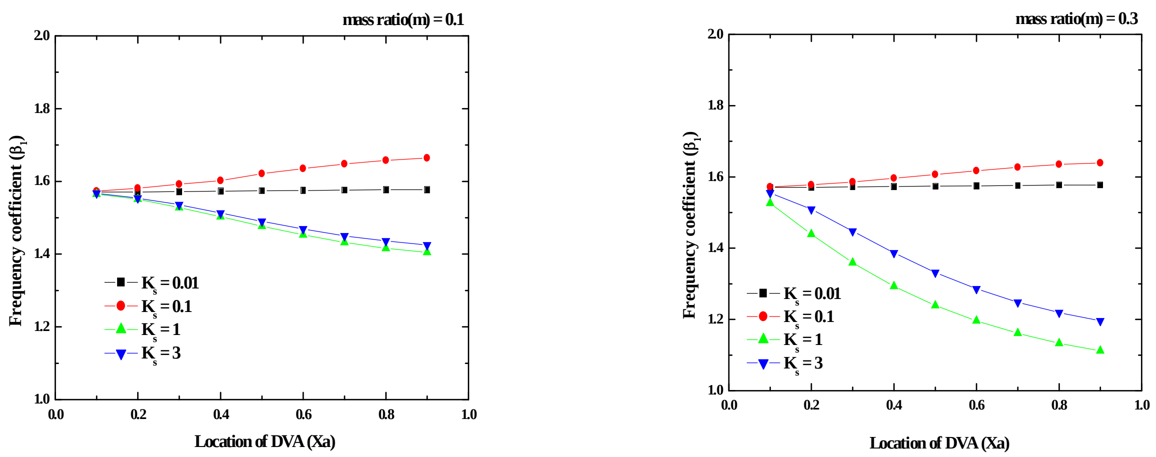
where $\beta = \alpha L$. Equation (16) is a transcendental equation in β and its roots must be obtained numerically. For two stiffness ratios (K_s = 0.01 and 1), the variations of the first resonance frequency coefficient (β_1) versus location of DVA for three cases of mass ratio (m = 0.1, 0.2, 0.3) were plotted in Fig. 2. For the low stiffness (K_s = 0.01) of DVA in Fig. 2 (A), the variation of the first resonance frequency coefficient (β_1) is found to be varied small along the location of a DVA but not sensitive to the mass of a DVA.

However in case of the high stiffness (K_s = 1) of DVA in Fig. 2 (B), the variation of the first resonance frequency coefficient (β_1) is found to be very sensitive to the location and mass of a DVA. It is noted that as the mass of a DVA is getting larger, the values of the first resonance frequency coefficient (β_1) are getting smaller. In Fig. 2(A), as the location of a DVA is close to the free end, the values of the first resonance frequency coefficient (β_1) are increasing gradually. But in Fig. 2(B), as the location of a DVA is close to the free end, the values of the first resonance frequency coefficient (β_1) are decreasing gradually. So it is noted that the DVA with a low stiffness gives the additional stiffness effect to the behaviour of the vibrating bar and the DVA with a high stiffness gives both effects of the additional mass and stiffness to the behaviour of the vibrating bar. In Fig. 3, the above mentioned notations might be proved. For two mass ratios (m = 0.1 and 0.3), the variations of the first resonance frequency coefficient (β_1) versus location of DVA for four cases of stiffness ratio (K_s = 0.01, 0.1, 1, 3) were plotted in Fig. 3. In cases of the low stiffness (K_s = 0.01, 0.1), the values of β_1 is increasing a little bit along the further distance from the fixed end of a bar. But in cases of the high stiffness (K_s = 1, 3), the values of β_1 is decreasing along the further distance from the fixed end of a bar.



(A) (B)

Fig. 2 Resonance frequency coefficient (β_1) vs. location of DVA : (A) K_s = 0.01 and (B) K_s = 1



(A) (B)

Fig. 3 Resonance frequency coefficient (β_1) vs. location of DVA : (A) m = 0.1 and (B) m = 0.3

B. Reduction of The Force Transmissibility (T_f)

The force transmissibility (T_f) of a vibrating bar is expressed as follows;

$$T_f = \frac{F_T}{F_o} \qquad (17)$$

where $F_T = EA \left. \frac{\partial u}{\partial x} \right|_{x=0}$

In Eq. (17), the structural damping (= 0.001) was introduced to avoid being infinite at the resonance. The reduction level of the force transmissibility (T_f) was evaluated in comparison with T_f of a bar without DVA and T_f of a bar with DVA. The ratio of T_f of a bar without DVA to T_f of a bar with DVA is expressed in terms of decibel [dB] as follows;

$$\text{Reduction of } T_f \text{ [dB]} = 10\,Log_{10}\left[\frac{\left|\mathrm{Re}[T_f]_{noDVA}\right|}{\left|\mathrm{Re}[T_f]_{DVA}\right|}\right] \qquad (18)$$

where 'Re' means the real part taking from the complex number of T_f. For the two arbitrary values of the stiffness (K_s = 0.1 and 2) ratio, Figs. 4 (A) and (B) shows the reductions of the force transmissibility (T_f) of two locations (X_a=0.8 and 0.2) of DVA versus the forcing frequency coefficient (β_f), respectively. As shown in Fig. 4, the levels of the reduction of T_f were observed to be good or bad irregularly along the forcing frequency coefficient (β_f). It means that a DVA with the arbitrary value of stiffness may not apply to a main system which is subjected to any forcing frequency.

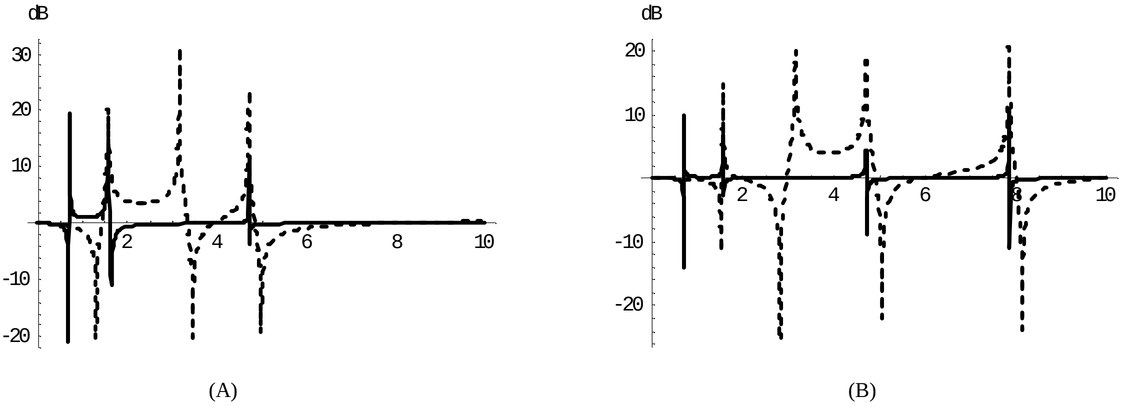


Fig. 4 Reduction of T_f vs. forcing frequency coefficient (β_f); m=0.2, Solid(K_s=0.1), Dot(K_s=2) (A) X_a=0.8 and (B) X_a=0.2

According to the previous study [13], the optimal design condition of a DVA was reported such that the natural frequency of DVA should be set equal to the forcing frequency of a main system. So the stiffness ratio can be given as;

$$K_s\big|_{optimal} \approx m \times \beta_f^2 \qquad (19)$$

Where β_f is the forcing frequency coefficient. For the given forcing frequency coefficient, the stiffness ratio of a DVA can be determined by using Eq. (19).

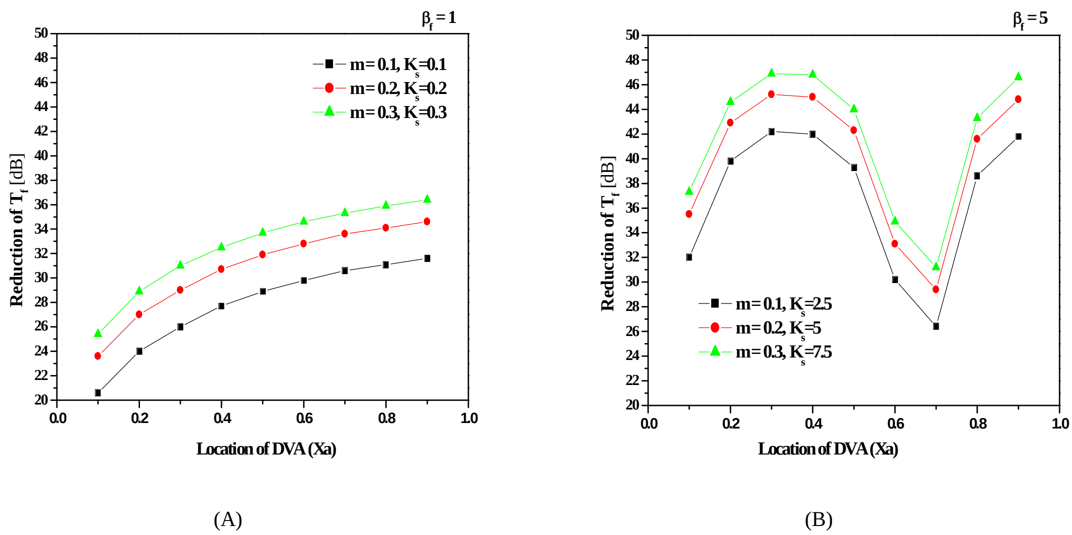
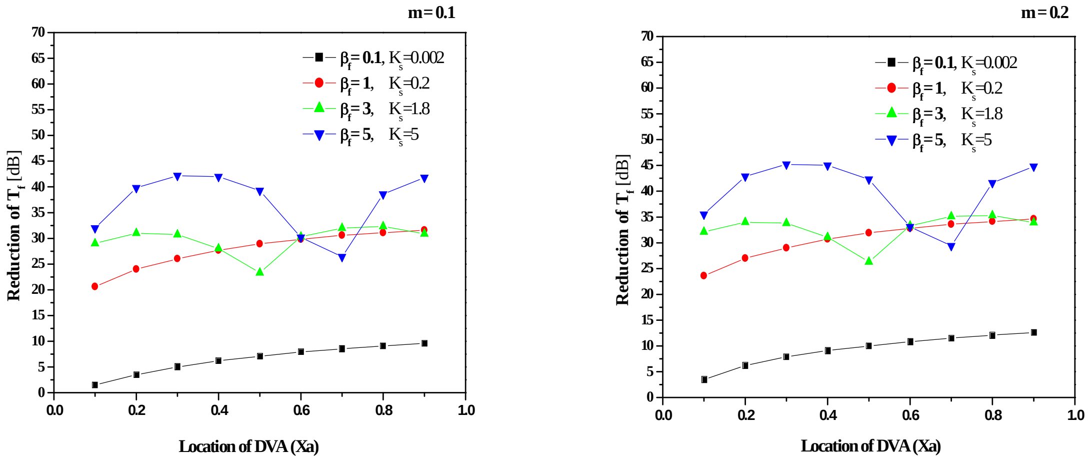


Fig. 5 Reduction of T_f vs. Location of DVA (X_a); (A) β_f=1 and (B) β_f=5

In Figs. 5 (A) and (B) for β_f =1 and 5, the reduction of T_f were evaluated along the location of DVA for three cases of mass ratio (m=0.1, 0.2, 0.3). As is the greater mass ratio, the higher reduction is observed. As observed in Fig. 5, the reduction of T_f are obtained over 20 dB at all locations of DVA based on the force transmissibility of a main system without DVA and the effect of DVA's location exhibits the spatial form of the normal modes corresponding to each natural frequency of the bar (β_1=1.57 and β_2 = 4.71).

In Figs. 6 (A) and (B), the reduction of T_f for four cases of the stiffness ratio are plotted versus location of DVA for two mass ratios (m = 0.1 and 0.2), respectively. As shown in Fig. 6, the reduction loci of four stiffness ratios are very similar in two mass ratios which look like the spatial form of the normal modes of the bar. As stated previously, the reduction levels are greater than 20 dB for the most cases except the low values of the stiffness ratio (K_s = 0.002) which is corresponding to the low forcing frequency coefficient (β_f = 0.1).



(A) (B)

Fig. 6 Reduction of T_f vs. Location of DVA (X_a); (A) m = 0.1 and (B) m = 0.2

IV. CONCLUSIONS

On the bases of the analyses in this study, the conclusions are obtained as follows,

- *Passive control of Dynamic Vibration Absorber (DVA) is observed to have the satisfactory results for the reduction of the force transmissibility which is the ratio of the force transmitted to a fixed end to the external force of the bar.*
- *The location and the stiffness of DVA are known to become the important factor for the control strategy.*
- *At the low stiffness of DVA, the effect of the DVA's mass is very weak.*
- *It is proven that the optimal stiffness of DVA is proportional to square of the forcing frequency and the mass of DVA.*

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