Terminal Sliding Mode Fuzzy-PDC Control for Nonlinear Systems

Yi-Jen Mon

Abstract: - The proposed method called terminal sliding Fuzzy Sliding Parallel Distributed Compensation (TSF-PDC) control is developed in this paper to deal with the nonlinear systems subjected to bounded disturbances. The TSF-PDC controller is combined with Fuzzy Parallel Distributed Compensation (F-PDC) control and terminal sliding mode (TS) control to deal with nonlinear systems. In the design of TSF-PDC, some F-PDC control gains are used to stabilize every F-PDC systems; and the TS control is used to cope with disturbances and uncertainties of systems. In simulation, the inverted pendulum is illustrated to verify the effectiveness, performances and robustness. The simulation results of this proposed TSF-PDC controller demonstrate better performance and robustness than that of F-PDC control.

Index Terms: - F-PDC control; Terminal sliding mode control; Non-linear system; pendulum system.

1 Introduction

The fuzzy logic (FL) controller and sliding-mode (SM) control schemes have been developed in many years which already have many successful research results can be search in web of Google Scholar [1-3]. These methodologies have abilities of model free, stability guaranteed and effective performances. In recent years, there have been some new mechanisms developed for improving SM control performance, such as the terminal sliding (TS) control has the ability to cope with the singularity of conventional SM control [4-6]. For another fuzzy methodology called Fuzzy Parallel Distributed Compensation (F-PDC) design approach had also been proposed in many years ago and also has been applied in many successful research achievements [7-9]. The F-PDC design is attractive since it is simple and natural, meanwhile, it can guarantee the system's global stability and robustness performance. In [8, 9], the author's previous work of the fuzzy sliding PDC (FS-PDC) control is proposed. In this paper, a new idea is proposed and called terminal sliding mode fuzzy parallel distributed compensation (TSF-PDC) which is by using the TS control methodology combined with the FS-PDC to deal with control problems of nonlinear systems. In simulation, the inverted pendulum system is illustrated to demonstrate the performance of this proposed scheme. The simulation results are revealed that the good robustness and performance are possessed.

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2 Terminal Sliding Fuzzy PDC Controller Design

Consider a nonlinear system includes disturbances:

$$\dot{\mathrm{x}}(t) = \mathrm{f}(\mathrm{x}) + \mathrm{g}(\mathrm{x})\mathrm{u} + \mathrm{d} \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n$ denotes the state vector, $\boldsymbol{u} = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in R^n$ denote the control input, $\boldsymbol{d} = [d_1(t), d_2(t), \cdots, d_n(t)]^T \in R^n$ denotes the unknown disturbances with a known upper bound $\|\boldsymbol{d}\| \le d_b$; $\|\cdot\|$ denotes the induced norm. The $\boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{x}(t)$ will all be abbreviated as $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{x}$ in the following discussions. These functions are all varied depending on $\boldsymbol{x}$ and $\|\boldsymbol{g}\| \le \beta$, where β is a bounded positive constant. Since the system parameters may be unknown or perturbed, the ideal controller u_{id} of sliding mode control cannot be implemented. To overcome this, a human knowledge based fuzzy parallel distributed compensation (F-PDC) methodology will be designed at first to approach the neighborhood of ideal controller. In addition, a terminal sliding (TS) controller is designed to compensate for the approximation error between the F-PDC controller and the ideal controller and to cope with disturbances. Suppose that a first order terminal sliding function for every F-PDC controllers is defined as [4]

$$s(t) = \dot{x} + kx^{\frac{q}{p}}. \tag{2}$$

where $k > 0$ and $p > q$, are positive constants. Then

$$\dot{s}(t) = \ddot{x} + kx^{\frac{q}{p}-1}\dot{x}. \tag{3}$$

Hence, the control law of *i-th* TSF-PDC controller can be defined as

$$u_i(t) = u_{F-PDCi}(t) + u_{TSi}(t) \tag{4}$$

where $u_{F-PDCi}(t)$ is the *i-th* F-PDC controller and $u_{TSi}(t)$ is the *i-th* TS controller. Thus, the TSF-PDC control system is shown in Fig. 1 where the inputs of every F-PDC controller are *i-th* state x_i of system and its derivative value $\dot{x}_i$; the inputs of every TS controller are *i-th* terminal sliding surface $s(t)$ of system and its derivative value $\dot{s}(t)$. The system dynamics can be captured by the feature of a Takagi-Sugeno fuzzy model to express the *i-th* local dynamics of each fuzzy rule by a linear system model, which locally represent the linear input-output relations of the system. The fuzzy system is as follows [3, 7, 8]:

$$\begin{aligned} &Rule\ i:\ IF\quad x_1(t)\quad is\quad M_{i1}\ \dots\ and\quad x_n(t)\quad is\quad M_{in} \\ &\qquad THEN\quad \dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i u(t) \end{aligned} \tag{5}$$

where $\boldsymbol{A}_i \in R^{n\times n}$ are system matrices; $\boldsymbol{B}_i \in R^{n\times n}$ are input matrices; *i*=1,2,3,...,*r*; *r* is the number of IF-THEN rules; M_{ij} are the fuzzy sets. Then the nonlinear system in (1) can be represented as the following fuzzy inference system [7-9]

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}+\boldsymbol{g}u+\boldsymbol{d} \\ &= \frac{\sum_{i=1}^{r}\mu_i(\boldsymbol{x})[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{B}_i u]}{\sum_{i=1}^{r}\mu_i(\boldsymbol{x})}+\Delta\boldsymbol{f}+\Delta\boldsymbol{g}+\boldsymbol{d} \\ &= \sum_{i=1}^{r} w_i(x)[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{B}_i u]+\Delta\boldsymbol{f}+\Delta\boldsymbol{g}+\boldsymbol{d} \\ &= \sum_{i=1}^{r} w_i(x)[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{B}_i u]+\boldsymbol{m}+\boldsymbol{d} \\ &= \sum_{i=1}^{r} w_i(x(t),t)[\boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{B}_i u]+\boldsymbol{n} \end{aligned} \tag{6}$$

where $\mu_i(\boldsymbol{x})=\prod_{j=1}^{n} M_{ij}(x_j)$; $w_i(\boldsymbol{x})=\dfrac{\mu_i(\boldsymbol{x})}{\sum_{i=1}^{r}\mu_i(\boldsymbol{x})}$.

The TSF-PDC controller shares the same fuzzy sets with the fuzzy system (6), the following fuzzy rules are employed to deal with the above control system design: [7-9]

$$\begin{aligned} &Rule\ j:\ IF\quad x_1(t)\quad is\quad M_{j1}\ \dots\ and\quad x_n(t)\quad is\quad M_{jn} \\ &\qquad THEN\quad u_{\boldsymbol{j}}(t)=-\boldsymbol{K}\boldsymbol{x}-\boldsymbol{F}\,\mathrm{sgn}(s(t)) \end{aligned} \tag{7}$$

for $j=1,2,\cdots,r$, where $\boldsymbol{K}\in R^{n\times n}$ is an optimized F-PDC gain matrix; $\boldsymbol{F}\in R^{n\times n}$ is a robust constant TS gain matrix. Defining a Lyapunov function $V=\frac{1}{2}s^2$; by some easily manipulations, the Lyapunov stability condition can be achieved and the stability of the fuzzy system in (6) is guaranteed. Based on the previous work [8, 9], the TSF-PDC is developed in this paper. From (7), the defined TS gains can be used to tune the control inputs for every F-PDC. Because every stabilities of subsystems are guaranteed, the total stability of u can be also guaranteed. Maybe the saturation can be added in front of the plant or not to limit the control input within a reasonable range.

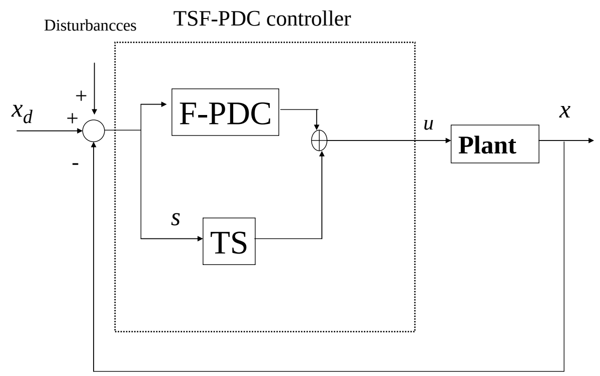

Fig. 1 The concept diagram of TSF-PDC control

3 Simulation Results

In this paper, the nonlinear example of pendulum system is illustrated as an example to verify the proposed TSF-PDC control. Consider the following system: [3, 10]

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1 \\ \dot{x}_2 &= \frac{g\sin(x_1)-amlx_2^2\sin(2x_1)/2-a\cos(x_1)u}{4l/3-aml\cos^2(x_1)}+d_2 \end{aligned} \tag{8}$$

where disturbance $\boldsymbol{d}=[d_1\quad d_2]^T=[0.1\cos(0.25t)\quad 0.1\sin(0.25t)]^T$, the initial conditions of states are $\pi/6$, 0 for $x_i\quad i=1,2$. The x_1 denotes the angle (in radians) of the pendulum from the vertical, and x_2 is the angular velocity. g = 9.8 $m/\sec^2$ is the gravity constant, *m* is the mass of the pendulum, *M* is the mass of the cart, 2l is the length of the pendulum, and u is the forced applied to the cart (in Newtons). *a* = l/(m + *M). We* choose *m* = 2.0 kg, *M* = 8.0 kg, 2l = 1.0 m in the simulations compared with F-PDC control. In this example, for simplicity, just two F-PDCs are designed. The first F-PDC1 is designed as four-rules fuzzy model as follows. Similarly, the second F-PDC2 can be also easily designed by the same steps by setting the initial conditions of states are $\pi/6$, 0 for $x_i\quad i=1,2$.

$$\begin{aligned} &Rule\ 1:\ IF\quad x_1\,is\quad about\quad 0\ and\quad x_2\,is\quad about\quad 0 \\ &\qquad THEN\quad \dot{\boldsymbol{x}}=\boldsymbol{A}_1\boldsymbol{x}+\boldsymbol{B}_1 u \end{aligned}$$

$$\begin{aligned} &Rule\ 2:\ IF\quad x_1\,is\quad about\quad 0\ and\quad x_2\,is\quad about\quad \frac{\pi}{6} \\ &\qquad THEN\quad \dot{\boldsymbol{x}}=\boldsymbol{A}_2\boldsymbol{x}+\boldsymbol{B}_2 u \end{aligned}$$

$$\vdots$$

$$\begin{aligned} &Rule\ 4:\ IF\quad x_1\,is\quad about\ \frac{\pi}{6}\ \ and\ \ x_2\,is\quad about\ \frac{\pi}{6} \\ &\qquad THEN\quad \dot{\boldsymbol{x}} = \boldsymbol{A}_4\boldsymbol{x} + \boldsymbol{B}_4 u \end{aligned} \tag{9}$$

where matrices of $\sim$ and $\boldsymbol{B}_i$ $i = 1,2,\cdots 4$ are linear system matrices which can be calculated by some suitable points which are located in their corresponding nonlinear points of fuzzy rules. A 'linmod' command of MATLAB™ software is used in this paper and got as follows:

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 17.294 & 0 \end{bmatrix}\ \boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 \\ 17.185 & 0 \end{bmatrix}\ \boldsymbol{A}_3 = \begin{bmatrix} 0 & 1 \\ 9.415 & 0 \end{bmatrix}$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}\quad \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}\quad \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ -0.1147 \end{bmatrix}$$

$$\boldsymbol{A}_4 = \begin{bmatrix} 0 & 1 \\ 9.423 & -0.127 \end{bmatrix}$$

$$\boldsymbol{B}_4 = \begin{bmatrix} 0 \\ -0.1147 \end{bmatrix}.$$

(10)

By comparison of the proposed TSF-PDC control and the F-PDC control for the pendulum system, the same conditions of simulations are proceeded. After the F-PDC gains are achieved, the other parameters of terminal sliding controller are set as $\boldsymbol{F} = [2 \quad 2]$, $p = 3$ and $q = 1$. The simulation results are shown in Fig. 2. These simulation results demonstrate that the states are controlled to achieve better transient state performances by using the proposed TSF-PDC control than F-PDC control.

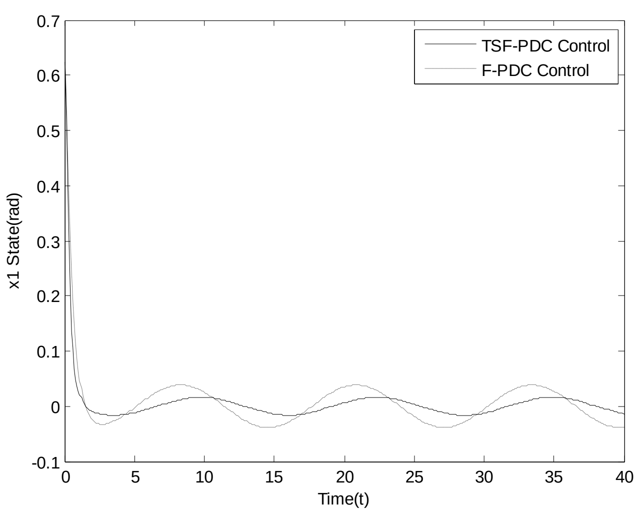

Fig. 2 (a). System states of x_1

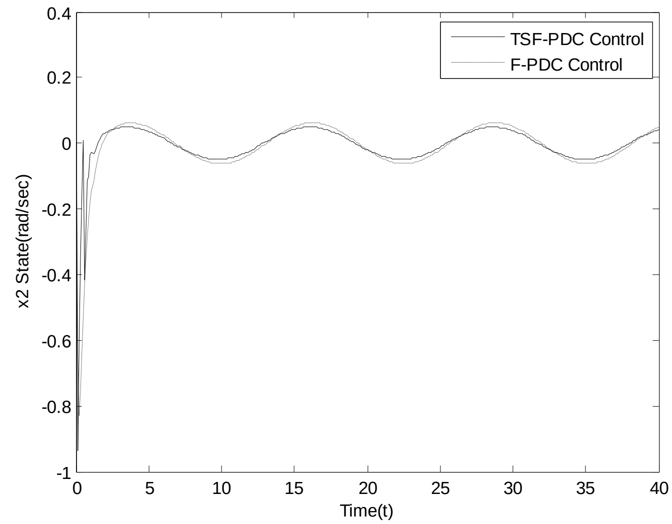

Fig. 2 (b). System states of x_2

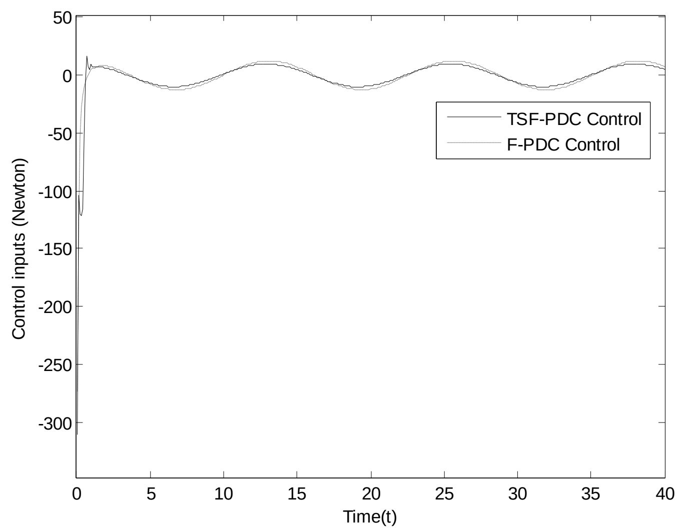

Fig. 2 (c). System control inputs

4 Conclusion

The novel terminal sliding parallel distributed compensation (TSF-PDC) control is proposed in this paper to deal with nonlinear control problems. This controller is composed of fuzzy parallel distributed compensation (F-PDC) control and terminal sliding (TS) control. The F-PDC control is based on human knowledge to divide nonlinear system into several linear model then using fuzzy inference getting control results. The TS control is used to cope with disturbances and system's uncertainties. In this paper, an inverted pendulum system is illustrated to verify the proposed scheme. In simulation, the TSF-PDC control possesses the better performances and robustness than F-PDC control.

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