# Exergy Analysis Of Gas Turbine-Burner Engine

Olaniyi Balogun, Changki Mo, A. K. Mazher

**Abstract**: The gas turbine-burner engine was first proposed by Professors Liu and Sirignano in 1999. A detail exergic analysis was performed on this engine. A turbofan engine core was used as a study system for this analysis. The first and second principles of thermodynamics were applied to each component of the system. An exergy balance was derived for each component of the system. An exergetic analytic study was performed to each component and the results were compared. Results show the combustor having the highest exergetic value. A comparison of the exergy of the conventional and the turbine-burner engines was completed based on ideal gas and reversible process assumption. Results shows less than 5% increase in the exergetic value of the turbine-burner to the conventional turbine.

**Index Terms**: Exergy Analysis, Entropy generation, Gas Turbine-Burner, Turbofan efficiency, Availability, Brayton cycle, Turbofan performance

---

## NOMENCLATURE

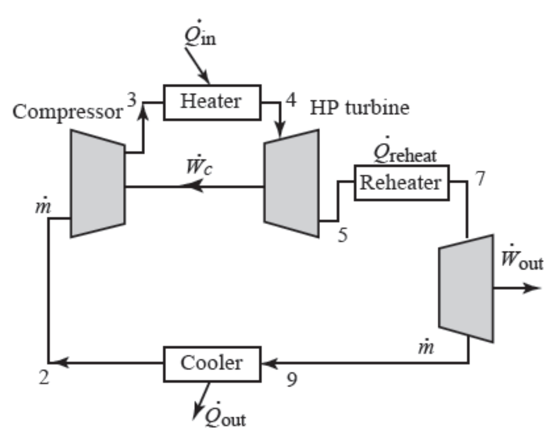
| | |
|---|---|
| $c_p$ | Specific heat (kJ/kg K) |
| G | Acceleration due to gravity (m/s$^2$) |
| $g_c$ | Newton's constant |
| H | Enthalpy (kJ/kg) |
| $\dot{m}$ | Mass flow rate (kg/s) |
| P | Pressure (kPa) |
| $\dot{Q}$ | Heat transfer rate |
| R | Universal gas constant ( kJ/kg K) |
| $\dot{S}$ | Rate of entropy change( KW/K) |
| T | Temperature (K) |
| V | Air speed ( m/s) |
| $\dot{W}$ | Power (KW) |
| Z | Potential height (m) |
| **Greek symbols** | |
| $\phi_{tb}$ | Turbine-burner additional power generation ( KW) |
| $\Phi$ | Power (KW) |
| $\eta_{tb}$ | Turbine-burner efficiency |
| $\Psi$ | Exergy (kJ/kg) |
| **Subscripts** | |
| A | Air |
| C | Compressor |
| Conv | Conventional engine |
| Ex | Exit |
| F | Fuel |
| Ftb | Fuel flow in turbine-burner engine |
| Gen | Entropy generation |
| In | Inlet |
| R | Fuel burn reaction |
| T | Turbine |
| Tb | Turbine-burner |
| Noz | Nozzle |

## 1. INTRODUCTION

The concept of the turbine-burner was first introduced by Sirignano and Liu in 1999 [1] where the authors describe a conventional gas turbine engine with a small heat-adder modification at the turbines.


- *Olaniyi Balogun is currently pursuing doctoral degree program in mechanical engineering at Washington State University E-mail: olaniyi.balogun@wsu.edu*
- *Changki Mo is currently an Assistant Professor in the School of Mechanical and Materials Engineering at WSU Tri-Cities. E-mail: changki.mo@tricity.wsu.edu*
- *A.K Mazher is currently a Professor in Nuclear Engineering Department at King Abdulaziz University, KSA. E-mail: amazher@hotmail.com*


The intent of adding heat to the turbine is to promote an efficient thermodynamic flow through the engine thereby improving the overall performance of the engine [2]. The Brayton power cycle, which similar to the Carnot cycle, is a thermodynamic model used for an ideal gas in turbine engines [3]. A conventional Brayton cycle is composed of a four stage cycle that includes isentropic compression, constant pressure heat addition, isentropic expansion and constant pressure heat rejection. Fig. 1 shows a modification of the Brayton cycle closely representing the turbine-burner's power cycle [3]. The cycle shows the main turbine driving the free-power turbine with reheat. The addition of reheat increases the power of the free turbine but reduces the thermal efficiency of this setup. A turbofan core with a modified turbine stage is appropriate for this study. Fig. 2 shows the corresponding T-S cycle for a turbine-burner and a conventional engine (dashed lines) [2]. From the figure it is clear that the conventional engine is similar to the Brayton modified cycle with the influence of afterburner addition. The negative effect of the afterburner is that although it increases the specific thrust it drastically increases the thrust specific fuel consumption of the engine. The main difference between the conventional and turbine-burner engine is at stage 04-05. Addition of a turbine-burner keeps the temperature in the burner constant as the gas expands and releases kinetic energy to turn the rotors. Typically, in commercial turbofan engines afterburners are not added, thus the cycle stage will go from 05 to 07 isentropically.



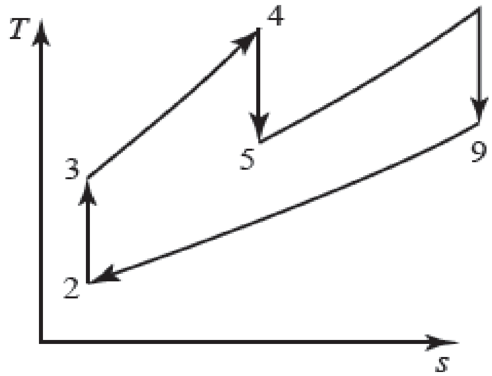



**Fig. 1**. Brayton Cycle with Reheat (*Figure adopted from [3] fig. 4.25*).

In 2005, Ebadi and Gorji-Bandpy completed [4] an exergetic analysis study of a turbine plant. Their studies show all components of the plant to be well above ninety percent efficiency with the exception of the combustion chamber, which was around sixty percent. A detailed exergy analysis of the gas turbine-burner engine is presented in this paper, which begins by looking at the application of the first law of thermodynamics to the turbine-burner engine. Because entropy and exergy are closely related, it is necessary to consider the entropy of the engine, so an entropy generation analysis of the turbine-burner is performed. With the entropic nature of the engine determined, exergy destruction becomes a straight forward topic to handle, thus leading to the change in exergy analysis. Finally, a simple study is presented to investigate the availability within each component of the turbine-burner engine. Much of the work presented in this paper has followed the analyses presented in Progress in Astronautics and Aeronautics [5].

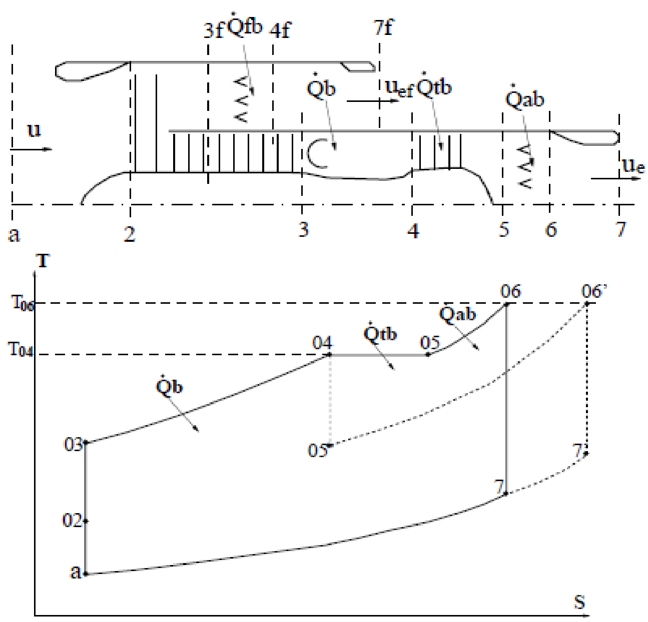



**Fig. 2**. Comparison of Thermodynamic Cycles with and without the Turbine Burner (*Figure adopted from [2] fig. 1*).

## 2 Mass and Energy balance

During steady flow through a control volume, the total amount of mass contained in the volume remains constant. Therefore, the conservation of mass principle requires that the total amount of mass entering a control volume must equal the total amount of mass leaving it. The assumed control surface shown below in Fig. 3 can be used to describe the First Law of Thermodynamics under steady state, steady flow, and quasi-one-dimensional flows [3]. The continuity equation is

$$\left(\dot{m}_a+\dot{m}_f\right)_{ex}=\left(\dot{m}_a+\dot{m}_f\right)_{in} \qquad (1)$$

where *ex* and *in* stands for exit and inlet flows.

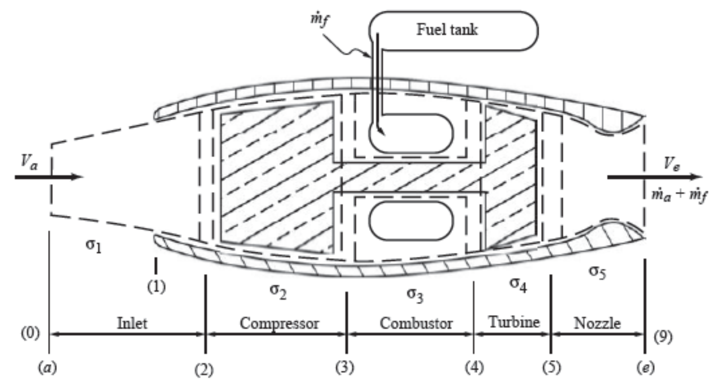



**Fig. 3**. Control Volume for Analyzing Turbine-Burner Engine (*Figure adopted from [3] fig. 2.4a*).

A consideration of the steady flow energy equation under the same assumptions previously stated takes into account the work interaction *W* and the heat interaction *Q* occurring at the boundary of the surface. The first law of thermodynamics can then be written as:

$$\dot{Q}-\dot{W}=\dot{m}\left(h+\frac{V^2}{2g_c}+\frac{gz}{g_c}\right)_{ex}-\dot{m}\left(h+\frac{V^2}{2g_c}+\frac{gz}{g_c}\right)_{in} \qquad (2)$$

where $g_c$ is Newton's constant.

### 2.1 Inlet and Nozzle

The inlet air's velocity is reduced as it enters the core because the inlet acts as a diffuser, while the nozzle air increases as it leaves the engine. If heat interactions are assumed to be negligible the steady flow equation (2) gives the result

$$\left(h+\frac{V^2}{2g_c}\right)_{ex}=\left(h+\frac{V^2}{2g_c}\right)_{in} \qquad (3)$$

For an assumption of ideal gas behavior the change in temperature of the component can be written as

$$T_{ex}-T_{in}=\frac{V_{in}^2-V_{ex}^2}{2g_c c_p} \qquad (4)$$

### 2.2 Compressor

The air velocity entering and leaving the compressor can be assumed to be the same under an adiabatic condition. The shaft power needed for driving the shaft and compressing the air can be derived from equation (2). The power of the conventional compressor is given as

$$\dot{W}_c = -\left(\dot{m}_a c_p (T_3 - T_2)\right) \quad (5)$$

Where the subscripts 2 and 3 match the surface descriptions in Fig. 3; the minus sign indicates that the compressor is delivering energy to the air. A different equation has been applied to the turbine-burner compressor

$$\dot{W}_c = -\left(\dot{m}_a c_p (T_3 - T_2) + \phi_{tb}\right) \quad (6)$$

Unlike with the conventional engine, for the turbine-burner compressor there is an extra term $\phi_{tb}$ that includes the additional power provided by the turbine-burner and that will be derived in the turbine-burner section.

### 2.3 Combustor

Within the combustion chamber heat is delivered by burning fuel. The heat value of the fuel $Q_R$ delivers energy to the air. There is no shaft work within the chamber. For simplicity, neglecting the enthalpy of the burning fuel, assuming equal velocity in and out of the chamber and an ideal burning efficiency, a virtual power can be described for the combustor

$$\dot{Q}_R = c_p\left(\dot{m}_a (T_4 - T_3) + \dot{m}_f\, T_4\right) \quad (7)$$

### 2.4 Turbine-Burner

The turbine runs the compressors. The energy delivered to the turbine in a conventional turbine component comes only from the compressed air and the burning in the combustor. Thus, applying the energy balance from equation (2) the power of the conventional turbine component becomes

$$\dot{W}_t = c_p\left(\left(\dot{m}_a + \dot{m}_f\right)(T_5 - T_4)\right) \quad (8)$$

For the turbine-burner component, the inclusion of a heat adder delivers extra energy that is used to deliver extra power to the compressor. Additional fuel is added to the turbine to generate this heat. The main assumption within the turbine-burner is that exit and inlet temperatures are the same

$$T_4 = T_5 \quad (9)$$

For simplicity, the specific heat value is assumed to be constant throughout the system (note, typically specific heat values vary through the system). If the inlet and exit velocities are the same and the enthalpy of the fuel burnt is negligible then the energy balance for a turbine-burner becomes

$$\dot{Q}_{tb} - \dot{W}_{tb} = c_p\left(\left(\dot{m}_a + \dot{m}_f + \eta_{tb}\dot{m}_{ftb}\right)\Delta T_{tb}\right) \quad (10)$$

Where

$$\Delta T_{tb} = T_4 - T_{5\_conv} \quad (11)$$

The heat-adder is only required to raise the temperature of the turbine-burner to $T_4$ from a minimum of $T_5$. As the gas expands within the turbine it cools down, however, additional heat is provided in order to raise the temperature. The net effect of this dynamic is that the temperature remains constant. For simplicity, the required delta temperature needed to keep the turbine-burner constant has been approximated as shown in equation (11). The total power driving the compressor is assumed to be the same as the total power driving the turbine-burner

$$\Phi_c = \Phi_{tb} \quad (12)$$

Comparing equations (6), (8), (10), (11), and (12) the maximum additional power delivered to the compressor is derived as

$$\phi_{tb} = \eta_{tb} c_p \dot{m}_{ftb}\, \Delta T_{tb} \quad (13)$$

## 3 Entropy Generation

Similar to the conventional energy approach, an entropy calculation can be completed for each component of the gas turbine-burner engine. Fundamental entropy equations have been taken from [8]. The total entropy generation of the engine is the sum of the entropy generation across each component within the engine. Thus, the inefficiency of the subsystem can be readily quantified and evaluated for optimization purposes. The Second Law of Thermodynamics can be written in the form of entropy generation as shown in equation (14) for the purpose of estimating the irreversibilities of the engine components

$$\dot{S}_{gen} = \frac{\partial s}{\partial t} + \frac{\dot{Q}}{T} + \sum\left(\dot{m}_{ex}\, s_{ex} - \dot{m}_{in}\, s_{in}\right) \geq 0 \quad (14)$$

For a steady state flow the first term drops out of the equation, and under adiabatic process the second term drops out. The problem is simplified to finding the mass flow rate entropy change. This is not a physical quantity that is easily measured. Using the Gibbs equation, entropy generation can be related to other thermodynamic properties of a substance. In this case an assumption of a steady flow ideal gas is made, thus providing the relationship below.

$$\dot{S}_{gen} = \dot{m} c_p \ln\frac{T_{ex}}{T_{in}} - \dot{m} R \ln\frac{P_{ex}}{P_{in}} + \sum\frac{\dot{Q}}{T_{av}} \quad (15)$$

As an estimation of the boundary temperature, the average temperature is used for the entropy generation.

### 3.1 Inlet and Nozzle

With no temperature change and pressure loss the entropy generation of the inlet and nozzle becomes zero.

$$\dot{S}_{gen_{in/noz}} = 0 \quad (16)$$

This is in accordance to the isentropic behavior of noted the in the figure 2. Note that the fact that the rate of entropy change in the inlet or nozzle means that entropy generation is constant in this component.

$$S_{gen_{in/noz}} = Const. = \frac{c_p\left[m_a\left(T_{ex} - T_{in}\right)\right]}{T_{av}} \quad (17)$$

### 3.2 Compressor

Assumptions similar to those outlined in the energy section are used for this section of the paper. There is a change in pressure, which is usually an engine design parameter. For this analysis a calorically perfect gas with a constant specific pressure is assumed. There is no heat addition with the compressor, so equation (15) is reduced to

$$\dot{S}_{gen_c} = \dot{m}_a\left(c_p \ln\frac{T_3}{T_2} - R\ln\frac{P_3}{P_2}\right) \quad (18)$$

The entropy generation derived here shows the initial value of irreversibilities within the system. Although on the T-s diagram the compressor shows no change in entropy, this does not mean the entropy generation is zero, only that the change in entropy is zero.

### 3.3 Combustor

There is no pressure change within the combustion chamber and the addition of fuel is considered. Again, using similar assumptions as those made in the energy section the rate of entropy generation is determined to be

$$\dot{S}_{gen_R} = \left(\dot{m}_a + \dot{m}_f\right)\left(c_p \ln\frac{T_4}{T_3}\right) - \frac{\dot{Q}_R}{T_{av}} \quad (19)$$

And $T_{av}$= average ($T_4$, $T_3$)

Note that the heating value can be calculated from equation (7).

### 3.4 Turbine-Burner

The turbine-burner main assumption is again shown below.

$$T_4 = T_5 \quad (20)$$

A turbine-burner fuel burn is introduced into the system and accounted for within the entropy generation calculation.

$$\dot{S}_{gen_{tb}} = \left(\dot{m}_a + \dot{m}_f + \dot{m}_{ftb}\right)\left(-R\ln\frac{P_5}{P_4}\right) - \frac{\dot{Q}_{tb}}{T_{av}} \quad (21)$$

*$T_{av}$= average ($T_4$, $T_5$)*

The entropy generations of the conventional engine turbine component will be

$$\dot{S}_{gen_t} = \left(\dot{m}_a + \dot{m}_f\right)\left(c_p \ln\frac{T_5}{T_4} - R\ln\frac{P_5}{P_4}\right) \quad (22)$$

There is no heat interaction considered and the temperature and pressure are not constant. Because by principle the entropy generated by a system must be a positive value, only the magnitudes of the calculated entropy should be used. It is apparent that results determined in the energy section can be used in the entropy section to find a quantifiable result for the entropy generation of any of the components. The total entropy generation of the system can be derived by adding all component entropy generations.

$$S_{gen_{tot}} = S_{gen_{in}} + S_{gen_c} + S_{gen_R} + S_{gen_{tb}} + S_{gen_{noz}} \quad (23)$$

## 4 Exergy Balance

The exergy analysis is done by combining the First and Second Laws of Thermodynamics. Fundamental exergetic equations used in the work are taken from [9]. Assuming a reversible process, the total exergy of an individual component is calculated by adding the work extracted and the heat within the component

$$\Psi = W_x + T_o S_{gen_x} \quad (24)$$

All chemical exergy considerations are assumed to be negligible.

### 4.1 Inlet and Nozzle

The exergy of the inlet and nozzle is primarily due to the mass flow of air. There are no heat or work flow differentials. The change in exergy of the nozzle/inlet is zero therefore there is no actual work done by the inlet/nozzle. This means that the total reversible work of the component is equal to the total irreversibility of the component, thereby leaving a net change of exergy as zero. Simplifying equation (24) for the inlet and nozzle component leads to the following equation

$$\Psi_{in/noz} = W_{rev} = Irrev = T_o S_{gen} \quad (25)$$

### 4.2 Compressor

Similar assumptions are made from the energy and entropy section. Simplifying equation (24) with the compressor component in consideration gives the results

$$\Psi_c = W_c + T_o S_{gen_C} \quad (26)$$

### 4.3 Combustor

As an approximation, the virtual power is used to calculate the exergy of the combustor component. The exergy balance of this component becomes

$$\Psi_R = \frac{\dot{Q}_R}{\left(\dot{m}_a + \dot{m}_f\right)} + T_o S_{gen_R} \quad (27)$$

**TABLE 1**
GAS TURBINE-BURNER ENGINE DATA FOR EXERGY ANALYSIS

| | Inlet | | | Compressor | | | Combustor | | | | Turbine-Burner | | | | Nozzle | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| in | P1 | T1 | ma'1 | P2 | T2 | ma'2 | P3 | T3 | ma'3 | mf'3 | P4 | T4 | ma'4 | mftb'4 | P5 | T5 | ma'5 |
| | kPa | K | kg/s | kPa | K | kg/s | kPa | K | kg/s | kg/s | kPa | K | kg/s | kg/s | kPa | K | kg/s |
| | 33.4 | 240.0 | 56.7 | 38.3 | 257.1 | 56.7 | 1824.5 | 812.1 | 56.7 | 1.0 | 1824.5 | 1475.6 | 57.7 | 0.4 | 1040.6 | 1475.6 | 58.1 |
| ex | P2 | T2 | ma'2 | P3 | T3 | ma'3 | P4 | T4 | ma'4 | | P5 | T5 | ma'5 | | P9 | T9 | ma'9 |
| | kPa | K | kg/s | kPa | K | kg/s | kPa | K | kg/s | | kPa | K | kg/s | | kPa | K | kg/s |
| | 38.3 | 257.1 | 56.7 | 1824.5 | 812.1 | 56.7 | 1824.5 | 1475.6 | 57.7 | | 1040.6 | 1475.6 | 58.1 | | 1020.0 | 903.4 | 58.1 |

| cp | R | To |
|---|---|---|
| KJ/kg K | KJ/kg K | K |
| 1.003 | 0.287 | 239 |

### 4.4 Turbine-Burner
The turbine-burner main assumption is again shown below.

$$T_4 = T_5 \tag{28}$$

Simplifying equation (24) with the turbine-burner component in consideration provides the result

$$\Psi = W_{tb} + T_o S_{gen_{tb}} \tag{29}$$

The conventional engine will be

$$\Psi = W_t + T_o S_{gen_t} \tag{30}$$

## 5 Exergetic Analysis Case Study
An exergetic analysis was completed in order to compare the work potential of a turbofan core with the modified turbine section. The analysis was performed by applying the equations derived in the above section with data provided in Table 1. The data in Table 1 shows the individual component of the engine with the corresponding inlet and exit pressure, temperature, and the mass flow. Note that the exit parameter of the previous component is the inlet parameter of the next component. Typically, engine designs will have bleed flows used as cooling flows for the blades and entire core. Also, there are inter-stage components where pressure and temperature losses occur before the flow enters the next components. For simplifying this analysis, these factors and interactions in the engine have been ignored. Extensive engine cycle data are provided by engine companies using numerical design tools like NPSS. Also, related values have been calculated to account for the burner within the turbine and presented in Table 1 as well. The data shown in Table 1 represents an engine in operation at 85% of max climb condition at 35000 feet.

## 6 Results And Discussion
A MATLAB code was written to generate the results of the analysis discussed above. The results presented in Fig. 4 shows the exergy value of each component over the total exergy of the system. As seen in the figure the combustor has the highest exergetic ratio, which is expected with the introduction of heat into the component through combustible fuel. The turbine-burner does have burning introduced to it which provided a higher exergetic value when compared to the compressor. A comparison between the conventional turbine and the turbine-burner is presented in Fig. 5. The analysis shows a 3.5% difference between the conventional versus the turbine-burner engine.

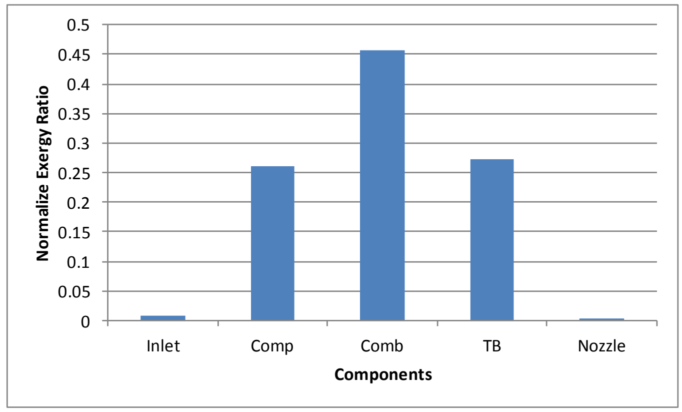



**Fig. 4.** Normalized Exergy of Engine Component

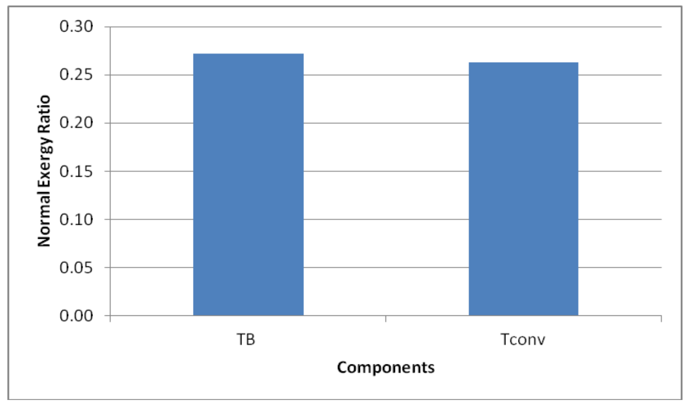



**Fig. 5**. Comparison of Turbine-Burner to Conventional Turbine

## 7 Conclusion
In this paper, a study of the work capacity potential of a gas turbine-burner engine was completed. The energetic and entropic nature of the engine was studied, and a representation for deriving a quantifiable value meeting the criteria of the first and second laws was presented. It was shown that under isothermal conditions within the turbine-burner component the exergetic value can still be calculated. This is the case with an ideal gas assumption and obeying the second law rule stating that entropy can only be zero for a reversible process or positive for an irreversible process. An exergetic comparison between the conventional engine and the turbine-burner shows a significant increase for the turbine-burner. From the results of this study, the advantage of the turbine-burner compared to the conventional turbine is less than 5%.

## References
[1] W. A. Sirignano and F. Liu, “Performance Increases for Gas Turbine Engines Through Combustion Inside The Turbine,” Journal of Propulsion and Power, Vol. 15, No. 1, January-February, 1999.

[2] F. Liu and W. A. Sirignano, “Turbojet and Turbofan Engine

Performance Increases Through Turbine Burners," Journal of Propulsion and Power, Vol. 17, No. 3, May-June, 2001.

[3] J. D. Mattingly, Elements of Propulsion Gas Turbines and Rockets, AIAA Educational Series, USA, pp.65 -260, 2006.

[4] M. J. E. Ebadi and M. Gorji-Bandpy, "Exergetic Analysis of Gas Turbine Plants,"International Journal of Exergy, Vol.2, No.1, 2005.

[5] J. A. Camberos, and D. J. Moorhouse (Eds.), Progress in Astronautics and Aeronautics: Exergy Analysis and Design Optimization for Aerospace Vehicles and Systems, Vol. 238, AIAA, Reston, VA, pp. 9-160, 2011.

[6] Y. A. Cengel and M. A. Boles, Thermodynamics An Engineering Approach, fifth Edition, McGraw-Hill, Boston, MA, pp. 423-550, 2006.

[7] G. Wall, Exergetics, http/exergy.se