# Unitary Pole Approximation For $^{16}$O ($S_{1/2}$state) And $^{40}$ca ($P_{3/2}$state) When Coulomb Interaction Is Included

A. Acharya, R.L.Nayak, T.Sahoo

**Abstract:** The form factor of a separable interaction between a pair of particles is an important input in a three body calculation for a transfer reaction. The three body equations of Alt, Grassberger and Sandhas have been solved for a system of three particles viz.(p, n and $^{16}$O)and (p, n and $^{40}$Ca) when coulomb interaction is included between the particle pairs. The input in this calculation i.e. the two body t-matrices representing the interaction between the pairs of particles is taken to be of a separable form conforming to the bound state of the pair. The form factors of the total interaction between the particle pairs are constructed using the prescription of Ueta and Bund.

**Index Terms:** Two-body matrices, form factors, AGS equations, UPA, BSA, Yamaguchi potential Pacs nos: 21.10.-k, 21.90.

## Introduction

Consider a transfer reaction of the kind

$$A + a\ (b + X) = B\ (A + X) + b$$

Where the particle a is composite, consisting of the outgoing particle b and the transmitted particle X. the interaction between the pairs of particles enter into the AGS form of Faddeev equations [1] through the two body transition operators in the three body space. The bound state approximation (BSA) implies that in the spectral resolution of $T_k(z)$ only the contribution of the bound state of the interacting pair is retained and that due to the continuum states of the latter is ignored. As a result, the matrix representing the operator $T_k(z)$ in the momentum representation becomes separable resulting in considerable simplification of an AGS equation when written in the angular momentum representation. These equations are solved provided one takes proper care of the singularities. Recent information about this can be obtained from Phys Rev C 71 024605 (2005) [15] The author Navratil say about the nuclear structure of few body system [16] and Bernard V.et.al derived the long range contributions to the three nucleon force at next to next-next to leading order in the chiral expansion in both momentum and co-ordinate space representations [17],[18].


- *Acharya is working as Assistant Professor in Department of physics, Veer Surendra Sai University of Technology Burla- 768018, Odisha, India*
- *Current interests: Nuclear structure and Reactions, Few body problems, Halo Nuclei, Super Heavy Elements with applications. E-mail: aacharya_phy@vssut.ac.in*
- *R.L.Nayak & T.Sahoo are working with Dr.A.Acharya as a Research Scholars.*


In the article new Horizon in Ab initio nuclear structure theory, the author Robert Roth et al. described about the recent key developments that facilitate ab initio calculations of ground and low-lying excited states of p- and sd-shell nuclei with full 3N interactions in the importance truncated no-core shell model using consistency [19],[20].

## AGS equation in one dimensional form

$$U_{ij}(z) = \left(1-\delta_{ij}\right)(z-H_0) + \sum_{k=1}^{3}(1-\delta_{ik})T_k(z)U_{kj}(z) \quad (1)$$

Where

$$T_k(z) = V_k + V_k G_k(z) V_k \quad \text{---- (2)}$$

And

$$G_k(z) = (z-H_k)^{-1} \quad (3)$$

Where $H_0$ is the free Hamiltonian, $T_k$ is the two-body t-matrices in three-body space and $G_k$ is the resolvant operator. The AGS equation in operator form can be written in integral equation form [4] using a suitable basis of representation known as Free State representation in momentum space. When the three-body AGS equation is written in momentum basis it would assume the form of coupled equation with six variables of integration and seven continuous parameters including one of energy. An angular momentum basis is chosen in order to reduce the parameters to get the solution in a more convenient form [8], [9].

$$|\alpha_i : JM> = |(L_iS_i)J_i : JM> = |(L_iS_i)\beta_i : JM> \quad \text{-- (4)}$$

$L_i$, $S_i$ denote the relative orbital momentum and spin quantum number of the i$^{th}$ pair whose total angular quantum number $J_i$ is coupled to the spin $S_i$ of the i$^{th}$ particle to give the channel spin $k_i$ which in turn coupled to $l_i$, the orbital angular momentum of i$^{th}$ particle with respect to the i$^{th}$ pair to give the total angular momentum JM of the three body system. The angular momentum basis state satisfies the twin requirements of completeness and orthogonality viz.

$$\sum \alpha_i \iint |p_i q_i \alpha_i : JM > p_i{}^2 q_i{}^2 dp_i dq_i < p_i q_i \alpha_i : JM| = 1 \quad \text{---- (5)}$$

And

$$< p_i q_i \alpha_i : JM | p_i' q_i' \alpha_i' : J' M' \geq$$

$$\delta(p_i - p_i')\delta(q_i - q_i')\delta_{\alpha\alpha'}\delta_{jj'}\delta_{mm'}/p_i{}^2 q_i{}^2 \quad \text{--- (6)}$$

By taking separable approximation for the two-body t-matrix in two-body space the two dimensional AGS equation is reduced to a set of one dimensional equation. When the potential is not separable, a separable approximation given by Lovelace [3] is that if bound or unbound states are present, the t-matrix may be approximately replaced by a set of separable terms each reproducing the behavior of t-matrix nears a bound state pole. Many workers tested BSA and showed that the neglect of continuum part in the spectral representation of $T_k(z)$ leads to poor and unacceptable description of the scattering process. Greven and Levin [10] also tested this approximation in Mitra's three-body model of stripping reaction and they concluded that this approximation was satisfactory only for incident energies closer to the break up threshold, which is anticipated in view of the fact that the neglect of the continuum terms in the BSA tend to become significant at energies greater than the threshold value, thus implies the agreement of BSA [2]. The singularities appearing in these derivations are overcome by the methods outlined by Sasakawa [13] and Kowalski [6]. The logarithmic singularity is taken care of by Doleschall's techniques [7]

**Unitary pole approximation for $^{16}$O and $^{40}$ca**

The t-matrix associated with the interaction between a pair of particles is dominated by the bound state pole of the pair. In this case, the interaction between the particles pair can be approximated by the separable potential

$$U = U_{Sep} = U|\Psi >< \Psi| \quad \text{---- (7)}$$

The two-body t-matrix corresponding to $U_{sep}$ is naturally separable and provides a good approximation to the exact t-matrix pertaining to the interaction. We consider U as being the single particle potential of the proton and a massive nucleus and one can put $U=V_s+V_c$, where $V_s$ is the short range part and $V_c$ is the coulomb part. For simplicity it is assumed that the short range part is already separable and acts in a specific $l_j$ orbit of the shell model. The long range part, coulomb potential is not separable. Let us consider the interaction consisting of only short range potential (say the interaction between a neutron and the nucleus). Then Schrodinger equation in momentum space is written as

$$\left(\frac{P^2}{2m} - E_n\right)\phi_n(p) = \iiint \frac{\lambda_{lj}}{2m} g_{lj}(p) g_{lj}(p')$$

$$\sum_{\mu} < p|\gamma_{lj\mu} >< \gamma_{lj\mu}|p' > \emptyset_n(p') d^3p' \quad \text{------ (8)}$$

Where p is the momentum of the proton having mass m and

$$< P\left|\gamma_{lj\mu} > = \sum_{m_l m_s}\left(lm_l \tfrac{1}{2} m_s : j\mu\right)\gamma_l(p)\right|\tfrac{1}{2} m_s > \text{-----(9)}$$

Now if we consider coulomb interaction in addition to the short range interaction i.e. the interaction between the closed shell nucleus and an orbital proton, then with this choice the two-body problem can be solved exactly. If $\varepsilon_{lj}$ is the energy of the bound state lj in the potential given by

$$U = V_s + V_c$$

And $k^2 = -2m\varepsilon_{lj}$, S= -2mz$e^2$/2

Then the corresponding bound state wave function in the momentum space is given by

$$\psi_{lj\mu}^{BS}(P) = N_{lj}\frac{1}{(p^2+k^2)}\left[g_{lj}(p) - V_{lj}^c(p)\right]\gamma_{lj\mu}(p) \quad \text{------(10)}$$

Where

$$V_{lj}^c(\mathrm{p}) = \frac{1}{p}\left[\frac{4pk^2}{(p^2+k^2)(\beta^2-k^2)}\right]^{l+1}$$
$$\sum_{n=l+1}^{\infty} \frac{\frac{s}{k}}{(n-\frac{s}{k})}\left[\frac{(\beta-k)}{(\beta+k)}\right]^n C_{n-l-1}^{l+1}\left[\frac{(p^2-k^2)}{(p^2+k^2)}\right] \quad \text{----(11)}$$

The separable potential $U_{Sep}$ which generates the same bound state as the potential given by Ueta and Bund [14] is

$$< p|U_{Sep}|p' \geq -\frac{\lambda}{2m} g_{lj}(p) g_{lj}^c(P')$$
$$\sum_{\mu} < p|\gamma_{lj\mu} >< \gamma_{lj\mu}|p' > \quad \text{-------- (12)}$$

Where

$g_{lj}^c(\mathrm{p}) = g_{lj}(p) - V_{lj}^c(p)$ And $\lambda_{lj}$ is determined by requiring that the bound state has the energy eigen value $\varepsilon_{lj}$ .

## Result and discussion

The AGS equation is written in solvable one dimensional coupled integral equation by choosing a suitable angular momentum basis and invoking a separable approximation to the two-body t-matrix. Following the methods outlined by Ueta and Bund, the t-matrix is written in separable form for an interaction which consists of short range as well as long range parts. With this prescription the three body AGS equations are reduced to a set of one-dimensional coupled integral equation which are solvable even when coulomb interaction is present. For $^{17}$F($S_{1/2}$state) and $^{41}$Sc($P_{3/2}$state) we first find the best values of the parameter $\beta$ of the separable interaction which produced the bound state wave function for $^{17}O$ and $^{41}Ca$ to match with the wave function found by adjusting Wood Saxon parameters. The same value of the parameter $\beta$ are used to delineate the strong (separable) interaction between p and $^{16}O$ in $^{17}F$, p and $^{40}Ca$ in $^{41}Sc$ nuclei respectively. In addition, coulomb interaction is also taken into account and the form factors of the total interaction between p and $^{16}O$, and between p and $^{40}Ca$ are constructed using the prescription of Ueta and Bund. It is also seen from the momentum versus form factor graphs (fig.1 and fig.2) that with the increasing value of momentum, the values of form factors $g_{lj}(p)$ and $g_{lj}^{c}(p)$ first increase and then decrease and tend to zero. The difference between the form factors which is termed as $V_{lj}^{c}(p)$ also shows the same behavior. Same features are observed in both cases of $^{17}O$ and $^{41}Ca$. one can calculate the form factor for other pair of particles provided appropriate input data is available. Our future plan is to extend the present calculation to other levels of $^{17}O$ as well as $^{41}Ca$ and further to apply to describe other values of reaction.

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## Figures

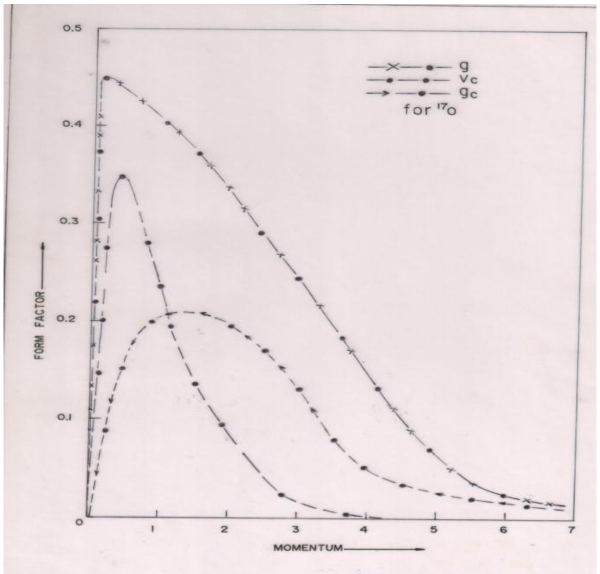


*Fig 1.* Form factor vs. Momentum for $^{17}O$

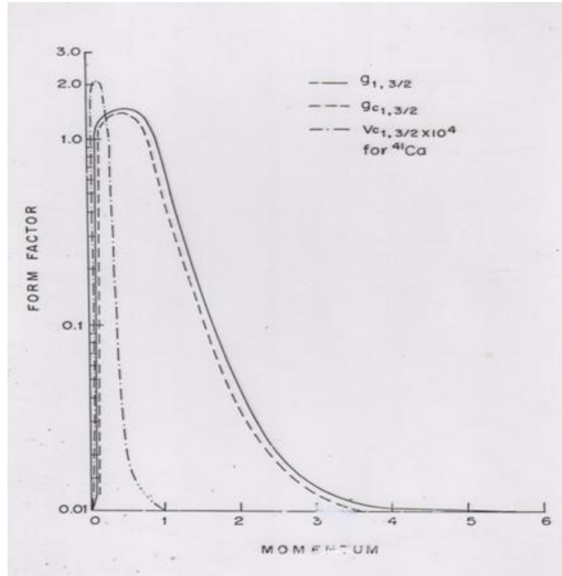


*Fig 2.* Form Factor vs. Momentum for $^{41}Ca$