Gα And Ag Closed Maps In Ideal Topological Spaces

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Abstract: In this paper some properties of gI±-I-closed maps, I±g-I- closed maps, wgI±-Iclosed, wI±g-I- closed functions are studied

1. INTRODUCTION AND PRELIMINARIES

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A ∈I and B ⊆ A implies B ∈ I, (2) A ∈ I and B ∈ I implies A∪B∈I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X , A*(I,τ)={x ∈ X : A ∩ U ∉I for every U∈τ (X,x)} is called the local function of A with respect to I and τ [4]. We simply write A* in case there is no chance for confusion. A kuratowski closure operator cl*(.) for a topology τ*(I, τ) called the *- topology ,finer than τ is defined by cl*(A) = A ∪ A* [9]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition1.1. A subset A of a topological space (X,τ) is called

1) αg-closed [5], if α cl (A) ⊆ U whenever A ⊆ U and U is open in (X,τ)
2) gα-closed [5], if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ)
3) wgα-closed [10], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
4) wαg-closed [10], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).

The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.2. [1] A subset A of an Ideal topological space (X, τ, I) is said to be αI-open if A ⊆ int(Cl*(int(A))). The complement of the αI-open set is called αI-closed.

Definition 1.3. A subset A of an ideal topological spaces (X, τ, I) is said to be

1) αg - I – closed [8], if α I cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2) gα - I – closed [8], if α I cl(A) ⊆ U whenever A ⊆ U and U is α - open in X.
3) wgα-I-closed [3], if αIcl (int(A)) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
4) wαg-I-closed [3], if αIcl (int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).
5) Ig-closed [7] , if A*⊆ U whenever A ⊆ U and U is open in X.

The complements of the above mentioned closed sets are called their respective open sets.

2. gαI – closed maps

Definition 2.1. A map f : (X, τ, I) → (Y, σ, J) is called gαI – closed map if for each closed set F of (X, τ, I), f(F) is a gαI - closed set of (Y, σ, J).

Example 2.2. Let X = Y = {a,b,c}, τ = σ = {∅, {a}, {b,c}, X} and I = {∅, {c}}. Define a map f : (X, τ, I) → (Y, σ, J) as follows. f(a) = a, f(b) = b, f(c) = c. Then for {b,c} which is closed in X, f ({b,c}) = {a,c} is not α - I - closed in Y. Hence f is not an αI-closed map. However, f is a gαI - closed map.

Definition 2.3. A map f : (X, τ, I) → (Y, σ, J) is said to be pre αI-closed, if f (U) is αI-closed in (Y, σ, J) for every αI-closed set U in (X, τ, I).

Definition 2.4. A map f: (X, τ, I) → (Y, σ, J) is said to be αI-irresolute, if the inverse image of every α-I set in Y is α-I-set in X.

Theorem 2.5. *Let f : (X, τ, I) → (Y, σ, J), h : (Y, σ, J) → (Z, η, K) be two mapping such that h o f : (X, τ, I) → (Z, η, K) is a gαI - closed map. Then*

1. *If f is continuous and surjective, then h is gαI-closed.*
2. *If h is αI-irresolute, Pre-αI- closed and injective, then f is gαI - closed.*

Proof.

(1) Let B be a closed set of Y. Then f-1(B) is closed in X. But (h o f) (f-1(B)) is gαI-closed in Z and hence h(B) is gαI - closed in Z. Therefore h is a gαI-closed map.

(2) Let A be closed in Y. Then (h o f) (A) is gαI-closed in Z. But h is αI-irresolute. Therefore h-1[(h o f) (A)] is gαI-closed in Y. This implies f(A) is gαI - closed in Y.

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Hence f is a gαI-closed map.

Theorem 2.6.

(1) *If f : (X, τ, I) → (Y, σ, J) is gαI- closed and A is αI-closed set of X then its restriction f / A : (A, τ / A, I) → (Y, σ, J) is gαI - closed.*

(2) *Let B be open and gαI-closed is a subset of Y. If f : (X, τ, I) → (Y, σ, J) is αI- closed then f / A : (A, τ / A, I) → (Y, σ, J) is gαI - closed. Where A = f-1(B).*

(3) *If B is αI – closed, f : (X, τ, I) → (Y, σ, J) is gαI-closed then f / A : (A, τ / A, I) → (Y, σ, J) is gαI - closed.Where A = f-1(B).*

Proof: To prove (1). Let F be a closed set of A. Since F is closed in X, (f / A) (F) = f(F) is gαI - closed in Y. Hence f / A is a gαI – closed map. To prove (2). Let F be a closed set of A. Then F = A ∩ H for some closed set H of X. But f (H) is αI-closed in Y, since f is an αI-closed map. We have (f / A) (F) = f (A ∩ H) = f(H) ∩ B. But f(H) ∩ B is gαI- closed in Y, since B is open and gαI- closed in Y.This implies that B is gαI- closed in Y, since f(H) is αI–closed in Y. Therefore f / A is a gαI- closed map. To prove (3). Let F be a closed set of A. Then F = A ∩ H for some closed set H of X. But f(H) is a gαI – closed set in Y, since f is a gαI – closed map. We have (f / A) (F) = f (A ∩ H) = f(H) ∩ B, since f(H) is gαI–closed and B is αI–closed. We have f(H) ∩ B is gαI–closed. Hence f / A is a gαI–closed map.

Definition 2.7. An Ideal topological space (X, τ, I) is said to be αI-normal if for every pair of disjoint closed sets A and B of (X, τ, I) there exists disjoint αI-open sets U and V such that A ⊂ U and B ⊂ V.

Theorem 2.8. *If f : (X, τ, I) → (Y, σ, J) is continuous, gαI - closed surjection and if (X, τ, I) is normal then (Y, σ, J) is αI–normal*.

Proof. Let A, B be disjoint closed set of Y. Since X is normal, there exist disjoint open sets U and V of X. Such that f-1(a) ⊂ U and f-1(B) ⊂ V. Then there exist gαI–open sets in G and H such that A ⊂ G, B ⊂ H and f-1(G) ⊂ U. f-1 (H) ⊂ V. Then we have f-1(G) ∩ f-1(H) = ϕ. Since G is gαI-open and A is αI – closed, G ⊃ A implies αIint(H) ⊃ A. Similarly αIint(H) ⊃ B. Therefore αIint(G) ∩ αIint(H) = G ∩ H = ϕ and hence Y is αI-normal.

3. αgI – closed maps

Definition 3.1. A map f : (X, τ, I) → (Y, σ, J) is called αgI-closed map if for each closed set F of (X, τ, I), f(F) is a αgI-closed set of (Y, σ, J).

Remark 3.2. Every αI-closed map, Ig-closed map and gαI-closed map are αgI-closed. However the converse are not true in general as seen from the following examples.

Example 3.3. Let X = Y = {a,b,c}, τ = {ϕ, {a}, X}, I = {ϕ, {c}} and σ = {ϕ, {b}, {c}, {b,c}, X}. Define f: (X, τ, I) → (Y, σ, J) as follows. f(a) = b, f(b) = c and f(c) = a. Then the image f{b,c} = {a,c} is not αI-closed. Hence f is not an αI-closed map. However f is an αgI-closed map.

Example 3.4. Let X = Y = {a,b,c}, τ = {ϕ, {a,b}, X}, I = {ϕ, {a}} and σ = {ϕ, {a}, {a,c}, X}. Define f : (X, τ, I) → (Y, σ, J) as follows. f(a) = a, f(b) = c, f(c) = b. Then the image f({c}) = {b} is not Ig-closed in (Y, σ, J). Hence f is not Ig-closed map. However f is αg-I-closed.

Example 3.5. Let X = Y = {a,b,c}, σ = τ = {ϕ, {a}, X}, I = {ϕ, {c}}. Define f : (X, τ, I) → (Y, σ, J) as follows. f(a) = b, f(b) = a, f(c) = c. Then the image f{b,c} = {a,c} is not gαI-closed in (Y, σ, I). Hence f is not gαI-closed. However f is αgI-closed.

Theorem 3.6. *A map f : (X, τ, I) → (Y, σ, J) is αgI-closed if and only if for each subset S of Y and for each open set V containing f-1(S), there exists an αgI-open set V of Y containing S and f-1(V) ⊂ U.*

Proof. Necessity. Let S be a subset of Y and U be open set of X such that f-1(S) ⊂ U. Then Y / f (X / U) say V, is an αgI-open set containing S such that f-1(V) ⊂ U. Sufficiency. Let F be a closed set of X. Then f-1 (Y / f (F)) ⊂ X / F. By taking S = Y / f(F) and V = X / F in hypothesis there exists an αgI-open set V of Y containing Y / f (F) and f-1(V) ⊂ X / F. Then we have F ⊂ X / f-1(V) and Y / V = f (F). Since Y / V is αgI-closed and thus f is an αgI-closed. Therefore f(F) is αgI-closed and thus f is an αgI-closed map.

Theorem 3.7. *1. If a map f : (X, τ, I) → (Y, σ, J) is continuous and Pre-αI-closed, then for every αgI-closed set F of X, f(F) is αgI-closed in Y. 2. If a map f : (X, τ, I) → (Y, σ, J) is continuous and αgI-closed for every Ig-closed set F of X, f(F) is αgI-closed.*

Proof. To prove (1) Let F be an αgI-closed set of (X, τ, I) . Let O be an open set of (Y, σ, J). Such that f(F) ⊂ O. Then F ⊂ f-1 (O) implies αICl(F) ⊂ f-1(O). since F is αgI-closed and f-1(O) is open. This implies f(αICl(F)) ⊂ O. Since f is Pre-αI-closed, f(αICl(F)) is αI-closed. Therefore αICl(f(αICl(F))) = f(αICl(F)) ⊂ O. This implies αICl(f(F))⊂ O and hence f(F) is αgI-closed in (Y, σ, J). To prove (2) let U an open set of Y such that f(F) ⊂ U . We claim that f(F) is αgI-closed. But F ⊂ f-1(V) and f-1(U) is open set of X and hence Cl(F) ⊂ f-1(U), since F is a Ig-closed set. Therefore f(Cl(F)) ⊂ U and f(Cl(F)) is an αgI-closed set, since f is an αgI-closed map and this implies αICl(f(αICl(F))) ⊂ U. Hence αICl(f(F)) ⊂ U. Therefore f(F) is an αgI-closed set.

Theorem 3.8. *Let f : (X, τ, I) → (Y, σ, J) and h : (Y, σ, J) → (Z, η, K) be two mappings such that h o f : (X, τ, I) →(Z, η, K) is a gαI-closed map. Thena) If f is a continuous, surjection then h is αgI-closed. b) If h is αI-irresolute, Pre-αI-closed injection then f is αgI-closed.*

Proof. To prove (1). Let F be closed in Y. Then f-1(F) is closed in X. Since f is continuous. By hypothesis (h o f) (f-1(F)) is gαI-closed set in Z. This implies h(F) is gαI-closed set and therefore h(F) is an αgI-closed set. Hence h is an αgI-closed map. To prove (2). Let F be closed in X. Then (h o f) (F) is gαI-closed in Z and h-1(h o f) (F) is gαI-closed in Y. Since f(F) is gαI-closed in Y. But every gαI-closed set is an αgI-closed set. Therefore f(F) is αgI-closed. Hence f is αgI-closed.

Theorem 3.9. *1. If f : (X, τ, I) → (Y, σ, J) is αgI-closed and h : (Y, σ, J) → (Z, η, K) is continuous and Pre-αI-closed then h o f : (X, τ, I) → (Z, η, K) is αgI-closed. 2. if f : (X, τ, I) → (Y, σ, J) is closed and h : (Y, σ, J) → (Z, η, K) is αgI-closed then h o f : (X, σ, I) → (Z, η, K) is αgI-closed.*

Proof. To prove (1). Let F be closed in (X, τ, I) , Then f(F) is αgI-closed in (Y, σ, J). By theorem 3.7 (2), h(f(F)) is αgI-closed in (Z, η, K). Therefore h o f is αgI-closed. To prove (2). Let F be closed in (X, τ, I). Then f(F) is closed in (Y, σ, J). Since h is αgI-closed map. h(f(F)) is αgI-closed in (Z, η, K). Therefore h o f is αgI-closed.

4. wαg-I-closed and wgα -I-closed function

Definition 4.1. A function f : (X, τ, I) → (Y, σ, J) is said to be wαg-I-closed function if the image of every closed set in (X, τ, I) is wαg-I-closed in (Y, σ, J). The complement of wαg-I-closed set is wαg-I-open.

Theorem 4.2. Every closed map is wαg-I-closed map but not conversely. Proof. Let f : (X, τ, I) → (Y, σ, J) be an closed map and V be a closed set in X. Then f(V) is closed and hence f(V) is wαg-I-closed in Y. Thus f is wαg-I-closed.

Example 4.3. Let X = Y = {a, b, c}, τ = {ϕ, {a}, {ac}, X}, σ = {ϕ, {ab}, X} and I = {ϕ, {a}}. Let f : X → Y be the identity function then f is wαg-I-closed but not closed.

Theorem 4.4. A function f : (X, τ, I) → (Y, σ, J) is wαg- I-closed if and only if for each subset B of Y and for each open set G containing f-1(B) there exists a wαg-I-open set F of Y such that f-1(F) ⊆ G.

Proof. Necessity. Let G open subset of (X, τ, I) and B be a subset of Y such that f-1(B) ⊆ G. Then F = Y – f (X – G) is wαg-I-open set containing B such that f-1(F) ⊆ G. Sufficiency. Let U be a closed subset of (X, τ, I) . Then f-1(Y – f(U)) ⊆ X – U and X – U is open. By hypothesis, there is a wαg-I-open set F of (Y, σ, J) such that Y – f(U) ⊆ F and f-1(F) ⊆ X – U. Therefore U ⊆ X – f-1(F). Hence Y – F ⊆ f(U) ⊆ f (X – f-1(F)) ⊆ Y – F. which implies f(U) = Y – F and hence f(U) is wαg-I-closed in (Y, σ, J). Therefore, f is wαg-I-closed function.

Theorem 4.5. *If a function f: (X, τ, I) → (Y, σ, J) is closed and a map g : (Y, σ, J) → (Z, η) is wαg-I-closed then their composition g o f : (X, τ, I) → (Z, η) is wαg-I-closed.*

Proof . Let G be a closed set in X. Then f(G) is closed in Y and (g o f) (H) = g (f(H)) is wαg-I-closed, as g is wαg-I-closed. Thus g o f is wαg-I-closed.

Definition 4.6. A function f : (X, τ, I) → (Y, σ, J) is said to be wgα-I-closed function if the image of every closed set in (X, τ, I) is wgα-I-closed in (Y, σ, J). The complement of wgα-I-closed set is wgα-I-open.

Theorem 4.7. *Every closed map is wgα-I-closed but not conversely.*

Proof. Let f : (X, τ, I) → (Y, σ, J) be a closed map and V be a closed set in X. Then f(V) is wgα-I-closed in Y. Thus f is wgα-I-closed.

Example 4.8. Let X = Y = {a, b, c}, τ = {ϕ,{ ab}, X}, σ = {ϕ, {a}, {ab}, X } and I = {ϕ, {a}}. Let f : X → Y be the identity function then f is wgα-I-closed but not closed.

Theorem 4.9. *A function f : (X, τ, I) → (Y, σ, J) is wgα- I-closed if and only if for each subset B of Y and for each open set G containing f-1(B) there exists a wgα-I-open set F of Y such that f-1(F) ⊆ G.*

Proof. Necessity. Let G open subset of (X, τ, I) and B be a subset of (Y, σ, J) such that f-1(B) ⊆ G. Let F = Y – f(X – G) since f is wgα-I-closed, F is wgα-I-open set containing B. Such that f-1(F) ⊆ G. Sufficiency. Let U be a closed subset of (X, τ, I) . Then f-1(Y – f(U) ⊆ X – U and X – U is open. By hypothesis, there is a wgα-I-open set F of (Y, σ, J) such that Y – f(U) ⊆ F and f-1(F) ⊆ X – U. Therefore U ⊆ X – f-1(F). Hence Y – F ⊆ f(U) ⊆ f (X – f-1(F)) ⊆ Y – F. which implies f(U) = Y – F and hence f(U) is wgα-I-closed in (Y, σ, J). Therefore, f is wgα-I-closed function.

Theorem 4.10. If a function f : (X, τ, I) → (Y, σ, J) is closed and a map g : (Y, σ, J) → (Z, η) is wgα-I-closed then their composition g o f : (X, τ, I) → (Z, η) is wgα-I-closed.

Proof: Let G be a closed in X. Then f(G) is closed in Y and (g o f) (G) = g(f(G)) is wgα-I-closed as g is wgα-I-closed. Thus g o f is wgα-I-closed.

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