Buckling Analysis Of Underwater Cylindrical Shells Subjected To External Pressure.

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Abstract— In the field of structural mechanics the word shell refers to a spatial, curved structural member. In shells, the external loads are carried by both membrane and bending response. The effect of such responses needs careful observation. This project fulfills the need on study of behavior of shell structures by both mathematical and numerical methods. In the present development, the Donnell's relation forms the basis of stability equation for circular cylindrical shells. The effect of stiffeners on cylindrical structures has been studied, by varying the geometry and orientation of stiffeners. The paper aims to analyze the cylindrical section of an underwater pressure hull, using finite element analysis and strengthen it accordingly. The project considers the finite element method for static structural and deflection analysis of underwater cylindrical structure by using ANSYS 15software.

Keywords— critical buckling load or load multiplier, Finite Element Analysis, Pressure Hull, Donell's relation, stiffened cylinders.

I. INTRODUCTION

Many theoretical studies have been done in cylindrical structures by considering the cylinder as thin, and the stress variation in the direction of thickness as negligible. This research project intends to combine the classic shell theories with the contemporary numerical approach and this thesis primarily focused into, large thickness variation by considering cylinder as thin, moderately thick as well as thick by varying, l/d and r/t ratio. This thesis evaluates the critical buckling load of cylindrical shells subjected to external pressure.

The Donnell's relation forms the basis for stability analyses in the literature than any other set of cylindrical shell equation. The finite element modelling of the structure can be done with Ansys Workbench 15. The check for accuracy can be made by comparing the results of both numerical and mathematical method.

The load carrying capacity of cylindrical structures can be improved by adding stiffeners, the analysis in variation of stiffener geometry and orientation can be done by considering the cylinder as externally stiffened, internally stiffened and spirally stiffened. The design of a pressure hull is made to withstand an external pressure of 65 bar (according to ASME standard) is analysed and an improved design of pressure hull is suggested to withstand such load with minimum deformation is obtained.

II. DONELL'S FORM OF THE LINEAR EQUILIBRIUM EQUATION

According to Donell's a cylindrical shell which is supported at its ends and subjected to uniform external pressure **Pc** in Newton per square metre[1]. Under such loading the buckling deformation of the shell is axisymmetric. In the interest of simplicity the axisymmetric bending is assumed to be cylindrical over its entire length the critical pressure **P$_{cr}$** is defined as the lowest pressure at which the body loses its stability and tends to buckle

$$\frac{P_e a}{Eh} = \frac{\left[\left(\pi a/L\right)^2 + n^2\right]^2}{n^2} \frac{\left(h/a\right)^2}{12\left(1-v^2\right)} + \frac{\left(\pi a/L\right)^4}{n^2\left[\left(\pi a/L\right)^2 + n^2\right]^2}$$

Pe =Pcr=critical buckling load in N/m^2,
A=radius of cylinder in m,
E=young's modulus of cylinder material N/m^2,
L=longitudinal length of cylinder m,
h=thickness in m,
n=wavelength parameter which can be choose by trial and error method.

III. FINITE ELEMENT MODELLING WITH ANSYS

Eigen value buckling analysis predicts the theoretical buckling strength of an ideal elastic structure. However, in real-life, structural imperfections and nonlinearities prevent most real-world structures from reaching their Eigen value predicted buckling strength which means that this method over-predicts the expected buckling loads. Ansys has the ability to formulate solutions for individual elements before putting them together to represent the entire problem. The solution of the problem strictly depends on the element size.

IV. CONVERGENCE OF MATHEMATICAL FORMULATION WITH NUMERICAL RESULTS

The solution from the finite element program is checked with a solution of increased accuracy especially by mathematical formulation. According to Donnell's solution, when a cylinder which is simply supported at its end and subjected to external lateral pressure, the critical buckling load can be obtained by the above equation, the cylinder of length 1.20 m, radius 0.60m, and thickness 0.03m which is made of aluminium alloy whose E=71GPa, shows the critical load as Pe=2.12*10^7 Pa, when n=4.

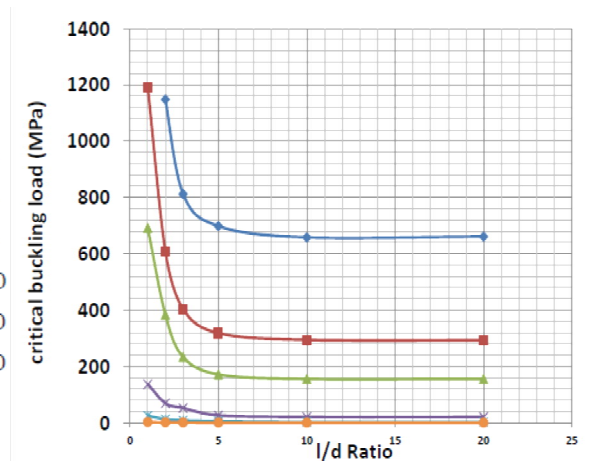The results according to numerical method using ansys workbench 15
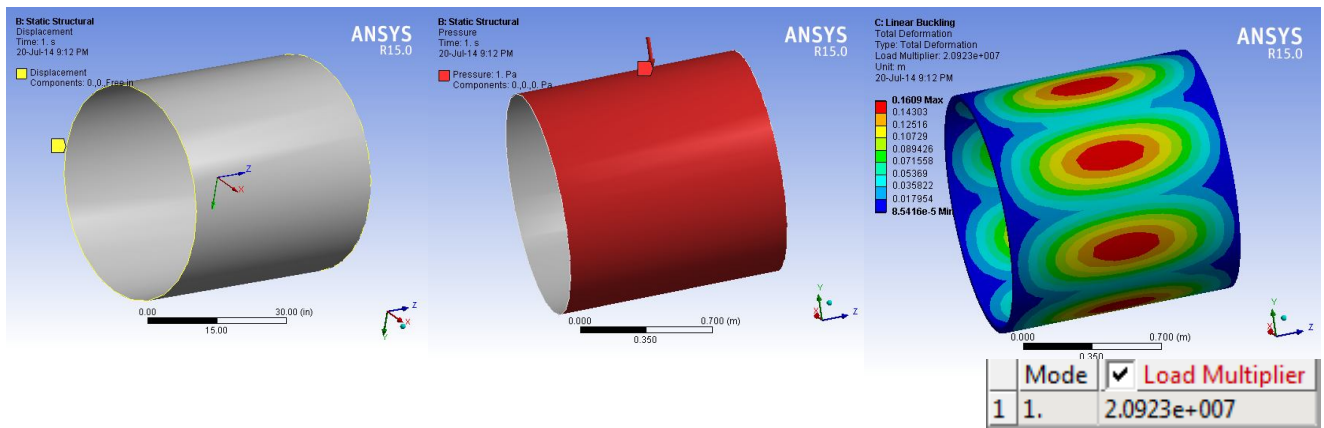
Size	Load	Supports	Buckling load (mathematical)	Buckling load (numerical)	Error
L=1.20 m D=.60 m T=3*10⁻²m	Pressure of 1MPa (normal)	Simply supported at its ends	$2.12*10^7$ Pa	$2.0923*10^7$ Pa	1.30%

The linear buckling analysis has been done by varying the l/d ratio as 1,2,3,5,10,20 and r/t ratio as 3,4,5,10,20,50 to obtain the critical buckling load with simply supported boundary condition and fixed boundary condition whose results is plotted graphically.
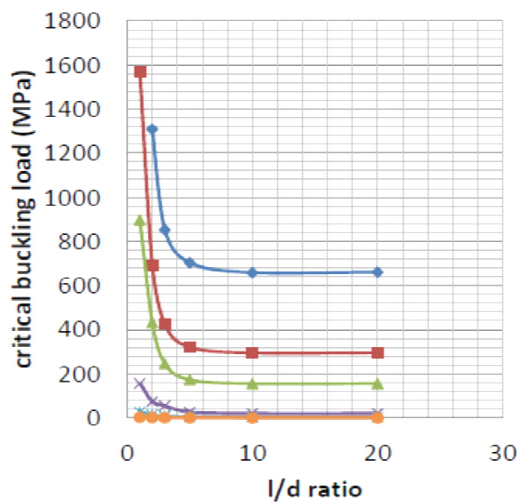
Graph 1 cylinder with fixed end condition   Graph 2. simply supported end condition

From the graph, it can be concluded that,after certain limit the l/d doesnot shows much vaiation under external pressure. And the error value (appendix 1) between the mathematical and numerical results, for thick shells and moderately thick shells are larger (less than 10%,hence its is accepted),but for thin shells, the error value is smaller, so the donnell's relation is well suited in the analysis of thin shells.

V.JUSTIFICATION FOR THE ERROR

In the general shell theory, it was assumed that flexural stress and membrane stress plays the same order of importance. If one of the above, mentioned types of stresses are negligible in comparison with the other one, then it is possible to introduce considerable simplifications in the general shell theory.

If the flexural stress are negligible compared with the membrane stress, then such type of state of stress is called a membrane state of stress. The governing equation of the membrane theory can be obtained directly from the general shell theory by neglecting the effect of bending; twisting and transverse shear effects and these types of stress state exist in thin shells.

If to the contrary, the membrane stresses are negligible in comparison with the flexural stresses, then such a type of stress is termed as pure flexural or moment state of stress.

Shells due to their small thickness are poorly adapted to restricting in bending. Therefore a pure flexural type of sate of stress is dangerous and technically disadvantaged for thin shells. Conversely the membrane state of stress, at which the shell is uniformly stressed cross its thickness, is most advantageous. So engineer design shin shells with membrane state of stress Donell's relation is developed for such thin shells and the application of Donell's relation in thick shells may shows some error but it is in acceptable limit.

VI.EFFECT OF STIFFENERS IN CYLINDRICAL STRUCTURES

Stiffeners are generally used to improve the load carrying capacity of cylindrical shells. For increasing the load carrying capacity, modifications in stiffeners are considered. Due to the easiness in application the variations considers in this paper are

Variation in stiffener orientation (external, internal and spiral stiffener),

Variation in stiffener geometry (height variation is considered).

Sl.No	Stiffener configuration	CRITICAL BUCKLING LOAD IN MPa		
		Internal stiffener	External stiffener	Spiral stiffener
1	T=.3cm H=0.3cm	3.3121`	3.1607	3.1652
2	T=.3cm H=0.6cm	6.8117	5.6466	5.5787
3	T=.3cm H=0.9cm	15.919	10.842	10.679
4	T=.3cm H=1.2cm	32.743	18.596	18.179
5	T=.3cm H=1.5cm	59.568	28.273	27.512
6	T=.3cm H=1.8cm	74.32	39.145	38.043
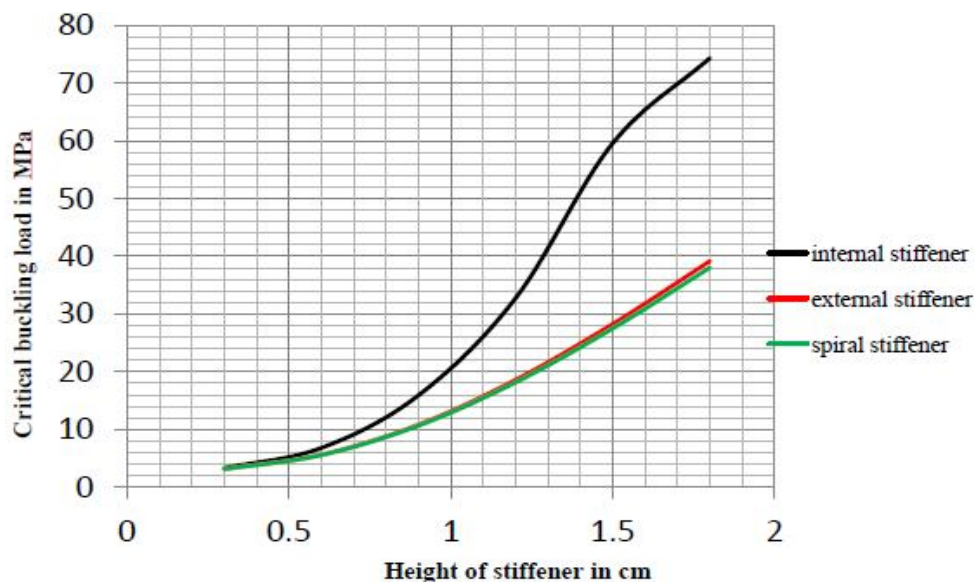
When the stiffeners spaced closely the construction will be difficult due to the increased height. Being an optimum parameter the stiffener spacing is made to be 5 cm for ring stiffeners and for spiral stiffeners the pitch is made as 5cm. The comparisons between calculations shows that for a thin cylindrical structure subjected to external pressure, internal stiffener withstand maximum load without buckling

VII.DESIGN OF A PRESSURE HULL FOR STATIC LOADING

Pressure hulls are the main load bearing structure of underwater submarines. Ring stiffened cylindrical shells are used in many structural applications, such as pressure vessels, submarine hulls, aircrafts, launch vehicles, and waterborne ballistic missiles. Submerged structures, are all subjected to external pressure and are required to operate in a variety of environments where they can be subjected to different loads and conditions. Therefore, the analysis of the dynamic characteristics of these shells under external water pressure is crucial to ensure safe and successful designs.

The pressure hull has been calculated and designed under the ASME rules, and Finite Element Method (FEM) simulations have been performed. Aluminium alloy has been specially selected due to its excellent mechanical properties and its high corrosion resistance
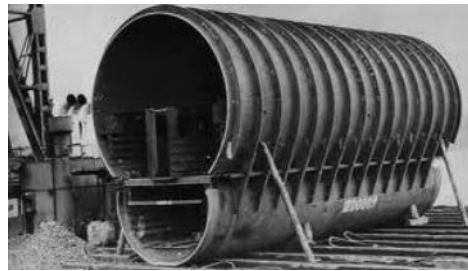

Fig 1.pressure hull image

Design specifications of Pressure Hull:
- Length overall = 120cm
- Pressure hull diameter = 12 cm
- Layout = double diameter ring stiffened cylinder.
- Stiffener spacing =5cm
- Stiffener thickness =0.3 cm

The material used for the construction of pressure hull is aluminum alloy. The mechanical properties are mentioned below: Young's Modulus (E) =2e5 MPa Poisson's Ratio = 0.3 Density = 7850 kg/mm3, Yield strength = 610 MPa The finite element model of the mounting structure assembly is shown below.
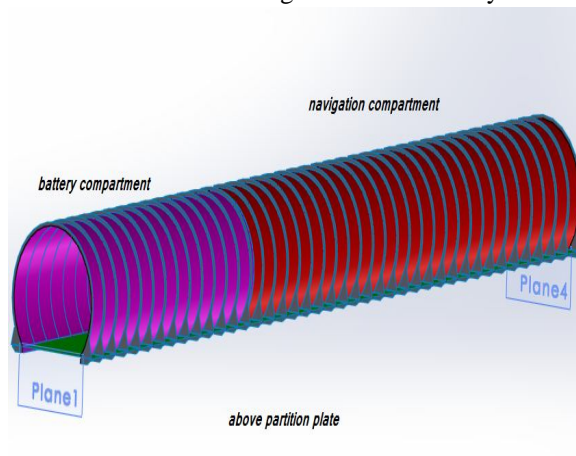

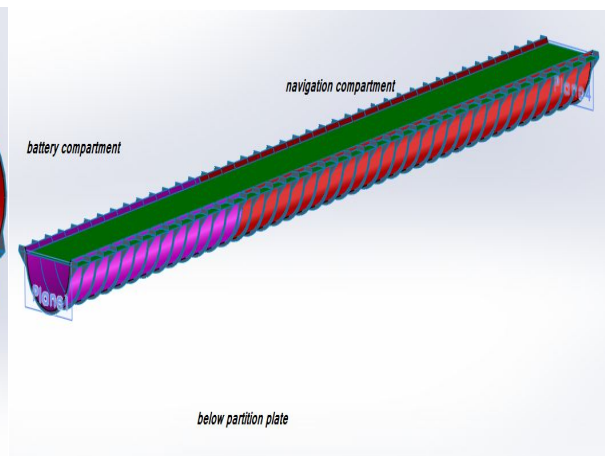Fig 2. Pressure hull above partition plate   Fig3.Pressure hull below partition plate

VIII.STRUCTURAL STATIC ANALYSIS OF PRESSURE HULL

Structure static analysis was done on the pressure hull for external pressure of 65 bars to determine the stresses and deflections. The ends of the pressure hull are fixed in all dof and the external pressure of 65bars is applied on the shells of the pressure hull. The deflection on the battery compartment and navigation compartment above partition plate and below partition plate is analysed. The boundary conditions and loading applied on the pressure hull is shown in the figure. To strengthen the pressure hull modifications are done by, adding gussets to improve the load carrying capacity and reduce the deflection.
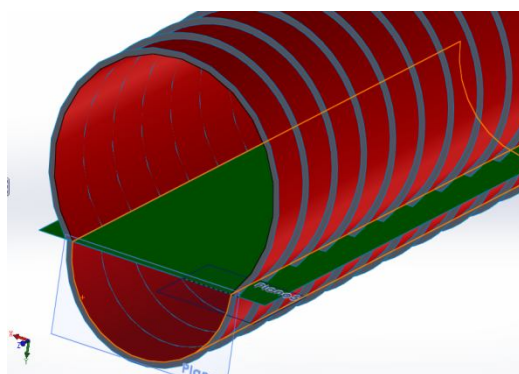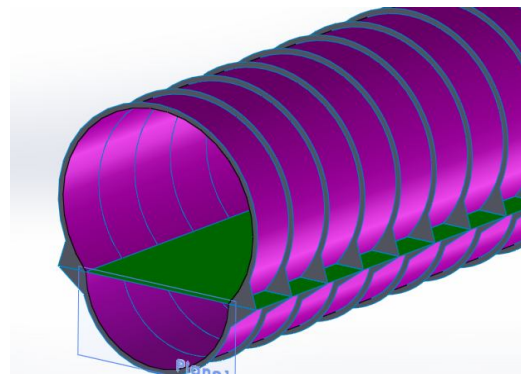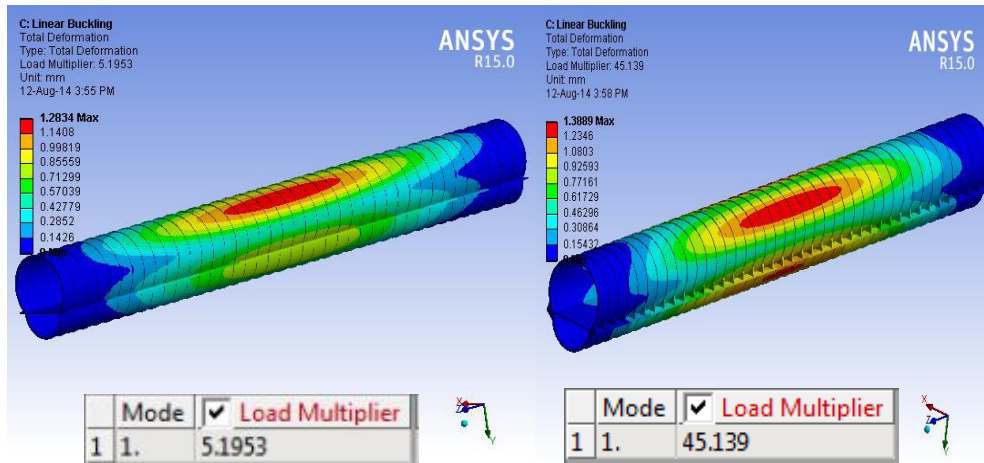

Fig 4 .pressure vessel without gussets   Fig 5.pressure vessel with gussets

IX.CONCLUSION

The primary goal of this project to analyse the performance of cylindrical stuctures subjected to external pressure. As the external loads are carried by membrane and bending responses, the variation in thickness can affect the load carrying capacity of engineering structure. This thesis primarily focused into, large thickness variation by considering cylinder as thin, moderately thick as well as thick by varying, l/d and r/t ratio.

The observation from the analysis shows that, the Donnell's relation can be applied to thin shells more precisely. Due to the increased bending or flexural effect in thick shells, the analysis shows some error value while comparing Donnell's relation with numerical solution.

To improve the load carrying capacity we can use stiffeners in thin shells. And the load carrying capacity of external and internal ring stiffeners and spiral stiffeners are compared by varying the height of stiffener.

	Without gussets	**With gussets**
CRITICAL BUCKLING LOAD	5.195 MPa	45.139MPa
BATTERY COMPARTMENT ABOVE PARTITION PLATE	6.7110	2.3978
NAVIGATION COMPARTMENT ABOVE PARTITION PLATE	8.1234	2.8791
BATTERY COMPARTMENT BELOW PARTITION PLATE	6.7110	2.4331
NAVIGATION COMPARTMENT ABOVE PARTITION PLATE	8.1129	2.9029

The comparisons between calculations show that, for a thin cylindrical structure subjected to external pressure, internal stiffener will withstand maximum load without buckling.

As ring stiffeners are used in pressure vessel design, the project focussed on the design and analysis of pressure vessel with Aluminium alloyT-6061 which has stiffened externally. Buckling and structural analysis shows that the load carrying capacity of pressure vessel is less standard design load of 6.5 MPa(ASME standard).
To improve the load carrying capacity modifications are done on the current design, by adding gussets in existing design.
The buckling analysis done on the modified design shows that the load carrying capacity has improved
The observations shows that the modified design of pressure vessel will withstand more buckling load and the deformations in the compartments are decreased.

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Appendix1.

Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error	Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error
L=120, D=120, T=20	3.1181 *10^9	3.334 *10^9	7.64%	L=120, D=60, T=10	1.245 *10^9	1.37 *10^9	10.48%
L=120, D=120, T=15.	1.4233 *10^9	1.50*10^9	5.30%	L=120, D=60, T=7.5.	5.20 *10^8	5.15*10^8	9.61%
L=120, D=120, T=12	8.2781 *10^8	8.30*10^8	3.64%	L=120, D=60, T=6.	3.0592 *10^8	3.25*10^8	6.15%
L=120, D=120, T=6	3.738*10^7	3.61*10^9	2.74	L=120, D=60, T=3.	6.5302*10^7	6.62*10^7	4.53%
L=120, D=120, T=3	2.092*10^7	2.11*10^7	0.94%	L=120, D=60, T=1.5.	9.781*10^7	9.96*10^7	1.80%
L=120, D=120, T=1.2	2.063*10^6	2.09*10^6	1.45%	L=120, D=60, T=0.6.	1.064*10^6	1.055*10^6	1.33%

Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error	Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error
L=120, D=40, T=6.6	1.0145 *10^9	1.08 *10^9	6.4%	L=120, D=24, T=4	7.7045 *10^8	8.53 *10^8	9.4%
L=120, D=40, T=5.	2.227*10^8	2.38*10^8	6.38%	L=120, D=24, T=3.	3.877*10^8	4.18*10^8	7.4%
L=120, D=40, T=4.	3.0592 *10^7	3.25*10^7	6.15%	L=120, D=24, T=2.4.	2.00 *10^8	2.10*10^8	6.15%
L=120, D=40, T=2.	3.407*10^7	3.59*10^7	5.09%	L=120, D=24, T=1.2.	2.52*10^7	2.64*10^7	4.54%
L=120, D=40, T=1.	8.076*10^6	8.18*10^6	1.27%	L=120, D=24, T=0.6.	3.714*10^6	3.76*10^6	1.27%
L=120, D=40, T=0.4.	6.60*10^5	6.52*10^5	1.22%	L=120, D=24, T=0.24.	4.73*10^5	4.80*10^5	1.45%
Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error	Dimension	Ansys (Pa)	Donnell's relation (Pa)	Error
L=120, D=12, T=2	7.976 *10^8	7.253 *10^8	10.0%	L=120, D=6, T=1.0	8.918 *10^8	9.70 *10^8	8.14%
L=120, D=12, T=1.5.	5.62*10^8	5.21*10^8	7.4%	L=120, D=6, T=0.75.	3.76 *10^8	4.02*10^8	6.40%
L=120, D=12, T=1.2.	1.98 *10^8	2.10*10^8	5.7%	L=120, D=6, T=0.6.	1.89 *10^8	2.02*10^8	6.24%
L=120, D=12, T=0.6.	2.50*10^7	2.63*10^7	4.94%	L=120, D=6, T=0.3.	2.473*10^7	2.60*10^7	4.88%
L=120, D=12, T=0.3.	2.208*10^6	2.23*10^6	0.98%	L=120, D=6, T=0.15.	3.13*10^6	3.09*10^6	1.29%
L=120, D=12, T=0.12.	3.201*10^6	3.23*10^6	0.89%	L=120, D=6, T=0.06.	2.019*10^6	2.04*10^6	1.02%

Appendix 1. Comparison between mathematical and numerical results