Generalized 2-Complement Of Set Domination

P. Sumathi, T. Brindha

Abstract: Let G=(V,E) be a simple, undirected, finite nontrivial graph. A set $S\subseteq V$ of vertices of a graph G = (V, E) is called a dominating set if every vertex $v\in V$ is either an element of S or is adjacent to an element of S. A set $S\subseteq V$ is a set dominating set if for every set $T\subseteq V-S$, there exists a non-empty set $R\subseteq S$ such that the subgraph <RUT> is connected. The minimum cardinality of a set dominating set is called set domination number and it is denoted by γ_s (G).Let $P=(V_1,V_2)$ be a partition of V,from E(G) remove the edges between V_1 and V_2 in G and join the edges between V_1 and V_2 which are not in G. The graph G_2^p thus obtained is called 2-complement of G with respect to 'P'.

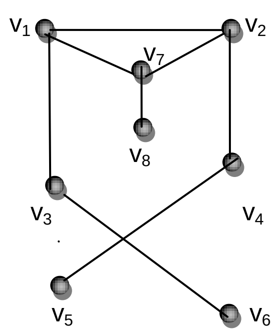
Key words: Dominating set, set dominating set, 2-complement of G.

Introduction: 1.1

Let G=(V,E) be a simple, undirected, finite nontrivial graph and $P= (V_1,V_2,\ldots, V_K)$ be a partition of V of order k>1.The k-complement G_k^p of G (with respect to P) is defined as follows: For all V_i and V_j in P $i\neq j$ remove the edges between V_i and V_j in G and join the edges between V_i and V_j which are not in G. The graph thus obtained is called the k-complement of G with respect to P. In this paper 2-complement is considered. Let G=(V,E) be a connected graph. A set $S\subseteq V$ is a set dominating set if for every set $T\subseteq V-S$, there exists a non-empty set $R\subseteq S$ such that the subgraph <RUT> is connected. The minimum cardinality of a set dominating set is called set domination number and it is denoted by γ_s (G). In the following example the set domination number γ_s is calculated.

Example:1.2

Consider the following graph G:



Let V= $\{v_1, v_2\ldots\ldots v_n\}$ be the vertices of G. S= $\{v_2, v_3,v_4,v_7\}$.For every $T\subseteq V-S$ there exists a nonempty set $R\subseteq S$ such that <RUT> is connected.

Here, γ_s (G) = 4.

Theorem: 1.3

When G=K_n ($n\geq3$), $\gamma_s (G_2^p)= 1 +\min \{|V_1|, |V_2|\}$.

Proof:-

Let $V(K_n)= \{v_1,v_2,\ldots\ldots v_n\}$ and (V_1,V_2) be a partition of G. Suppose

$V_1= \{v_1\}$ & $V_2= \{v_2, v_3\ldots.v_n\}$.Here < $\{v_2, v_3\ldots\ldots v_n\}$> form a complete graph with n-1 vertices in G_2^p.Also v_1 is not adjacent to v_i's, $2 \leq i \leq n$. Here a

γ_s – set is $\{v_1, v_2\}$. Therefore $\gamma_s (G_2^p) = 2=1+1=1+ \min\{1,n-1\}=1+ \min \{|V_1|, |V_2|\}$.

Suppose $V_1 = \{v_1, v_2\}$ & $V_2= \{v_3, v_4\ldots.v_n\}$. Then in G_2^p , < $\{v_3, v_4\ldots\ldots v_n\}$ form a complete graph with n-2 vertices. Also v_1 & v_2 are adjacent. And in G_2^p , < v_1, v_2> and < $v_3, v_4\ldots.v_n$> are disjoint. Here γ_s – set is $\{v_1, v_2, v_3\}$for G_2^p.Hence $\gamma_s (G_2^p)$ =3=1+2=1+ min{2,n-2}=1+ min $\{|V_1|, |V_2|\}$.

Suppose $V_1 = \{v_1, v_2, v_3\}$ & $V_2 = \{v_4, v_5\ldots\ldots v_n\}$.Then the induced subgraph < $\{v_1, v_2, v_3\}$> form a complete graph with 3 vertices the induced subgraph <$\{v_4,v_5\ldots\ldots v_n\}$> form a complete graph with n-3 vertices. And in G_2^p , < v_1, v_2, v_3> and < $v_4, v_5\ldots\ldots v_n$ > are disjoint. Here a γ_s – set is $\{v_1, v_2, v_3, v_4\}$. Here $\gamma_s (G_2^p)$ =4=1+3=1+ min{3,n-3}=1+ min $\{|V_1|, |V_2|\}$.if $n\geq6$(that is $n-3\geq3$)or $\{v_1\}\cup V_2$ for a γ_s – set if $3\leq n<6$)

Suppose $V_1 = \{v_1,v_2,\ldots,v_m\}$&$V_2=\{v_{m+1},v_{m+2}\ldots\ldots v_n\}$.(m>3).Without loss of generality $|V_1|< |V_2|$.

Then the induced subgraph < $\{v_1, v_2,\ldots, v_m\}$> and <$\{v_{m+1},v_{m+2}\ldots\ldots v_n\}$> form a complete graph in G_2^p. And in G_2^p , < $\{v_1, v_2,\ldots, v_m\}$> and< $v_{m+1},v_{m+2}\ldots\ldots v_n$ > are disjoint .Here a γ_s – set is $\{v_1, v_2, \ldots, v_m,v_{m+1}\}$. Here $\gamma_s (G_2^p)$ =m+1=1+m==1+ min $\{|V_1|, |V_2|\}$

Suppose $|V_1| > |V_2|$ then $\{ v_1,v_{m+1},v_{m+2},\ldots v_n\}$ form a complete graph in G_2^p. $\{v_1\}\cup V_2$ is a γ_s – set Therefore by the same procedure it can be proved that if $V_1 = \{v_1,v_2\ldots\ldots v_{n-3}\},V_2 = \{v_{n-2}, v_{n-1}, v_n\}$ .Then in G_2^p, a γ_s – set is $\{v_1, v_{n-2}, v_{n-1}, v_n\}$.Hence $\gamma_s (G_2^p)$ = 4=1+3=1+ min{3,n-3}=1+ min $\{|V_1|, |V_2|\}$ if $|V_1| > |V_2|$.

Suppose $|V_1| < |V_2|$ then $V_1\cup\{v_n\}$ is a γ_s – set. Therefore $\gamma_s (G_2^p)$ = n-3+1=1+n-3=1+ min $\{|V_1|, |V_2|\}$.

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If $V_1=\{v_1, v_2........v_{n-2}\}$ and $V_2=\{v_{n-1}, v_n\}$.Then in G_2^p, $<\{v_1, v_2........v_{n-2}\}>$ is a complete graph with n-2 vertices and v_{n-1} and v_n are adjacent. Hence γ_s – set is $\{v_1, v_{n-1}, v_n\}$. Therefore $\gamma_s(G_2^p)=3=1+2=1+\min\{2,n-2\}=1+\min\{|V_1|, |V_2|\}$ if $|V_1|>|V_2|$.

Suppose $|V_1|<|V_2|$ then $V_2\cup\{v_1\}$ is a γ_s – set. Therefore $\gamma_s(G_2^p)=n-2+1=1+n-2==1+\min\{|V_1|, |V_2|\}$.

If $V_1=\{v_1, v_2..........v_{n-1}\}$ and $V_2=\{v_n\}$ then in G_2^p,$<\{v_1, v_2,.........,v_{n-1}\}>$ is a complete graph with n-1 vertices and v_n is not adjacent to $v_1, v_2,.........,v_{n-1}$,the corresponding γ_s – set is $\{v_1, v_n\}$. Therefore $\gamma_s(G_2^p)=2=1+1=1+\min\{n-1,1\}=1+\min\{|V_1|, |V_2|\}$. Hence $\gamma_s(G_2^p)=1+\min\{|V_1|, |V_2|\}$.

Theorem : 1.4

Let G be a Complete bipartite graph with partition (V_1,V_2), where $V_1=\{u_1,u_2, ...u_m\}$ and $V_2=\{v_1,v_2,\ldots,v_n\}$and $m\le n$. Let (W_1,W_2)be a partition of $V(G_2^p)$ then

$$\gamma_s(G_2^p)=\begin{cases} 1 & \text{if } |W_k|=V_1-\{u_i\} \text{ or } W_k=V_2-\{u_j\} \text{ where } 1\le i\le m \text{ and } 1\le j\le n \text{ and } k=1,2 \\ n+m & \text{if } W_k=V_1 \text{ or } V_2 \text{ for } k=1,2 \\ 2 & \text{Otherwise.}\end{cases}$$

Proof:
Let $m\le n$.
case 1:

If $W_1=\{u_1,u_2, ...u_{m-1}\}$ and $W_2=\{u_m,v_1,v_2,\ldots,v_m\}$ then in G_2^P, um is adjacent to all other vertices. Therefore γ_s-set is {um}. Hence

$\gamma_s(G_2^p)=1$.

If $W_1=\{v_1,v_2, ...v_{n-1}\}$ and $W_2=\{v_n,u_1,u_2,\ldots,u_m\}$ then in G_2^P, v_n is adjacent to all other vertices. Therefore γ_s-set is $\{v_n\}$. Hence $\gamma_s(G_2^p)=1$.

Case:2

If $W_1=\{u_1,u_2, ...u_m\}$ and $W_2=\{v_1,v_2,\ldots,v_n\}$ then G_2^P is the disjoint union of an isolated vertices. Therefore γ_s-set is $\{u_1,u_2, ...u_m,v_1,v_2,\ldots,v_n\}$. Hence $\gamma_s(G_2^p)=n+m$

Case:3

If $W_1=\{u_1\}$ and $V_2=\{u_2,u_3,\ldots u_m,v_1,v_2,\ldots,v_n\}$ then u_1 is adjacent to $u_2,u_3,\ldots u_m$ and u_2 is adjacent to $v_1,v_2,\ldots v_n$. Therefore γ_s-set is $\{u_1,u_2\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{u_2\}$ and $V_2=\{u_1,u_3,u_4\ldots u_m,v_1,v_2,\ldots,v_n\}$ then u_2 is adjacent to $u_1,u_3,\ldots u_m$ and u_1 is adjacent to $v_1,v_2,\ldots v_n$. Therefore γ_s-set is $\{u_1,u_2\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this if $W_1=\{u_m\}$ and $V_2=\{u_1,u_2,\ldots u_{m-1},v_1,v_2,\ldots,v_n\}$ then u_m is adjacent to $u_1,u_2,\ldots u_{m-1}$ and u_1 is adjacent to $v_1,v_2,\ldots v_n$ and u_{m-1} is adjacent to $v_1,v_2,\ldots,v_n$. Therefore a γ_s-set is $\{u_1,u_m\}$. Hence

$\gamma_s(G_2^p)=2$.

And if $W_1=\{u_1,u_2\}$,$W_2=V\backslash W_1$ then u_1 is adjacent to $u_3,u_4\ldots u_m$ and u_2 is adjacent to $u_3,u_4\ldots u_m$. Also u_3 is adjacent to $v_1,v_2,\ldots,v_n\ldots u_m$ is adjacent to $v_1,v_2,\ldots,v_n$. Therefore a γ_s-set is $\{u_1,u_3\}$. Hence $\gamma_s(G_2^p)=2$.

And if $W_1=\{u_1,u_3\}$, $W_2=V\backslash W_1$then u_1 is adjacent to $u_4,u_5\ldots u_m$ and u_2 is adjacent to $v_1,v_2,\ldots,v_n$. Therefore a γ_s-set is$\{u_1,u_2\}$. Hence $\gamma_s(G_2^p)=2$.

And if $W_1=\{u_1,u_4\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to $u_2,u_3\ldots u_{m-1}$ and u_2 is adjacent to $v_1,v_2,\ldots,v_n$ Therefore a γ_s-set is$\{u_1,u_2\}$. Hence $\gamma_s(G_2^p)=2$.

And if $W_1=\{u_2,u_3\}$, $W_2=V\backslash W_1$ then u_2 is adjacent to $u_1,u_4,u_5\ldots u_m$ and u_3 is adjacent to $v_1,v_2,\ldots,v_n$ Therefore a γ_s-set is $\{u_2,u_3\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this, if And if $W_1=\{u_{m-1},u_m\}$, $W_2=V\backslash W_1$ then u_{m-1} is adjacent to $u_1,u_2\ldots u_{m-2}$ and u_1 is adjacent to $v_1,v_2,\ldots,v_n,u_m$ .Therefore a γ_s-set is$\{u_1,u_{m-1}\}$. Hence $\gamma_s(G_2^p)=2$.

Also,If $W_1=\{u_1,v_1\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to $v_1,u_2,u_3,\ldots u_m$ and u_2 is adjacent to $v_1,v_2,\ldots,v_n$ Therefore a γ_s-set is$\{u_1,u_2\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{u_1,u_2,v_1\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to $u_3,u_4,\ldots u_m$ and u_3 is adjacent to $u_1,u_2,v_2,v_3,\ldots,v_n$ Therefore a γ_s-set is $\{u_1,u_3\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{u_1,u_2,v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to $u_3,u_4,\ldots u_m,v_{n-1},v_n$ and u_3 is adjacent to u_1,u_2,v_1,v_2 Therefore a γ_s-set is $\{u_1,u_3\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{u_1,u_m,v_{n-1},v_1\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to u_2,u_3,v_{n-1},v_n and u_2 is adjacent to$v_1,v_2,u_1,u_4,u_5,\ldots,u_m$ Therefore a γ_s-set is $\{u_1,u_2\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this if $W_1=\{u_{m-1},u_m,v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then u_{m-1} is adjacent to $u_1,u_2,\ldots u_{m-2},v_{n-1},v_n$ and u_1 is adjacent to $v_1,v_2,\ldots v_{n-2}$ .Therefore a γ_s-set is $\{u_1,u_{m-1}\}$. Hence

$\gamma_s(G_2^p)=2$.

The above discussion have explained all the possibilities and the theorem is proved.

Theorem:1.5

Let G be a star ($K_{1,n}$ where $n\ge3$).Let u be the star center and $u_1,u_2,\ldots,u_n$ be the pendant of G. Let (W_1,W_2) be the partition of G_2^p.

Then γ_s (G_2^p)= 1 if $W_k=\{u,u_j\}$, $1\le j\le n$, k=1,2
n+1 if $W_k=\{u\}$,k=1,2
2 otherwise

Proof:
case:1

If $W_1=\{u,u_1\}$, $W_2=V\backslash W_1$ then u_1 is adjacent to all other vertices. Also , G_2^p is again a star with u_1 is the star center. Therefore a γ_s-set is$\{u_1\}$. Hence γ_s (G_2^p)= 1.

If $W_1=\{u,u_2\}$, $W_2=V\backslash W_1$ then u_2 is adjacent to all other vertices. Also , G_2^p is again a star with u_2 is the star center. Therefore a γ_s-set is$\{u_2\}$. Hence γ_s (G_2^p)= 1.

Proceeding like this, if $W_1=\{u,u_n\}$, $W_2=V\backslash W_1$ then u_n is adjacent to all other vertices. Also , G_2^p is again a star with u_n is the star center. Therefore a γ_s-set is$\{u_n\}$. Hence γ_s (G_2^p)= 1.

case:2

If $W_1=\{u\}$ and $W_2=\{u_1,u_2,\ldots u_n\}$ then G_2^p is the disjoint union of an isolated vertices. G_2^p is a disconnected graph. Therefore a γ_s-set is$\{u,u_1,u_2,\ldots u_n\}$. Hence γ_s (G_2^p)=n+ 1.

case:3

If $W_1=\{u_1,u_2\}$, $W_2=V\backslash W_1$ then u_2 is adjacent to $u_3,u_4\ldots u_n$ and u_2 is adjacent to $u_3,u_4\ldots u_n$ .Also u is adjacent to $u_3,u_4\ldots u_n$. Therefore a

γ_s-set is $\{u,u_n\}$. Hence γ_s (G_2^p)= 2.

If $W_1=\{u_2,u_3\}$, $W_2=V\backslash W_1$ then u_2 is adjacent to $u_1,u_4,u_5\ldots u_n$ and u_3 is adjacent to $u,u_4,u_5\ldots u_n$ Also u is adjacent to u_1,u_4, $u_5\ldots u_n$. Therefore a γ_s-set is $\{u,u_n\}$. Hence γ_s (G_2^p)= 2.

Proceeding like this,

If $W_1=\{u_{n-1},u_n\}$, $W_2=V\backslash W_1$ then u_{n-1} is adjacent to $u_1,u_2,\ldots u_{n-2}$ and u_n is adjacent to $u_1,u_2\ldots u_{n-2}$ Also u is adjacent to $u_1,u_2,\ldots u_{n-2}$. Therefore a

γ_s-set is $\{u,u_{n-2}\}$. Hence γ_s (G_2^p)= 2.

Also, if $W_1=\{u_1\}$ and $V_2=V\backslash W_1$ then u_1 is adjacent to $u_2,u_3,\ldots u_m$ and u is adjacent to $u_2,u_3,\ldots u_n$ Therefore γ_s-set is $\{u,u_1\}$. Hence

γ_s (G_2^p)= 2.

If $W_1=\{u_2\}$ and $V_2=V\backslash W_1$ then u_2 is adjacent to $u_1,u_3,u_4,\ldots u_m$ and u is adjacent to $u_1,u_3,u_4,\ldots u_n$. Therefore γ_s-set is $\{u,u_2\}$. Hence

γ_s (G_2^p)= 2.

Proceeding like this , if $W_1=\{u_n\}$ and $V_2=V\backslash W_1$ then u is adjacent to $u_1,u_2,\ldots u_{n-1}$ and u_n is adjacent to $u_1,u_2,\ldots u_{n-1}$. Therefore γ_s-set is $\{u,u_n\}$. Hence γ_s (G_2^p)= 2.

Theorem: 1.6
Let G be a Cycle with n vertices (n=4) say $v_1, v_2.........v_n$.

Then γ_s $(G_2^{\ p})$= 1 if $W_k=\{v_j\}$, $1\le j\le n$, k=1,2
2 if $W_k=\{v_j\cup N(v_j)\}$, $1\le j\le n$, k=1,2
n if W_k has alternative vertices for k=1,2

Proof:
case:1
If $W_1=\{v_1\}$ and $V_2=\{v_2,v_3,v_4\}$ then v_3 is adjacent to all other vertices. Therefore γ_s-set is $\{v_3\}$. Hence γ_s (G_2^p)= 2.

If $W_1=\{v_2\}$ and $V_2=V\backslash W_1$ then v_4 is adjacent to all other vertices. Therefore γ_s-set is $\{v_4\}$. Hence γ_s (G_2^p)= 1.

If $W_1=\{v_3\}$ and $V_2=V\backslash W_1$ then v_1 is adjacent to all other vertices. Therefore γ_s-set is $\{v_1\}$. Hence γ_s (G_2^p)= 1.

If $W_1=\{v_4\}$ and $V_2=V\backslash W_1$ then v_2 is adjacent to all other vertices. Therefore γ_s-set is $\{v_2\}$. Hence γ_s (G_2^p)= 1.

case:2

If $W_1=\{v_1,v_2\}$, $W_2=V\backslash W_1$ then G_2^p is again a cycle with four vertices. Here v_1 and v_2 are adjacent vertices. Therefore a γ_s-set is $\{v_1,v_2\}$. Hence γ_s (G_2^p)= 2.

If $W_1=\{v_2,v_3\}$, $W_2=V\backslash W_1$ then in G_2^p v_2 and v_3 are adjacent vertices. Therefore a γ_s-set is $\{v_2,v_3\}$. Hence γ_s (G_2^p)= 2.

case:3

If $W_1=\{v_1,v_3\}$, $W_2=V\backslash W_1$ then G_2^p is union of isolated vertices. Therefore a γ_s-set is $\{v_1,v_2 ,v_3,v_4\}$. Hence γ_s (G_2^p)= 2.

Theorem: 1.7
Let G be a Cycle with n vertices (n=5) say $v_1, v_2.........v_n$.

Then γ_s $(G_2^{\ p})$= 1 if $W_k=\{v_j\cup N(v_j)\}$, $1\le j\le n$, k=1,2
2 if $W_k=\{v_j\}$, $1\le j\le n$, k=1,2
3 if W_k has alternative vertices for k=1,2

Proof:
case:1

If $W_1=\{v_1,v_2\}$, $W_2=V\backslash W_1$ then v_4 is adjacent to all other vertices. Also v_1 and v_2 are adjacent vertices. Therefore a γ_s-set is $\{v_4\}$. Hence

γ_s (G_2^p)= 1.

If $W_1=\{v_2,v_3\}$, $W_2=V\backslash W_1$ then v_5 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_5\}$. Hence γ_s (G_2^p)= 1.

If $W_1=\{v_3,v_4\}$, $W_2=V\backslash W_1$ then v_1 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_4\}$. Hence γ_s (G_2^p)= 1.

If $W_1=\{v_4,v_5\}$, $W_2=V\backslash W_1$ then v_2 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_2\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_1,v_5\}$, $W_2=V\backslash W_1$ then v_3 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_3\}$. Hence $\gamma_s(G_2^p)=1$.

case:2

If $W_1=\{v_1\}$ and $V_2=V\backslash W_1$ then there exists a path from v_2 to v_5 and v_1 is adjacent to v_3 and v_5. Therefore γ_s-set is $\{v_3,v_4\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_2\}$ and $V_2=V\backslash W_1$ then there exists a path from v_3 to v_5 and v_1 is adjacent to v_5.Also v_2 is adjacent to v_4 and v_5. Therefore γ_s-set is $\{v_4,v_5\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_3\}$ and $V_2=V\backslash W_1$ then v_3 is adjacent to v_1 and v_5.Also v_1 is adjacent to v_2 and v_5. Also v_4 is adjacent to v_5. Therefore γ_s-set is $\{v_1,v_5\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_4\}$ and $V_2=V\backslash W_1$ then there exists a path from v_1 to v_3 and v_4 is adjacent to v_1 and v_3.Also v_1 is adjacent to v_5. Therefore γ_s-set is $\{v_1,v_2\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_5\}$ and $V_2=V\backslash W_1$ then there exists a path from v_1 to v_4 and v_5 is adjacent to v_2 and v_3. Therefore γ_s-set is $\{v_2,v_3\}$. Hence $\gamma_s(G_2^p)=2$.

case:3

If $W_1=\{v_2,v_4\}$, $W_2=V\backslash W_1$ then v_3 is an isolated vertex. And v_1 is adjacent to v_4 and v_5.Also v_2 is adjacent to v_5. Therefore a γ_s-set is $\{v_1,v_3,v_5\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_1,v_3\}$, $W_2=V\backslash W_1$ then v_2 is an isolated vertex. And v_1 is adjacent to v_4 .Also v_4 is adjacent to v_5 and v_3 is adjacent to v_5. Therefore a γ_s-set is $\{v_2,v_4,v_5\}$. Hence $\gamma_s(G_2^p)=3$.

Theorem: 1.8
Let G be a Cycle with n vertices ($n\geq 6$) say $v_1, v_2.........v_n$.

Then $\gamma_s(G_2^p)=$
- 1 if $W_k=\{v_s,v_{s+1},v_{s+2}\}$, $1\leq j\leq n$, k=1,2
- 2 if $W_k=\{v_1,v_2\}$or $\{v_{n-1},v_n\}$ or $\{v_1,v_n\}$ or $\{v_s,v_{s+1},v_{s+2},v_{s+3}\}$ $1\leq j\leq n$, k=1,2
- 3 if $W_k=\{v_j\}$, $1\leq j\leq n$, k=1,2 or $\{v_r,v_s\}$ where v_r and v_s are alternative vertices.
- 4 if W_k contains alternative vertices when n is o odd for k=1,2
- 5 if W_k contains alternative vertices when n is e even for k=1,2

Proof:

Let $v_1, v_2.........v_n$ are such that v_i and v_{i+1} are adjacent vertices where

$1\leq i\leq n-1$ and v_n is adjacent to v_1.

If $W_1=\{v_1,v_2,v_3\}$, $W_2=V\backslash W_1$ then v_2 is adjacent to all other vertices of W_2. Therefore a γ_s-set is $\{v_2\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_2,v_3,v_4\}$, $W_2=V\backslash W_1$ then v_3 is adjacent tov_2 and v_4 and all other vertices of W_2. Therefore a γ_s-set is $\{v_3\}$. Hence $\gamma_s(G_2^p)=1$.

Proceeding like this

If $W_1=\{v_{n-2},v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then v_{n-1} is adjacent tov_{n-2} and v_n and all other vertices of W_2. Therefore a γ_s-set is $\{v_{n-1}\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_{n-1},v_n,v_1\}$, $W_2=V\backslash W_1$ then v_n is adjacent tov_{n-1} and v_1 and all other vertices of W_2. Therefore a γ_s-set is $\{v_n\}$. Hence $\gamma_s(G_2^p)=1$.

Case:2

If $W_1=\{v_1,v_2\}$, $W_2=V\backslash W_1$ then v_1 is adjacent to $v_3,v_4,\ldots,v_{n-1}$ and v_2 is adjacent to $v_4,v_5,\ldots v_n$. Also there exists a path from v_3 to v_n and v_1 is adjacent to v_2. Therefore a γ_s-set is $\{v_1,v_2\}$.Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_2,v_3\}$, $W_2=V\backslash W_1$ then v_2 is adjacent to $v_3,v_4,\ldots,v_n$ and v_3 is adjacent to $v_1,v_5,v_6\ldots v_n$. Therefore a γ_s-set is $\{v_2,v_3\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this,

If $W_1=\{v_1,v_n\}$, $W_2=V\backslash W_1$ then v_1 is adjacent to $v_3,v_4,\ldots,v_{n-1}$ and v_n is adjacent to $v_2,v_3,\ldots v_{n-1}$. Also there exists a path from v_2 to v_6 and v_1 is adjacent to v_n. Therefore a γ_s-set is $\{v_1,v_n\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then v_{n-1} is adjacent to $v_2,v_3,\ldots,v_{n-3}$ and v_n is adjacent to $v_2,v_3,\ldots v_{n-2}$. Also there exists a path from v_1 to v_{n-2} and v_{n-1} is adjacent to v_n. Therefore a γ_s-set is $\{v_{n-1},v_n\}$. Hence $\gamma_s(G_2^p)=2$.

Also if $W_1=\{v_2,v_3,v_4,v_5\}$, $W_2=V\backslash W_1$ then v_3 is adjacent to $v_1,v_2,v_4,v_6,v_7,\ldots v_n$ and v_4 is adjacent to $v_1,v_3,v_5\ldots v_n$. Also there exists a path from v_2 to v_5 and v_6 to v_n. Therefore a γ_s-set is $\{v_3,v_4\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{v_3,v_4,v_5,v_6\}$, $W_2=V\backslash W_1$ then v_4 is adjacent to $v_1,v_2,v_3,v_5,v_7,v_8\ldots,v_n$ and v_5 is adjacent to $v_1,v_2,v_4,v_6,v_7,\ldots v_n$. Therefore a γ_s-set is $\{v_4,v_5\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this, if $W_1=\{v_{n-3},v_{n-2},v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then v_{n-2} is adjacent to $v_1,v_2,v_3,\ldots,v_{n-4},v_{n-3},v_{n-1}$ and v_{n-1} is adjacent to

$v_1,v_2,v_3,\ldots,v_{n-4},v_{n-2},v_n$. Therefore a γ_s-set is $\{v_{n-2},v_{n-1}\}$. Hence

$\gamma_s(G_2^p)=2$.

case: 3

If $W_1=\{v_1\}$ and $V_2=V\backslash W_1$ then v_1 is adjacent to $v_3,v_4,\dots,v_{n-1}$ and there exists a path from v_2 to v_n. Therefore a γ_s-set is $\{v_1,v_3,v_{n-1}\}$. Hence

$\gamma_s(G_2^p)=3$.

If $W_1=\{v_2\}$ and $V_2=V\backslash W_1$ then v_2 is adjacent to $v_4,v_5,\dots,v_n$ and there exists a path from v_3 to v_n. Also v_1 is adjacent to v_n Therefore a γ_s-set is $\{v_2,v_4,v_n\}$. Hence $\gamma_s(G_2^p)=3$.

Proceeding like this if $W_1=\{v_n\}$ and $V_2=V\backslash W_1$ then v_n is adjacent to $v_2,v_3,\dots,v_{n-2}$ and there exists a path from v_1 to v_{n-1}. Therefore a γ_s-set is $\{v_2,v_{n-2},v_n\}$. Hence $\gamma_s(G_2^p)=3$.

Also if $W_1=\{v_1,v_3\}$, $W_2=V\backslash W_1$ then in G_2^p, v_2 is an isolated vertex. And v_1 is adjacent to $v_4,v_5,\dots,v_{n-1}$ and v_3 is adjacent to $v_5,v_6,\dots v_n$. Therefore a γ_s-set is $\{v_1, v_2,v_3\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_3,v_5\}$, $W_2=V\backslash W_1$ then in G_2^p, v_4 is an isolated vertex. And v_3 is adjacent to $v_1,v_6,v_7\dots,v_n$ and v_5 is adjacent to $v_1,v_2,v_7,v_8,\dots v_n$. Also v_1 is adjacent to v_n. Therefore a γ_s-set is $\{v_3, v_4,v_5\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_2,v_4\}$, $W_2=V\backslash W_1$ then in G_2^p, v_3 is an isolated vertex. And v_2 is adjacent to $v_5,v_6,\dots,v_n$ and v_4 is adjacent to $v_1,v_6,v_7,\dots v_n$. Also v_1 is adjacent to v_n. Therefore a γ_s-set is $\{v_2, v_3,v_4\}$. Hence $\gamma_s(G_2^p)=3$.

Proceeding like this if $W_1=\{v_{n-3},v_{n-1}\}$, $W_2=V\backslash W_1$ then in G_2^p, v_{n-2} is an isolated vertex. And v_{n-3} is adjacent to $v_1,v_2,v_3\dots,v_{n-5}, v_n$. Also v_{n-1} is adjacent to $v_1,v_2,v_3,\dots v_{n-4}$. Also v_1 is adjacent to v_n. Therefore a γ_s-set is $\{v_{n-1}, v_{n-2},v_{n-3}\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_{n-2},v_n\}$, $W_2=V\backslash W_1$ then in G_2^p, v_{n-1} is an isolated vertex. And v_{n-2} is adjacent to $v_1,v_2,v_3\dots,v_{n-4}$ and v_n is adjacent to $v_2,v_3,v_4,\dots v_{n-4}, v_{n-3}$. Therefore a γ_s-set is $\{v_{n-2}, v_{n-1},v_n\}$. Hence $\gamma_s(G_2^p)=3$.

case: 4
n is odd

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to $v_4,v_6,\dots,v_{n-1}$ and v_3 is adjacent to $v_6,v_7,\dots v_n$. Also v_5 is adjacent to

$v_2,v_7,v_8,\dots, v_n$. Therefore a γ_s-set is $\{v_1,v_3, v_5,v_6\}$. Hence $\gamma_s(G_2^p)=4$.

If $W_1=\{v_1,v_3,v_5,v_7\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to $v_4,v_6,\dots,v_{n-1}$ and v_3 is adjacent to $v_6,v_8,\dots v_n$. Also v_5 is adjacent to $v_2,v_8,v_9,\dots, v_n$. And v_7 is adjacent to $v_2,v_4,\dots,v_{n-5},v_n$. Therefore a γ_s-set is $\{v_1,v_3, v_5,v_6\}$. Hence $\gamma_s(G_2^p)=4$.

If $W_1=\{v_1,v_3,v_5,\dots,v_n\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to v_n. And v_1 is adjacent to $v_4,v_6,\dots v_{n-1}$. Also v_3 is adjacent to $v_6,v_8,v_{10},\dots, v_{n-1}$ and v_5 is adjacent to $v_2,v_8,v_{10},\dots,v_{n-1}$. v_n is adjacent to $v_2,v_4,v_6,\dots v_{n-3}$. Therefore a γ_s-set is $\{v_1,v_2, v_3,v_5\}$. Hence $\gamma_s(G_2^p)=4$.

case: 5
n is even

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to $v_4,v_6,\dots,v_{n-1}$ and v_3 is adjacent to $v_6,v_7,\dots v_n$. Also v_5 is adjacent to $v_2,v_7,v_8,\dots, v_n$. Therefore a γ_s-set is $\{v_1,v_3, v_5,v_6,v_7\}$. Hence $\gamma_s(G_2^p)=5$.

If $W_1=\{v_1,v_3,v_5,v_7\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to $v_4,v_6,\dots,v_{n-2},v_{n-1}$ and v_3 is adjacent to $v_6,v_8,v_9\dots v_n$. Also v_5 is adjacent to

$v_2,v_8,v_9,\dots, v_n$. And v_7 is adjacent to $v_2,v_4,v_9,v_{10},\dots,v_n$. Therefore a γ_s-set is $\{v_1,v_2, v_3,v_4,v_7\}$. Hence $\gamma_s(G_2^p)=5$.

If $W_1=\{v_1,v_3,v_5,\dots,v_{n-3}\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to $v_4,v_6,\dots v_{n-1}$. Also v_3 is adjacent to $v_6,v_8,v_{10},\dots, v_{n-1},v_n$ and v_5 is adjacent to $v_2,v_8,v_{10},\dots,v_n$. v_{n-3} is adjacent to $v_2,v_4,v_6,\dots v_{n-6},v_{n-1}$. Therefore a

γ_s-set is $\{v_1,v_3, v_5,v_{n-4},v_{n-1}\}$. Hence $\gamma_s(G_2^p)=5$.

If $W_1=\{v_1,v_3,v_5,\dots,v_{n-1}\}$, $W_2=V\backslash W_1$ then in G_2^p, γ_s-set has 5 vertices to satisfy the set dominating set. Hence $\gamma_s(G_2^p)=5$.

Note: 1.9

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ then $\gamma_s(G_2^p)=3$ if n=8.

Proof:

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ then in G_2^p, v_1 is adjacent to v_4,v_6,v_7. Also v_3 is adjacent to v_6,v_7,v_8 and v_5 is adjacent to v_2,v_7,v_8. Therefore a γ_s-set is $\{v_1,v_3,v_5\}$. Hence $\gamma_s(G_2^p)=3$ if n=8.

Theorem: 2.0
Let G be a path on n vertices ($n\geq5$) say $v_1, v_2,\dots\dots,v_n$. Let v_1 and v_n are pendant vertices and $v_2,v_3,\dots,v_{n-1}$ are vertices of degree 2 then

Then $\gamma_s(G_2^p)=$
- 1 if $W_k=\{v_j\cup N(v_j)\}$, j=1,n, k=1,2 or $W_k=\{v_s,v_{s+1},v_{s+2}\}$, where v_s,v_{s+1},v_{s+2} are of degree 2.
- 2 if $W_k=\{v_j\}$ where j=1,2 or $\{v_s,v_{s+1}\}$ or $\{v_1,v_n\}$
- 3 if $W_k=\{v_j\}$, $2\leq j\leq n-1$, k=1,2 or $W_k=\{v_j,v_r\}$ wher i=1,n
- 4 if $W_k=\{v_i,v_r,v_s\}$ where v_i is a pendant vertex and v_r,v_s are non adjacent, non pendant vertices for k=1,2

5 if W_k has all alternative vertices for

k=1,2 where $n \geq 7$

Proof:
case :1

If $W_1=\{v_1,v_2\}$, $W_2=V\backslash W_1$ then v_1 is adjacent to v_2 and there exists a path from v_3 to v_n. Moreover v_1 is adjacent to v_2 and all other vertices of W_2. Therefore a γ_s-set is $\{v_1\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_{n-1},v_n\}$, $W_2=V\backslash W_1$ then there exists a path from v_1 to v_{n-2}. Moreover v_n is adjacent to v_{n-1} and all other vertices of W_2. Therefore a γ_s-set is $\{v_n\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_1,v_2,v_3\}$, $W_2=V\backslash W_1$ then there exists a path from v_4 to v_n. Moreover v_2 is adjacent to v_1 and v_3 and all other vertices of W_2. Therefore a γ_s-set is $\{v_2\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_2,v_3,v_4\}$, $W_2=V\backslash W_1$ then v_3 is adjacent to v_2 and v_4 and all other vertices of W_2. Therefore a γ_s-set is $\{v_3\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_3,v_4,v_5\}$, $W_2=V\backslash W_1$ then v_4 is adjacent to v_3 and v_5 and all other vertices of W_2. Therefore a γ_s-set is $\{v_4\}$. Hence $\gamma_s(G_2^p)=1$.

Proceeding like this,

If $W_1=\{v_{n-3},v_{n-2},v_{n-1}\}$, $W_2=V\backslash W_1$ then there exists a path from v_1 to v_{n-4}. Moreover v_{n-2} is adjacent to v_{n-3} and v_{n-1} and all other vertices of W_2. Therefore a γ_s-set is $\{v_{n-2}\}$. Hence $\gamma_s(G_2^p)=1$.

case: 2

If $W_1=\{v_1\}$ and $V_2=V\backslash W_1$ then v_1 is adjacent to $v_3,v_4,\ldots,v_n$ and there exists a path from v_2 to v_n. Therefore a γ_s-set is $\{v_1,v_2\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{v_n\}$ and $V_2=V\backslash W_1$ then v_n is adjacent to $v_1,v_2,\ldots,v_{n-2}$ and there exists a path from v_1 to v_{n-1}. Therefore a γ_s-set is $\{v_n,v_{n-1}\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_2,v_3\}$ and $V_2=V\backslash W_1$ then v_2 is adjacent to $v_3,v_4,\ldots,v_n$ and v_3 is adjacent to $v_1,v_5,v_6\ldots,v_n$ Therefore a γ_s-set is $\{v_2,v_3\}$. Hence

$\gamma_s(G_2^p)=2$.

If $W_1=\{v_3,v_4\}$ and $V_2=V\backslash W_1$ then there exists a path from v_5 to v_n. And v_3 is adjacent to $v_5,v_6,\ldots,v_n$ and v_4 is adjacent to all other vertices of W_2. Therefore a γ_s-set is $\{v_3,v_4\}$. Hence $\gamma_s(G_2^p)=2$.

Proceeding like this,

If $W_1=\{v_i,v_j\}$ where $i\neq j$ and $I,j \neq 1,n$ and $W_2=V\backslash W_1$ then v_i is adjacent to $v_1,v_2,\ldots,v_{i-2},v_{i+2},\ldots v_n$ and v_j is adjacent to $v_1,v_2,\ldots v_{j-2},v_{j+2},\ldots\ldots,v_n$ Therefore a γ_s-set is $\{v_i,v_j\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_1,v_n\}$ and $V_2=V\backslash W_1$ then there exists a path from v_2 to v_{n-1}. And v_1 is adjacent to $v_3,v_4,\ldots,v_{n-1}$ and also v_n is adjacent to $v_2,v_3,\ldots,v_{n-2}$ Therefore a γ_s-set is $\{v_1,v_n\}$. Hence $\gamma_s(G_2^p)=2$.

case: 3

If $W_1=\{v_2\}$ and $V_2=V\backslash W_1$ then v_1 is an isolated vertex and there exists a path from v_3 to v_n. Also v_2 is adjacent to $v_4,v_5,\ldots,v_n$ Therefore a γ_s-set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_3\}$ and $V_2=V\backslash W_1$ then there exists a path from v_4 to v_n. Also v_3 is adjacent to $v_1,v_5,v_6,\ldots,v_n$. And v_1 is adjacent to v_2.Therefore a γ_s-set is $\{v_1,v_3,v_{n-1}\}$. Hence $\gamma_s(G_2^p)=3$.

Proceeding like this,

If $W_1=\{v_i\}$ where $i\neq1,2,n-1,n$ and $W_2=V\backslash W_1$ then there exists a path from v_1 to v_{i-1} and v_{i+1} to v_n. Also v_i is adjacent to $v_1,v_2,\ldots,v_{i-2},v_{i+2},\ldots v_n$ Therefore a γ_s-set is $\{v_{i-2},v_i,v_{i+2}\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_1,v_3\}$ and $W_2=V\backslash W_1$ then v_2 is an isolated vertex and there exists a path from v_4 to v_n. Also v_1 is adjacent to $v_4,v_5,\ldots,v_n$ Therefore a γ_s-set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_2^p)=3$.

Proceeding like this

If $W_1=\{v_r,v_n\}$ and $W_2=V\backslash W_1$ then there exists a path from v_1 to v_{n-3} and v_{n-2} to v_n. Also v_r is adjacent to $v_1,v_2,\ldots,v_{r-2},v_{r+2}$. And v_n is adjacent to $v_1,v_2,\ldots,v_{n-2}$.Therefore a γ_s-set is $\{v_r,v_n,v_3\}$. Hence $\gamma_s(G_2^p)=3$.

case: 4

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ then in G_2^p ,v_1 is adjacent to $v_4,v_6,v_7\ldots,v_n$ and v_3 is adjacent to $v_6,v_7,\ldots v_n$. Also v_5 is adjacent to$v_2,v_7,v_8,\ldots, v_n$. Therefore a γ_s-set is $\{v_1,v_3, v_5,v_{n-1}\}$. Hence $\gamma_s(G_2^p)=4$.

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ and n=5 then in G_2^p ,v_3 is an isolated vertex. And v_1 is adjacent to v_4 and v_5 is adjacent to v_2. Therefore a γ_s-set is $\{v_1,v_4, v_3,v_2\}$. Hence $\gamma_s(G_2^p)=4$.

If $W_1=\{v_1,v_3,v_5\}$, $W_2=V\backslash W_1$ and n=6 then in G_2^p ,v_1 is adjacent to v_4,v_6 and v_3 is adjacent to v_6. Also v_5 is adjacent tov_2. Therefore a γ_s-set is $\{v_1,v_2, v_5,v_6\}$. Hence $\gamma_s(G_2^p)=4$.

If $W_1=\{v_1,v_3,v_{n-1}\}$, $W_2=V\backslash W_1$ then in G_2^p ,v_1 is adjacent to $v_4,v_5,\ldots,v_n$ and v_3 is adjacent to $v_5,v_6,\ldots v_n$. Also v_{n-1} is adjacent to

$v_2,v_4,\ldots, v_{n-3}$. Therefore a γ_s-set is $\{v_1,v_{n-1}, v_4,v_n\}$. Hence $\gamma_s(G_2^p)=4$.

case: 5

If $W_1=\{v_1,v_3,v_5,v_7\}$, $W_2=V\backslash W_1$ then in G_2^p ,v_1 is adjacent to $v_4,v_6,v_7\ldots,$

v_{n-3},v_{n-1} and v_3 is adjacent to v_{n-1}. Also v_5 is adjacent tov_2 and v_7 is adjacent to $v_2,v_4,\ldots,v_{n-3}$. Therefore a γ_s-set is $\{v_1,v_2, v_4,v_{n-1},v_n\}$. Hence

$\gamma_s(G_2^p)=5$.

If $W_1=\{v_1,v_3,v_5,v_7,v_9\}$, $W_2=V\backslash W_1$ then in G_2^p ,v_1 is adjacent to $v_4,v_6,v_8\ldots,v_{n-3},v_{n-1},v_n$ and v_3 is adjacent to $v_6,v_8\ldots,v_{n-3},v_{n-1},v_n$. Also v_5 is adjacent to $v_2,v_8,v_{10}\ldots,v_{n-1},v_n$ and v_7 is adjacent to $v_2,v_4,v_{10}\ldots,v_n$. And v_9 is adjacent to $v_2,v_4\ldots,v_{n-4},\ldots,v_n$ Therefore a γ_s-set is $\{v_1,v_2, v_3,v_5,v_8\}$. Hence $\gamma_s(G_2^p)=5$.

Proceeding like this,

If $W_1=\{v_1,v_3,v_5,\ldots v_{2n-1}\}$, $W_2=V\backslash W_1$ then in G_2^p ,v_1 is adjacent to $v_4,v_6,v_8\ldots,v_n$ and v_3 is adjacent to $v_6,v_8\ldots,v_n$. Also v_5 is adjacent to $v_2,v_8,v_{10}\ldots,v_n$ … v_{n-1} is adjacent to $v_2,v_4,v_6\ldots,v_{n-4}$.Therefore a γ_s-set is $\{v_1,v_2, v_3,v_{n-1},v_n\}$. Hence $\gamma_s(G_2^p)=5$.

Theorem: 2.1

Let G be a path on n vertices (n=3,4) say $v_1, v_2.........v_n$ .Let v_1 and v_n are pendant vertices and $v_2,v_3,\ldots,v_{n-1}$ are adjacent vertices of degree 2 then

$\gamma_s(G_2^p)=$ k=1,2
- 1 if $W_k=\{v_j\}$,or $W_k=\{v_j\cup N(v_j)\}$ where j=1,n,
- 2 if $W_k=\{v_j\}$ where $j\neq1,n$ or $W_k=\{v_s,v_{s+1}\}$where v_s,v_{s+1} are of degree 2
- 3 if $W_k=\{v_1,v_3\}$ for k=1,2

Proof:
case:1

If $W_1=\{v_1\}$ and $V_2=V\backslash W_1$ then v_3 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_3\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_n\}$ and $V_2=V\backslash W_1$ then v_2 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_2\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_1,v_2\}$ and $V_2=V\backslash W_1$ then v_1 is adjacent to all other vertices. Therefore a γ_s-set is $\{v_1\}$. Hence $\gamma_s(G_2^p)=1$.

If $W_1=\{v_{n-1},v_n\}$ and $V_2=V\backslash W_1$ then v_n is adjacent to v_{n-1} and all other vertices of W_1. Therefore a γ_s-set is $\{v_n\}$. Hence $\gamma_s(G_2^p)=1$.

case:2

If $W_1=\{v_2\}$ and $V_2=V\backslash W_1$ then v_1 is an isolated vertex .Also v_4 is adjacent to v_2,v_3. Therefore a γ_s-set is $\{v_1,v_4\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_3\}$ and $V_2=V\backslash W_1$ then v_4 is an isolated vertex. And v_1 is adjacent to v_2,v_3 . Therefore a γ_s-set is $\{v_1,v_4\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_2,v_3\}$ and $W_2=V\backslash W_1$ then v_2 is adjacent to v_3,v_4.And v_3 is adjacent to v_1.Therefore a γ_s-set is $\{v_2,v_3\}$. Hence $\gamma_s(G_2^p)=2$.

If $W_1=\{v_1,v_n\}$ and $W_2=V\backslash W_1$ then v_1 is adjacent to v_3. And v_2 is adjacent to v_3.Therefore a γ_s-set is $\{v_2,v_3\}$. Hence $\gamma_s(G_2^p)=2$.

case: 3

If $W_1=\{v_1,v_3\}$ and $W_2=V\backslash W_1$ and n=3 then G^2_p is a union of an isolated vertices. Therefore a γ_s-set is $\{v_1,v_2,v_3\}$. Hence $\gamma_s(G_2^p)=3$.

If $W_1=\{v_1,v_3\}$ and $W_2=V\backslash W_1$ and n=4then $v_2$and v_3 are isolated vertices. And v_1 is adjacent to v_4.Therefore a γ_s-set is $\{v_1,v_2,v_3\}$. Hence

$\gamma_s(G_2^p)=3$.

Theorem: 2.2

If G and G_2^p are connected graphs then $2\leq\gamma_s(G)+\gamma_s(G_2^p)\leq n+2$.

Proof:

Let G and G_2^p are connected graphs then $\gamma_s(G)\geq1$ and $\gamma_s(G_2^p)\geq1$.Therefore $2\leq\gamma_s(G)+\gamma_s(G_2^p)$.

Also we can justify $\gamma_s(G)+\gamma_s(G_2^p)\leq n+2$ with the following examples.

Example: 2.21
Consider the path with 6 vertices with v_1 and v_6 are pendant vertices and $v_2,v_3,\ldots v_5$ are vertices of degree 2. Take the partition $V_1=\{v_2,v_5\}$,$V_2=\{v_1.v_3,v_4,v_6\}$ then $\gamma_s(G_2^p)\leq4$.

And $\gamma_s(G)=4$.

Therefore $\gamma_s(G)+\gamma_s(G_2^p)\leq8=n+2$

Example: 2.22
Consider Star with n vertices ($n\geq3$) .Take the partition $V_1=\{v_1,v_2\}$, $V_2=\{v_3,v_4,\ldots v_n\}$ then $\gamma_s(G_2^p)\geq1$ and $\gamma_s(G)=1$.

Therefore $\gamma_s(G)+\gamma_s(G_2^p)\geq2$.

Remark: 2.3
If any one of the partitions of G_2^p contains an isolated vertex then $\gamma_s(G_2^p)\geq2$.

Conjecture: 2.4
G_n^P has n components if n-complementary of complete graphs.

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