Effect of Thermal Gradient and Thickness Variation on Vibration of Visco-Elastic Plate

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Abstract— Plates and plate-type structures have gained a special importance and notably increased engineering applications in recent years. A large number of structural components in engineering structures can be classified as plates. Plates are also indispensable in ship building and aerospace industries. The wings and a large part of the fuselage of an aircraft, for example, consist of a slightly curved plate skin with an array of stiffened ribs. The hull of a ship, its deck and its super structure are further examples of stiffened plate structures. Plates with variable thickness are also have a great importance in a wide variety of engineering applications i.e. nuclear reactor, aeronautical field, naval structure, submarine, earth-quake resistors etc. A mathematical model is presented for the use of engineers and research workers in space technology have to operate under elevated temperatures. In this paper, the thickness varies linearly in x-direction and thermal effect is varying linearly in two directions. Rayleigh Ritz method is used to evaluate the fundamental frequencies. Both the modes of the frequency are calculated for the various values of taper parameters and thermal gradient.

Index Terms— Visco-elastic, Vibration, Thickness, Square plate, Thermal gradient, Taper constant, Frequency.

1. INTRODUCTION

Plates with variable thickness are frequently used in order to economize on the plate materials or to lighten the plates, especially when used in wings for high-speed, high-performance aircrafts. By carefully designing the thickness distribution, a substantial increase in stiffness, buckling and vibration capacities of the plate may be obtained over its uniform thickness counterpart. In the aeronautical field, analysis of non-homogeneous plates with thermal gradient and variable thickness has been of great interest due to their utility in aircraft wings. The research in the field of vibration is quite mesmerizing and continuously acquiring a great attention of scientists and design engineers because of its unbounded effect on human life. In the engineering we cannot move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications.

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Study of effect of vibration can't be restricted only in the field of science but, our day to day life is also affected by it. Whether it be a constructive aspect e.g. aircraft, space shuttle, satellite or design engineering to the destructive aspect, e.g. tsunami, earthquake etc., none of these are remained untouched with the effect of vibrations. Structures of plates have wide applications in ships, aircrafts, bridges, etc. A thorough dynamic study of their behavior and characteristics is essential to assess and use the full potentials of plates. In the aeronautical field, analysis of plates with variable thickness has been of great interest due to their utility in aircraft wings.
Vibration of plates of various shapes, homogeneous or non-homogeneous, orthotropic or isotropic, with or without variation in thickness, have been studied by various authors, with or without considering the effect of temperature. Recently, A.K. Gupta and Anupam Khanna [1], studied the Thermal Effect on Vibrations of Parallelogram Plate of Linearly Varying Thickness. A.K. Gupta and Anupam Khanna [2], studied the Vibration of clamped visco-elastic rectangular plate with parabolic thickness variations. A.K. Gupta and Anupam Khanna [3], has been studied on Free vibration of clamped visco-elastic rectangular plate having bi-direction exponentially thickness variations. A.K.Gupta and A. Khanna [4], also studied the, Vibration of Visco-elastic rectangular plate with linearly thickness variations in both directions. Anupam Khanna, Ashish Kumar Sharma [5], studied the Study of free Vibration of Visco-Elastic Square Plate of Variable Thickness with Thermal Effect. Anupam Khanna, Ashish Kumar Sharma [6], has been studied on Vibration Analysis of Visco-Elastic Square Plate of Variable Thickness with Thermal Gradient. In the recent past, there has been increasingly great interest in high strength, corrosion resistance and high temperature performance materials for structural components used in mechanical, aerospace, ocean engineering, electronic and optical equipments. Modern engineering structures are based on different types of design, which involve various types of anisotropic and non-homogeneous materials in the form of their structure components. Depending upon the requirement, durability and reliability, materials are being developed so that they can be used to give better strength and efficiency. The equipment used in air-jet, communications and in other similar technological industries take into consideration such materials,

which not only reduce the weight and size but also are reliable in terms of efficiency, strength and economy. Plate type structural components of varying thickness fabricated out of modern composite materials such as glass epoxy, boron epoxy, Kevlar and graphite etc. fulfill these requirements which are lighter, stiffer and stronger than any other conventional material used earlier and help the designer to reduce the weight and size of the structure. Visco-elasticity, as its name implies, is a generalization of elasticity and viscosity. The ideal linear elastic element is the spring. When a tensile force is applied to it, the increase in distance between its two ends is proportional to the force. The ideal linear viscous element is the dashpot. The plate type structural components in aircraft and rockets have to operate under elevated temperatures that cause non-homogeneity in the plate material i.e. elastic constants of the materials becomes functions of space variables. In an up-to-date survey of literature, authors have come across various models to account for non-homogeneity of plate materials proposed by researchers dealing with vibration but none of them considers non-homogeneity on orthotropic visco-elastic plates. The aim of present investigation is to study two dimensional thermal effect on the vibration of visco-elastic square plate whose thickness vary linearly in one direction and thermal effect vary linearly in two directions. It is assumed that the plate is clamped on all the four edges and its temperature varies linearly in both the directions. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity. For various numerical values of thermal gradient and taper constants. Frequency for the first two modes of vibration is calculated and results are shown in Graphs.

2. Equation of Motion and Analysis

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is given [1]:

$$[D_1\left(W_{,xxxx}+2W_{,xxyy}+W_{,yyyy}\right)+2D_{1,x}\left(W_{,xxx}+W_{,xyy}\right)+2D_{1,y}\left(W_{,yyy}+W_{,yxx}\right)+$$
$$D_{1,xx}(W_{,xx}+\nu W_{,yy})+D_{1,yy}(W_{,yy}+\nu W_{,xx})+2(1-\nu)D_{1,xy}W_{,xy}]-\rho h p^2 W=0$$

(2.1)

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness.

Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3/12(1-\nu^2) \quad (2.2)$$

and corresponding two-term deflection function is taken as [5]

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2$$
$$[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \quad (2.3)$$

Assuming that the square plate of engineering material has a steady two dimensional temperature distribution i.e.

$$\tau=\tau_0(1-x/a)(1-y/a) \quad (2.4)$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this

$$E = E_0(1-\gamma\tau) \quad (2.5)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variation (2.5) become

$$E=E_0[1-\alpha(1-x/a)(1-y/a)] \quad (2.6)$$

where, $\alpha=\gamma\tau_0\,(0\le\alpha<1)$ thermal gradient.

It is assumed that thickness also varies linearly in one direction as shown below:

$$h = h_0(1+\beta_1 x/a) \quad (2.7)$$

where, β_1 is taper parameter in x direction respectively and $h=h_0$ at x=y=0.

Put the value of E & h from equation (2.6) & (2.7) in the equation (2.2), one obtain

$$D_1=[E_0[1-\alpha(1-x/a)(1-y/a)]h_0^3(1+\beta_1 x/a)^3]/12(1-\nu^2) \quad (2.8)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^*-T^*)=0 \quad (2.9)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{aligned} W=W_{,x}=0,\ x=0,a \\ W=W_{,y}=0,\ y=0,a \end{aligned}\right\} \quad (2.10)$$

Now assuming the non-dimensional variables as

$$X=x/a, Y=y/a, \overline{W}=W/a, \overline{h}=h/a \quad (2.11)$$

The kinetic energy T* and strain energy V* are [2]

$$T^*=(1/2)\rho p^2\overline{h_0}a^5\int_0^1\int_0^1[(1+\beta_1 X)\overline{W^2}]dYdX \quad (2.12)$$

and

$$V^*=Q\int_0^1\int_0^1[1-\alpha(1-X)(1-Y)](1+\beta_1 X)^3\{(\overline{W}_{,XX})^2+(\overline{W}_{,YY})^2$$
$$+2\nu\overline{W}_{,XX}\overline{W}_{,YY}+2(1-\nu)(\overline{W}_{,XY})^2\}dYdX \quad (2.13)$$

where, $Q=E_0h_0^3a^3/24(1-\nu^2)$

Using equations (2.12) & (2.13) in equation (2.9), one get

$$(V^{**}-\lambda^2T^{**})=0 \quad (2.14)$$

where,

$$V^{**}=\int_0^1\int_0^1[1-\alpha(1-X)(1-Y)](1+\beta_1 X)^3\{(\overline{W}_{,XX})^2+(\overline{W}_{,YY})^2$$
$$+2\nu\overline{W}_{,XX}\overline{W}_{,YY}+2(1-\nu)(\overline{W}_{,XY})^2\}dYdX \quad (2.15)$$

and

$$T^{**}=\int_0^1\int_0^1[(1+\beta_1 X)\overline{W^2}]dYdX \quad (2.16)$$

Here, $\lambda^2=12\rho(1-\nu^2)a^2/E_0h_0^2$ is a frequency parameter.

Equation (2.16) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**}-\lambda^2T^{**})/\partial A_n \quad , n = 1, 2 \quad (2.17)$$

On simplifying (2.17), one gets

$$bn_1A_1+bn_2A_2=0 \quad , n=1, 2 \quad (2.18)$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (2.18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \qquad (2.19)$$

With the help of equation (2.19), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1(Mode1) & λ_2(Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3 Result And Discussion

All calculations are carried out with the help of latest Matrix Laboratory computer software. Computation has been done for frequency of visco-elastic square plate for different values of taper constants β_1 and β_2, thermal gradient α, at different points for first two modes of vibrations have been calculated numerically. In Fig I: - It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for β_1 =0.0 and β_1 =0.8 for both modes of vibrations. In Fig II :- Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_1 from 0.0 to 1.0 for α=0.0 and α=0.8 respectively. Note that value of frequency increased.

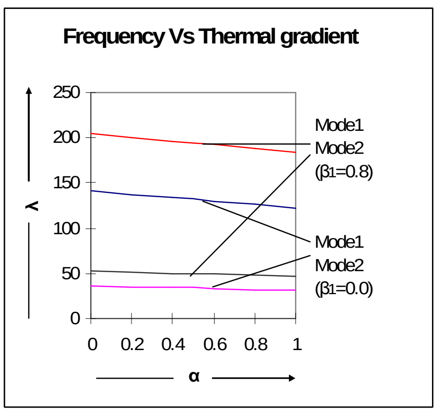

Fig I: - Frequency vs. thermal gradient

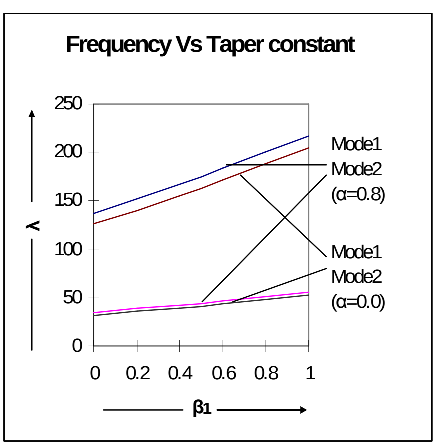

Fig II: - Frequency vs. Taper constant

4. Conclusion

The objective of this paper is to clarify the characteristics of vibration of plates with variable thickness. It shows that the proposed results have a good convergence and satisfactory accuracy compared with experimental results. The effects of the thickness of plates, size of the defect on free vibration behavior of plates also investigated. So, main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

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