Beam And Rotating Disc Brake With Different Contact Location

Ammar A. Yousif Mohammed, Inzarulfaisham Abd Rahim

ABSTRACT: - In this paper a matix27 of ANSYS/General is used to simulate the contact between beam-disc systems. The beam was changed to different location and by using complex eigenvalue the stability was detected. An attempted is made to investigate the effect of system parameters, such as friction coefficient of the contact interaction between the beam and the disc, disc young's modulus and contact stiffness. The main idea of the complex eigenvalue method involves asymmetry arguments of the stiffness matrix which formulate the friction coupling. This idea is more efficient and provides more insight to the friction induced dynamic instability in the disc brake system. The simulation performed in this work present a guideline to reduce the squeal noise of disc brake system and showed that the most important source of nonlinearity is the contact frictional sliding between the disc and the beam. ANSYS allows for a convenient contact interface by specifying the contact surface and the properties of the interfaces. The complex eigenvalue is solved by using modal analysis with QRDAMP method. The analysis determined the stability of the system. When the system is unstable the real part of the eigenvalue becomes positive and squeal occurs. If the damping ratio is negative, the system becomes unstable, and vice versus. The result showed that even the beam and disc at natural frequency, no squeal could generated if the beam at anti-node position.

INDEX TERMS: - QRDAMP method, matix27, eigenvalue, contact stiffness, modal coupling, and dynamic instability

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1 INTRODUCTION

Squeal is the most common problem which it's defined as a noise has frequency content at 1000 Hz or higher that occurs when a system experiences high vibrations amplitude [1]. However, there is yet neither a complete understanding of the squeal generating nor a general theory to describe the complexity of the system. The researchers tried to find an easy way to represent the brake system in order to pass these complexities and measure experimentally the effect of some parameters that generate the squeal. Many researchers tried to represent the brake system by using beam on disc, while other tried to represent the system by using plate on disc as Ammar [2]. An early experimental investigation by Masayaki and Mikio [3] was done by using beam on disc system. They found different in sound pressure between high and low frequency. These noises generated are depending on the friction coefficient, sliding distance and young's modulus. In this paper the authors didn't mention any information about the effect of Poisson's ratio on the squeal. At 1980 Masayaki and Mikio [4] return to run an attempted to study the effect of the beam length on squeal frequency. They found that increase the beam length can establish a fundamentals squeal mode at lower system frequency. At 1981 Masayaki and Mikio [5] study the contact's beam roughness and the squeal. They found that increase the roughness will generated higher noise and pressure level due to increase of sub-harmonic frequency level.

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Also at 1981 Masayaki and Mikio [6] they observed the effect of contact's beam angle on the squeal. Changing the contact angle can generated higher squeal, but the effect of the beam length on the squeal was more than the effect of changing the contact's angle. Masayaki and Mikio [7] found that the lost of the contact between the beam and disc is related to surface roughness. Increase the roughness can increase the loss of contact and squeal generated. Akey et al at [8], return to use the beam and the disc system to study the squeal. He represents the system inside finite element software and predicts the system instability with eigenvalue method. Massi and Baillet [9] used beam on disc with two different numerical methods linear and non-linear approach. In this study the authors approved that the beam on disc system with linear approach can match the same result exactly with non-linear real approach of the system. As can observe from the previous work, many factors can affect the squeal frequency occurrence. As the contact angle can affect the squeal, the different of beam location on the disc may be able to have some influence on the squeal also. In this study we tried to find the relation between the beam location and the squeal generated.

2 CONTACT'S STIFFNESS

The stress-strain relationship for the pad friction material was used to determine the contact stiffness. The contact stiffness is a function for the young's modulus of the elastic material (friction material). The contact area between the pad and the rigid surface (rotor) and the length L is given in the equation 1

$$K = \frac{AE}{L} \quad \text{, Junior [10],} \qquad \text{equation (1)}$$

The contact stiffness can be calculated using two different approaches: from a previous knowledge of the young's modulus of the friction material and through measurement of the stress-strain relationship under normal load, [10]. The effect of braking pressure was introduced into the finite element model through the variation of the contact stiffness between the rotor and beam. The use of the stress–strain relationship method to determine this contact stiffness, allows an evaluation of the effect of the braking pressure on the contact stiffness. The increases in braking pressure lead to high values for contact stiffness, as showed by Junior [10].

3 PERPENDICULAR'S BEAM ON ROTATING DISC

A perpendicular beam on rotating disc is modeled to study disc brake squeal. The system consists of disc which is rotate about the axis of the wheel. The beam is in contact with the disc by using matrix27. The disc outer diameter is 0.4m, while the inner diameter is 0.1m and the thickness is 0.05m. The disc density and young's modulus are 210GN/m^2 and 7800 Kg/m^3, respectively. These values have been taking from Giannini [11] paper. The disc inner radius and outer radios is free-free. The beam dimensions are 0.05×0.05×0.5 m. The beam is free –free also. The beam density, and young's modulus are 100 GN/m^2, 2500 kg/m^3. These values have been also taking from Giannini et al [11] paper. The FE mesh is generated using three dimension sweep mesh, as illustrated in figure 1. It is assumed that equal magnitude of the contact stiffness between the beam-disc interface surfaces. Note that in these numerical examples, the disc thickness is deliberately taken to be very small in order to reduce the amount of computing work. However, this will not affect the qualitative features of the results or the conclusion. The first seven natural (diametral) frequencies of the disc are 273.31, 650.02, 1118.7, 1679.8, 2328.6, 3059.3 and 3866.3 Hz The beam and the disc element are modeled with solid 45. Matrix27 has a dimension 12×12 and can relating two coincident nodes used in this simulation. The beam and the disc mesh (node) must be coincident to enable the generation of the node-to-node contact element. The inclusion of the friction force causes stiffness matrix (matrix27) to be unsymmetrical. This means that the friction force causes the results of eigenvalue to be complex values. Using matrix27 has showed how the contact stiffness and friction coefficient affects the mode shape and natural frequency. Axial symmetric mesh should be applied to the disc because the non-axial symmetric mesh causes the double modes of the disc to split even in free conditions.

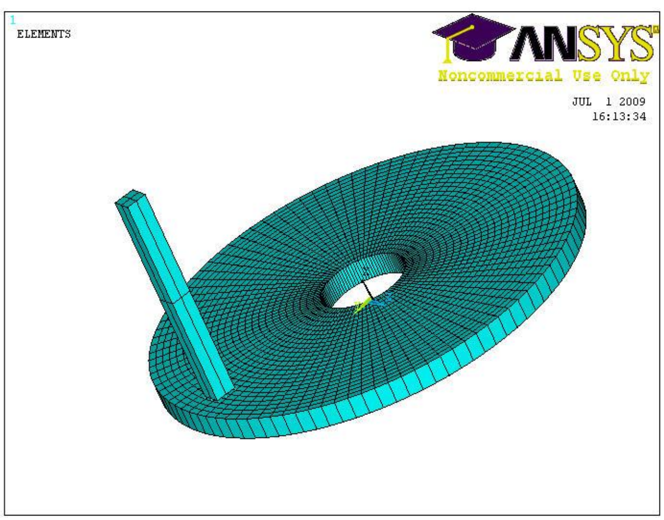


Figure 1 Geometry illustrated the mesh of perpendicular beam with rotating disc

The mating surface of the beam and the disc is assumed to have some flexibility in which they can move relative to each other. The formula which represents the contact between the beam and the disc can be derived as fellow.

Normal force on the disc, Fx1=-K(x1-x2)

Normal force on the beam, Fx2=K(x1-x2)

Friction force on the disc, Fz1=μFx1=-μK(x2-x1)

Friction force on the beam, Fz2=μFx2=μK(x1-x2)

The above equation can be rearranged in matrix as follows.

$$\begin{bmatrix} Fx1 \\ Fz1 \\ Fx2 \\ Fz2 \end{bmatrix} = \begin{bmatrix} K & 0 & -K & 0 \\ -\mu K & 0 & \mu K & 0 \\ -K & 0 & K & 0 \\ \mu K & 0 & -\mu K & 0 \end{bmatrix} \begin{bmatrix} x1 \\ z1 \\ x2 \\ z2 \end{bmatrix}$$

(Interface friction matrix)

The interface friction matrix can be used as the input to the interface element in the finite element method using ANSYS. The interface used in this analysis is known as matrix27. Matrix27 represents an arbitrary element whose geometry is undefined but whose elastic kinematics response can be specified by the stiffness, damping, or mass coefficients. The matrix assumed to related, each nodes, each with six degree of freedom per node, translations in the nodal x, y and z directions and rotations about the x, y and z axes. The effect of the contact stiffness between the beam and the disc on the system stability has been investigated by simulation the model with different contact values. The complex eigenvalue is performed between 1000 to 12000 Hz which is the range of the squeal occurrence by using modal analysis. The analysis is carried out by changing the values of the contact stiffness while retaining the respective typical values for the others. The line of the instability is conducted as in Figure 2 by applying a range of contact stiffness value from 250 to 1100 MN/m with increment of 150 MN/m.

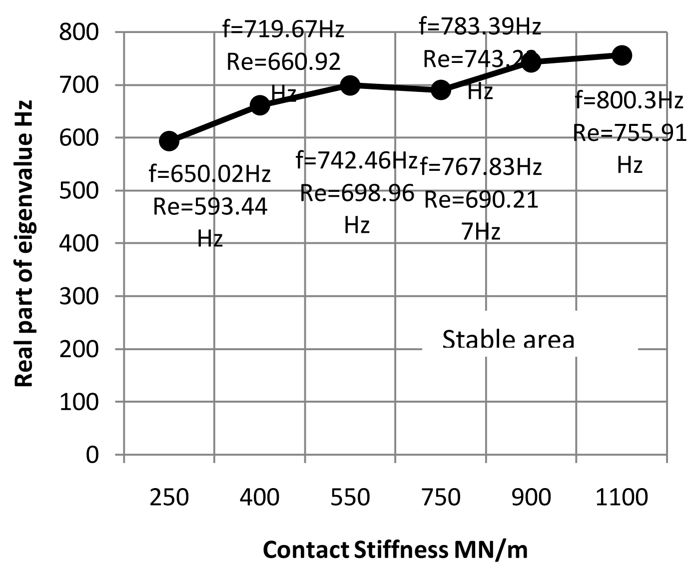
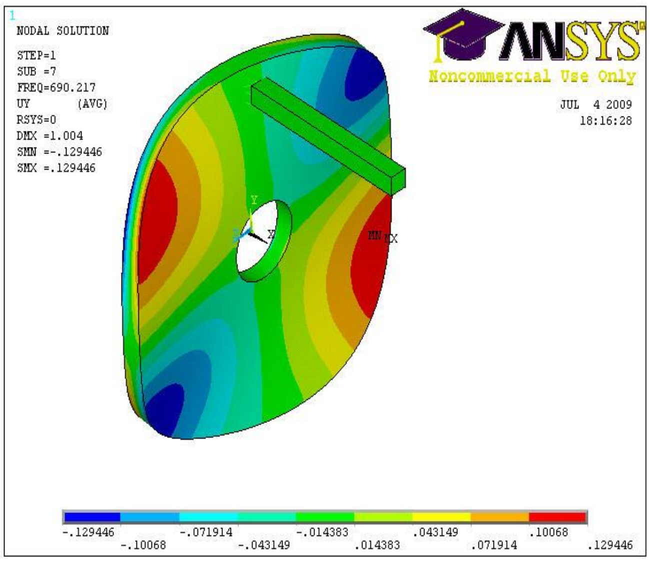


Figure 2 instability versus contact stiffness for the 3rd diametral mode shape, μ=0.4

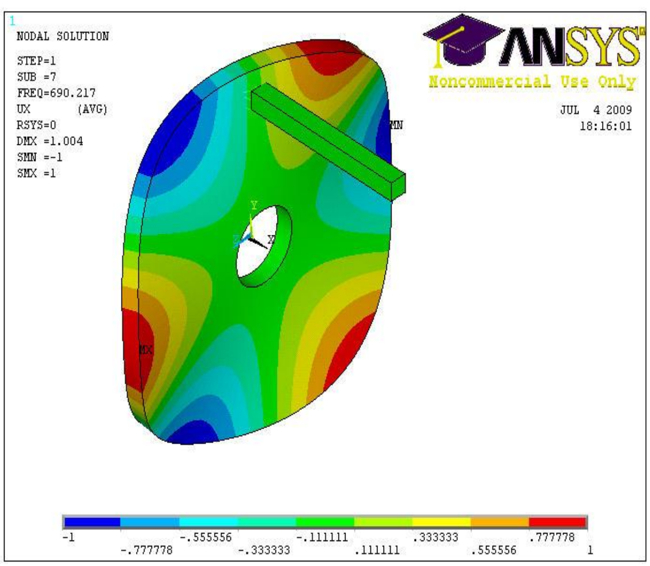
Figure 2 shows that increase the contact stiffness can increase the instability. This means that most fundamental method of eliminating brake squeal is by changing the contact stiffness for each frequency or rotation speed in order to transfer the system to the stable frequency.

4 SYSTEM MODE SHARE

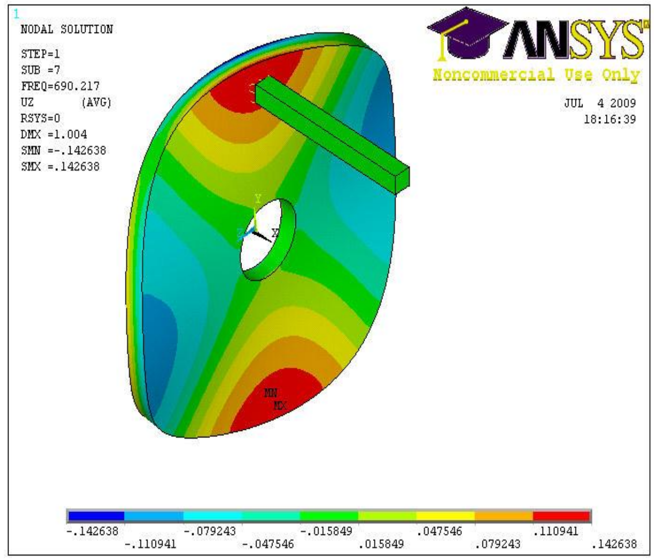
The purpose of the free-free finite element modal analysis is to identify the natural frequencies and mode shapes of the disc and pad as separate components. For, more information about the modal analysis for free-free method can be found in Ammar's paper [2] The mode shape at contact stiffness 750 MN/m is conducted to understand more about the relation between the instability and mode shape as in figure 3.



Mode shape in y-direction



Mode shape in x-direction



Mode shape in z-direction

Figure 3 Mode shape at contact stiffness750 MN/m

The mode shape in the x-direction showed that the disc in the third diametral mode shape. In y-direction and z-direction (normal to the beam) the mode shape is a second diametral mode. Figure 3 indicates that the coupling between in-plane, transverse, and vertical displacements of the rotor generates dynamic instabilities and brake squeal. The effect of the contact stiffness on the squeal propensity showed two significant unstable diametral. The first is 3rd diametral mode which is already presented in the figure 2 and the second is the 7th diametral mode which is presented in the figure 4. The analysis in figure 4 is carried out by changing the values of the contact stiffness while retaining the respective typical values for the others. The area of stability is illustrated in figure 4 by applying a range of contact stiffness values from 250 to 1100 MN/m with increment of 150 MN/m.

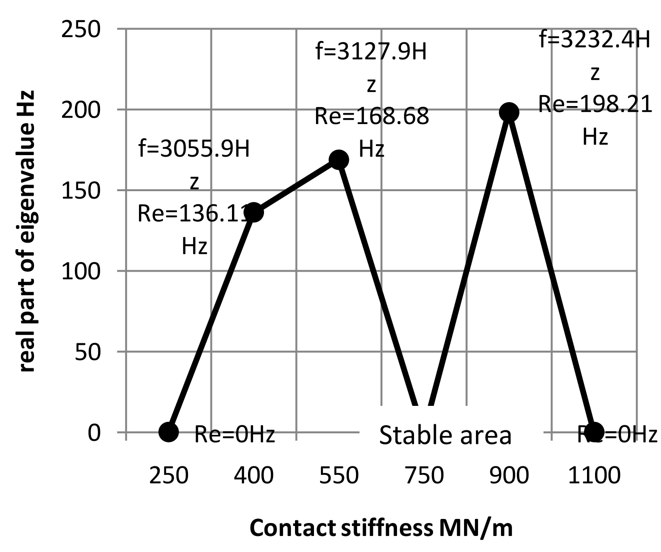


Figure 4 Instability versus contact stiffness for the 7th diametral mode shape, μ=0.4

Figure 4 shows that with certain contact stiffness 750 MN/m the system become stable for different disc rotation velocity. By comparing Figures 4 and 2 the instability in the figure 4 is higher than Figure 2, because the occurrence of the unstable frequencies is in the high frequency rang. This increment can be explained by change the pad stiffness. The value of the real part in the figure 4 is lower than Figure 2. This indicates that with higher diametral mode the system damping decreased. The effect of friction coefficient on the system stability is studied by running the simulation with different coefficient of friction values. The instability is conducted as in Figure 5 by applying a range of friction coefficient value from 0 to 0.05 with increment of 0.01.

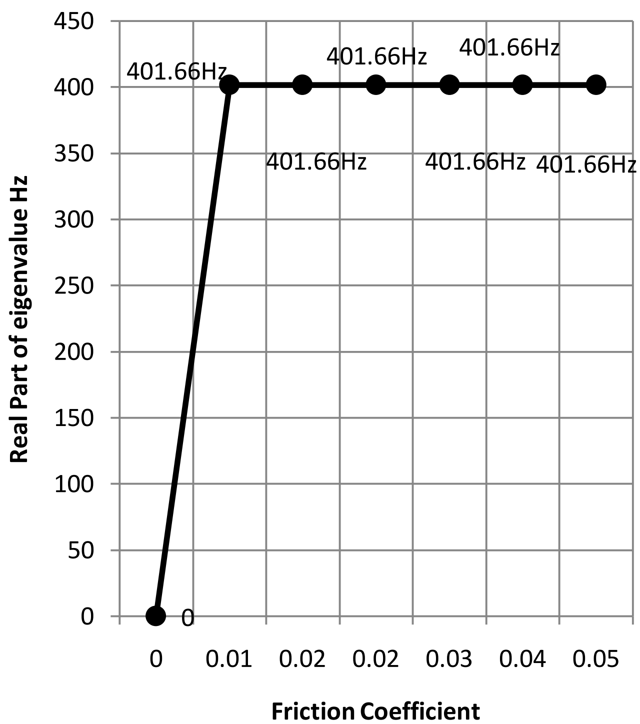


Figure 5 Real part of eigenvalue as a friction coefficient function, K=400 MN/m

The Figure 5 shows that the squeal stated with starting the friction coefficient value. The result indicates that the instability does not change for different friction coefficient values. This indicates that with high frequency the effects of friction coefficient will be constant. The value of the real part is approximately 401.66 Hz which stay constant with different friction coefficient. The sixth diametral mode shape is the mode which is showed in the figure 5. By changing the friction coefficient most of the other modes did not show any effect for increase the friction coefficient, in simple word the others mode shape did not tend to be unstable mode by increasing the friction coefficient. It can be said that the squeal generated due to the friction coefficient is related to the sixth diametral mode shape only. Figure 6 shows the effect of changing beam young's modulus on the system instability. The beam young's modulus is changed from 6 to 10 GN/m^2 with increments of 1 GN/m^2.

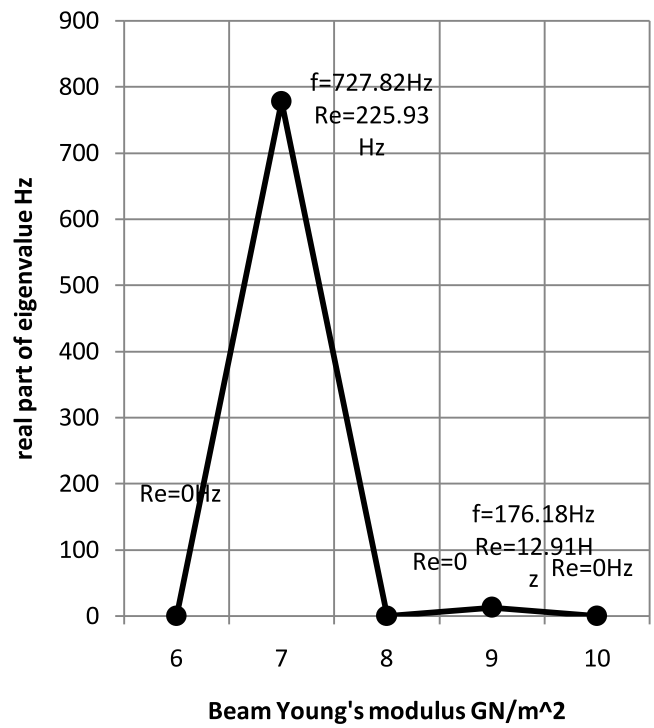


Figure 6 Instability as a function for beam young's modulus, K=600MN/m, μ=0.01

The Figure 6 showed that with increase young's modulus the instability will increase until it reaches to 225.93 Hz (real part). The system returned to be stable when the young's modulus reached to 8 GN/m^2. The system after that showed little instability. The results indicate that with increase young's modulus the system tends to be stable system. The result for the figure 6 is for the third and second diametral mode shape. An increase in the Young modulus of the beam affects the natural frequency of the mode with the node at the contact point, because it introduces an increasing stiffness at the contact point. The system natural frequency of the mode at the unstable point (225.93Hz, 727.82 Hz) decreased if its compared with the disc free mode, because the stiffness at the contact point increased due to increment of the young's modulus. The beam at this frequency showed penetration into the disc, as in the figure 7. This penetration make the system frequency decreased. It can said that the beam due to increase the contact stiffness trying to eliminate the disc rotation.

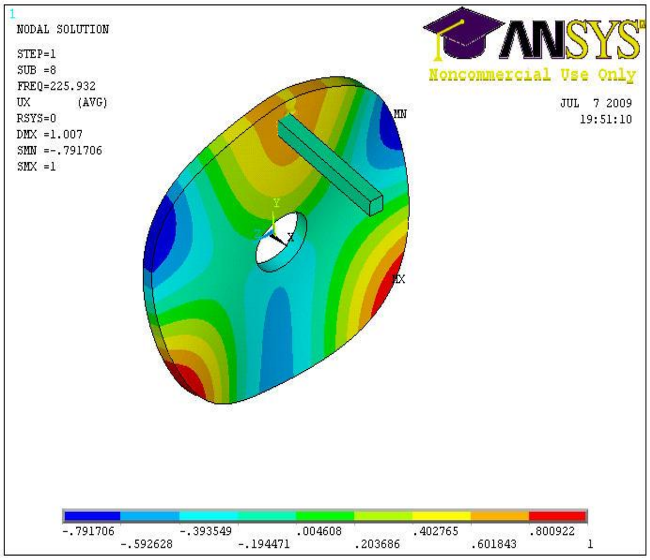


Figure 7 disc mode shape in the x-direction

The complex eigenvalue analysis as a function of the young's modulus was presented in Massi et al paper [11] at a friction coefficient 0.01. The mode at 3 kHz referred to the first mode of the beam. The mode at 3.5 kHz is characterized by a tangential translation of the support summed to its bending vibration (beam second mode shape). The natural frequency of the disc, third mode shape (0, 3+) increased up to the natural frequency of the support at 3 kHz and then to the one at 3.5 kHz. Likewise, the natural frequency of the forth mode shape (0, 4+) increases up to the frequency of the third beam mode at 5 kHz. Giannini et al [11] result indicates that the mode which responsible for the squeal due to the change beam young's modulus is the third and the forth mode shape, while the result in the Figure 6 indicates that the mode shape which is responsible about the squeal is the second and the third mode shape. This different in the result is due to the difference in the disc and beam dimensions, the number of the contact element (matrix27) and the value of the contact stiffness. Beside that Massi et al [11] did not considered one important factor in the matrix27 calculations. The distance between the beam and the disc will affect the result, which is not deal with it in the authors' work. This different in the dimension and the application matrix27 made the different for the mode which has a relation with generated noise.

5 HORIZONTAL BEAM CONTACT TOSTUDY THE EFFECT OF THE LATERAL VIBRATION PARAMETERS ON DISC BRAKE SQUEAL

A horizontal beam on rotating disc is modeled to study disc brake squeal. The system consists of disc which is rotate about the axis of the wheel. The beam is in contact with the disc by using matrix27. The beam is model to be parallel to the z-axis. The beam and the disc young's modulus, density, dimensions, boundary condition, mesh and element type is the same as the section 3. The system was run between the 1000 to 12000 Hz which represents the squeal frequency. The beam and the disc mesh must be coincident to enable the generation of the node-to-node contact element. The inclusion of the friction force causes stiffness matrix (matrix27) to be unsymmetrical. The system geometry is illustrated in the Figure 8.

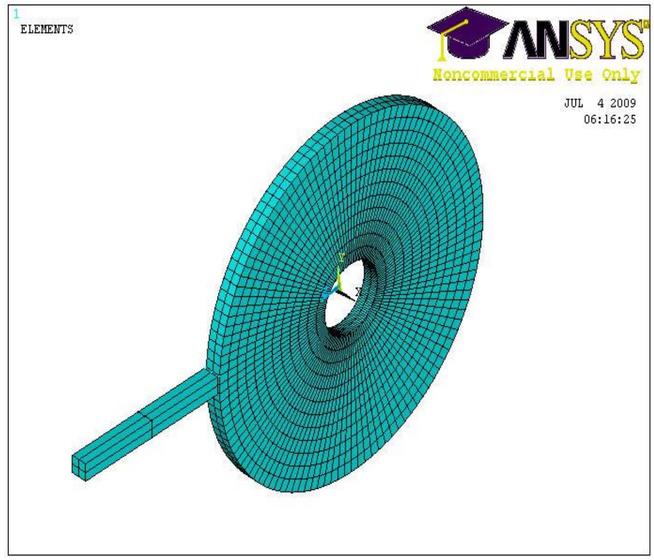


Figure 8 Geometry illustrate the mesh of horizontal beam with rotating disc

Matrix27 was developed in this paper in order to achieve the requirement of the beam horizontal contact with the disc. The out of the plane or diametral motion of the disc acts as a friction force on the horizontal beam which should be including in the contact matrix. The formula which represents the contact between the beam and the disc can be derived as fellow.

Normal force on the disc, $F_{z1}=K\,(z_1-z_2)$

Normal force on the beam, $F_{z2}=-K\,(z_1-z_2)$

Friction force on the disc, $F_{x1}=\mu F_{z1}=\mu K\,(z_1-z_2)$

Friction force on the beam, $F_{x2}=-\mu F_{z2}=\mu K\,(z_2-z_1)$

Friction force on the disc, $F_{y1}=-\mu F_{z1}=-\mu K\,(z_1-z_2)$

Friction force on the beam, $F_{y2}=\mu F_{z2}=-\mu K\,(z_2-z_1)$

The above equation can be arranged in matrix as follows.

$$\begin{bmatrix} Fx1 \\ Fy1 \\ Fz1 \\ Fx2 \\ Fy2 \\ Fy3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \mu K & 0 & 0 & -\mu K \\ 0 & 0 & -\mu K & 0 & 0 & \mu K \\ 0 & 0 & K & 0 & 0 & -K \\ 0 & 0 & -\mu K & 0 & 0 & \mu K \\ 0 & 0 & \mu K & 0 & 0 & -\mu K \\ 0 & 0 & -K & 0 & 0 & K \end{bmatrix} \begin{bmatrix} x1 \\ y1 \\ z1 \\ x2 \\ y2 \\ z2 \end{bmatrix}$$

The effect of the contact stiffness between the beam and the disc on the system stability has been investigated by simulation the model with different contact values. The analysis is carried out by changing the values of the contact stiffness while retaining the respective typical values for the others. The area of instability is conducted as in Figure 9 by applying a range of contact stiffness value from 200 to 1000 MN/m with increment of 200 MN/m.

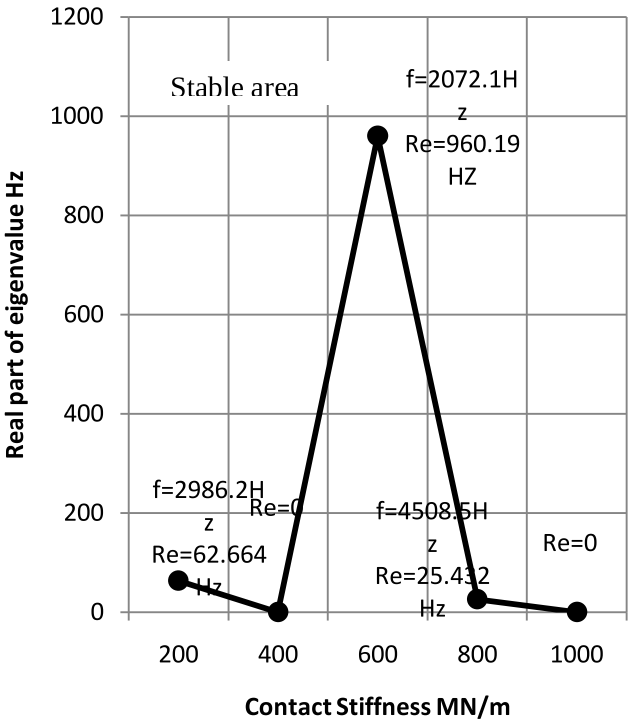


Figure 9 Real part of eigenvalue versus contact stiffness, μ=0.4

Figure 9 shows that the instability started to increase when the contact stiffness reach to 400 MN/m. This increments was linearly until it reach to 600 MN/m. the instability decrease until it diminish with contact stiffness 1000 MN/m. By comparing, figures 2 and 9 (perpendicular and horizontal beam position) at contact stiffness 600MN/m. The instability in the figure 2 was investigated with the third diametral mode (at frequency 767.83Hz) while in the figure 9 the instability is investigated with the sixth diametral mode (at frequency 2072.1 Hz). The contact stiffness is 600MN/m in both figures showed approximately same real part with different frequency that mean the damping ratio decreased with high frequency in the figure 9. Thus it can be said that the horizontal beam with lower contact stiffness made the squeal appeared at higher frequency than the perpendicular beam. This result showed that changing the pad position may lead to an increase in the unstable system frequency. The system in the figure 8 can vibrates until higher frequency range without any squeal because the mode which is responsible about the squeal is shafted to the higher frequency. The result in the figure 9 investigates a wide range of the area without any noise especially when the contact stiffness increases.

6 SYSTEM MODE SHAPE

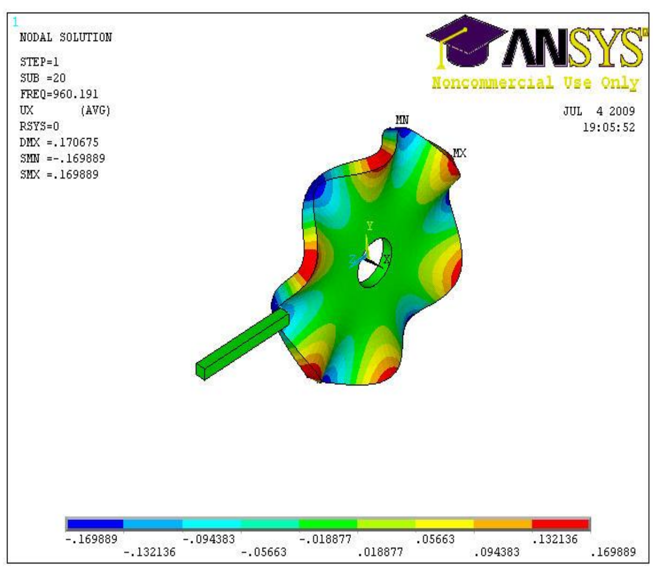
For a particular mode the eigenvalue pair is$S_{i1,2} = \sigma_i \mp j\omega_i$, σ_i is the real part and ω_i is the imaginary part for the ith mode. The motion for each mode can be described in terms of the complex conjugate eigenvalue and eigenvector:

$$\{u_i\} = \{\emptyset_i\}\, e^{\sigma_i t}(e^{j\omega_i t} + e^{-j\omega_i t}) \qquad \text{Equation (2)}$$

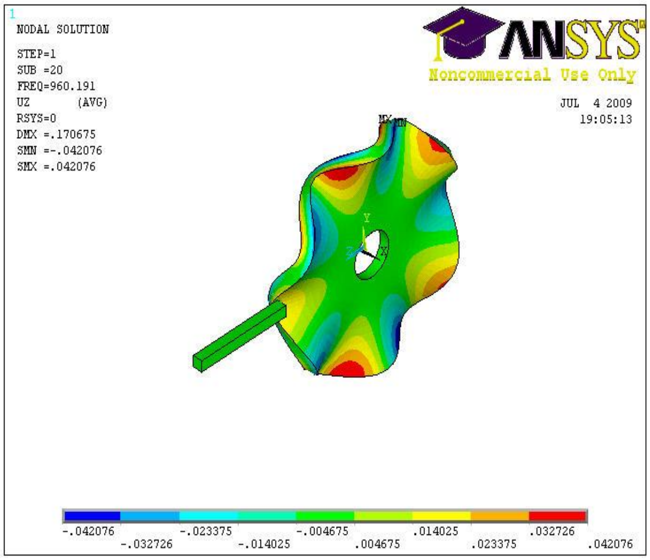
The displacement can be rewritten as a damped sinusoidal wave:

$$\{u_i\} = \{\emptyset_i\}\, e^{\sigma_i t}\cos\omega_i t \qquad \text{Equation (3)}$$

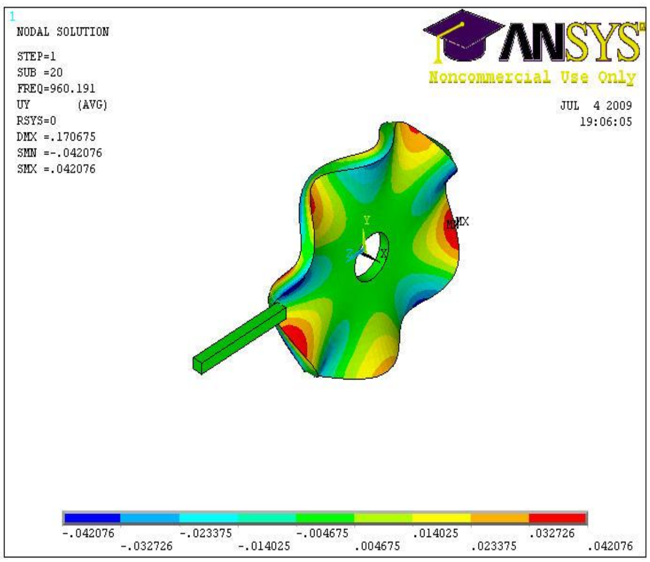
Thus, σ_i and ω_i are the damping coefficient and damped natural frequency describing damped sinusoidal motion. If the damping coefficient is negative, decaying oscillations typical of a stable system result. A positive damping coefficient, however, causes the amplitude of oscillations to increase with time. Therefore the system is not stable when the damping coefficient is positive. By examining the real part of the system eigenvalues the modes that are unstable and likely to produce squeal are shown in the figure 10 The mode shape at contact stiffness 600 MN/m is conducted (the real part showed positive value) to understand more about the relation between the instability and mode shape, figure 10. The mode shape is conducted in three directions to understand the relation between the lateral, diametral and vertical mode in producing brake squeal. The mode shape in the x-direction (normal to the beam) showed that the disc in the sixth diametral mode shape. In y-direction and z-direction the mode shape is the fifth diametral mode. Figure 10 indicates that the coupling between in-plane, transverse, and vertical displacements of the rotor generates dynamic instabilities and brake squeal. By comparing figures 3 and 10, the lateral mode is the higher diametral mode. Thus it can be said that the conclusion for the disc brake should be built on the lateral vibration of the disc. The figures 3 and 10 indicates that the horizontal vibration have the higher displacement than diametral and vertical mode.



Mode shape in x- direction



Mode shape in z-direction



Mode shape in y-direction

Figure 10 Mode shape with contact stiffness 600 MN/m

Figure 10 shows that the disc has three modes shape at the squeal frequency with rotating angle (phase) equal to 45 degree. That rotating angle related to the rule of the symmetric system. The effect of the horizontal beam density on the system stability has been investigated by simulation the model with different density values. The complex eigenvalue is performed in the range of the squeal occurrence. The analysis is carried out by changing the values of the density while retaining the respective typical values for the others. The instability which was conducted with the contact stiffness 600 MN/m as in the figure 9 is studied. The instability line is conducted in Figure 11 by applying a range of density value from 1500 to 3000 Kg/m^3 with increment of 1500 Kg/m^3.

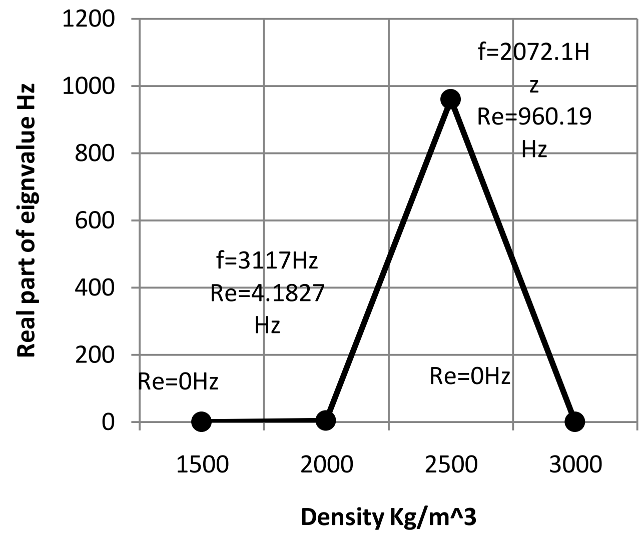


Figure 11 showed the instability as density function

Figure 11 indicates that by increasing the density of the beam or decrease it, the system tends to be stable again. The results showed that the pad density have significant effect on the system instability. Figure 12 shows the effect of changing horizontal beam young's modulus on the system stability. The beam young's modulus is changed from 0 to 2 GN/m^2. The result does not show any instability with the young's modulus value above 2 GN/m^2 until 10 GN/m^2.

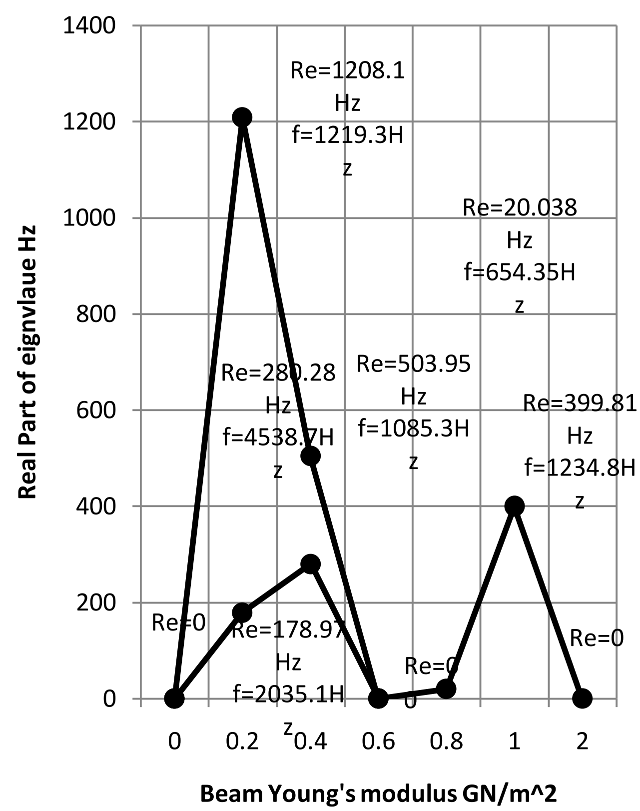


Figure 12 Instability depending on beam young's modulus, K=600 MN/m, μ=0.01

Figure 12 showed that with increase young's modulus the degree of the instability increased until the frequency reach to the 1219.3 Hz. The system returned to be stable when young's modulus reached to 0.6 GN/m^2 (no squeal is found). The system after that showed little increase in the degree of instability, until 2GN/m^2. The results indicate that with increase of young's modulus the system tends to be stable system. By comparing figures 6 and 12 the results indicate that with the horizontal beam position the instability appeared with the lower value of young's modulus than the perpendicular beam. The maximum value of the real part in the figure 6 appeared with the third diametral mode shape while the maximum value of the real part in the figure 12 appeared with the fifth diametral mode shape. This indicates that the beam in the horizontal position can shaft the brake noise to high frequency because the mode shape which represents the instability is increased. By comparing figures 11 and 12, the high unstable frequency which companied the effect of the density 2500 Kg/m^3 is 2072.1 Hz while the high unstable frequency companied with the effect of young's modulus is 1208.1 Hz. This result indicates that the effect of changing the density values on the instability can be higher than the effect of changing young's modulus on the instability because the unstable frequency is higher. The system showed two frequencies at beam young's modulus 0.2 and 0.4 GN/m^2. The mode at the frequency 2035.1Hz is fifth diametral mode and at frequency 4538.7Hz is eight diametral mode. The system can rotate at 0.2 GN/m^2 between these two frequencies (2035.1Hz, fifth diametral mode and 1219.3Hz, fourth diametral mode) without noise, but with this limitation it is difficult to control the noise.

The effect of friction coefficient on the system stability is studied by running the simulation with different coefficient of friction values. The area of stability is conducted as in Figure 13 by applying a range of friction coefficient value from 0.3 to 0.7 with increment of 0.1

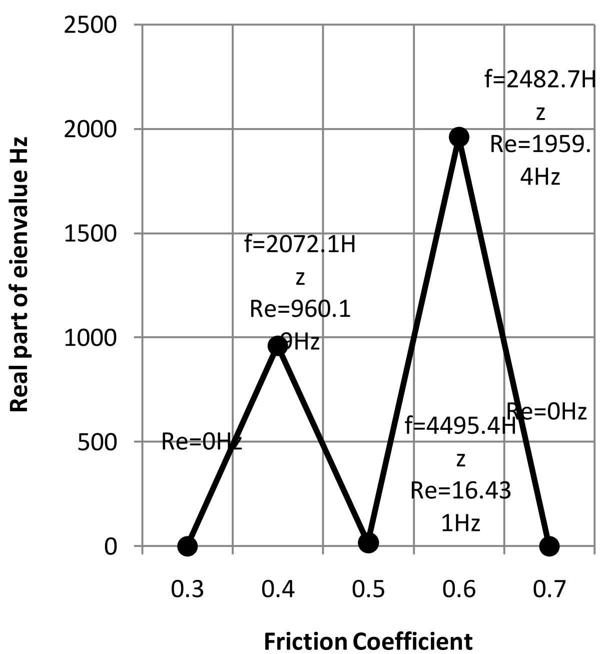


Figure 13 Instability of the system as a function for the friction coefficient, K=600 MN/m

Figure 13 shows that the squeal stated with friction coefficient value 0.3. The result showed that the real part increased until it reached to 960.19 Hz with friction coefficient 0.4 then decrease linearly until it reaches to 16.431Hz at friction coefficient 0.5. The real part stated to increase again until it reached to 1959.4Hz then decrease linearly to zero value. The mode shape of the figure 13 at real part 960.19Hz is fifth diametral mode with while the sixth and eight diametral modes appeared with the real part 1959.4Hz and 16.413Hz, respectively. By comparing figures 13 and 5 it could be seen that the line of the instability in figure 13 returned to be zero value at friction coefficient 0.3 and 0.7, while there is no stability point in the figure 5. This stability point appeared due to change of beam location relative to disc node. This means that the system tends to be noisier at certain frequencies than the system in the figure 5. It should be noted that even the friction coefficient is constants the verity of the friction forces is still possible. This indicates that the change in friction force value results from the verity of the normal force, thus sometimes there is no noise with real part frequency 2482.7Hz.

7 VERTICAL BEAM CONTACT TO STUDY THE EFFECT OF THE VERTICAL VIBRATION PARAMETERS ON DISC BRAKE SQUEAL

A vertical beam on rotating disc is modeled to study disc brake squeal. The system consists of disc which is rotate about the axis of the wheel. The beam is in contact with the disc by using matrix27. The beam is model to be parallel to the y-axis. The beam and the disc young's modulus, density, dimensions, boundary condition, mesh and element type is the same as the section 2. The system was run between the 1000 to 12000 Hz which represents the squeal frequency. The beam and the disc mesh (node) must be coincident to enable the generation of the node-to-node contact element. The inclusion of the friction force causes stiffness matrix (matrix27) to be unsymmetrical. The system geometry mesh is illustrated in the Figure 14. The aim of this study is to determine the effects of the node and antinode on brake squeal. The position of the pad should be considered to reduce the noise. Feildhouse [12] showed experimentally that the noise frequency can effect by the number of the node and antinode under the pad. However this can explain why the brake sometimes show a squeal at certain frequency and does not showing any noise at the same certain frequency.

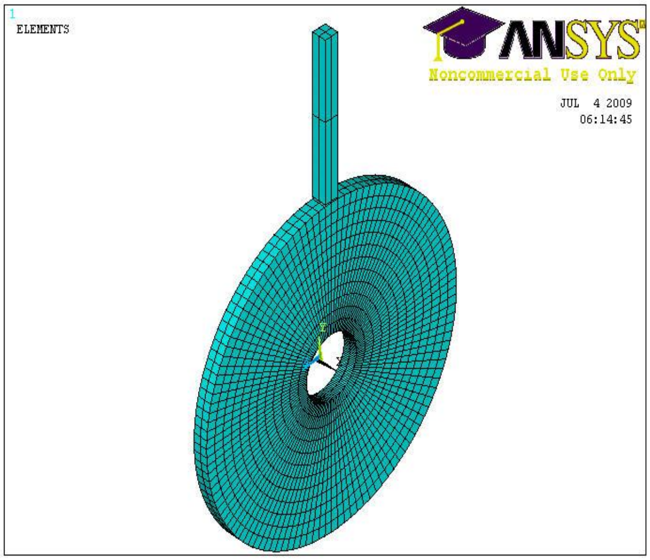


Figure 14 Geometry illustrates the mesh of vertical beam on rotating disc

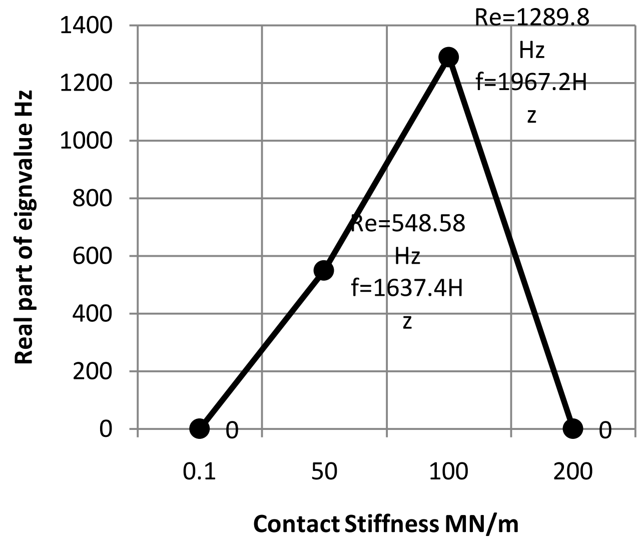


Figure 15 Real part of egienvalue versus contact stiffness MN/m, µ=0.4

The formula which represents the contact between the beam and the disc can be derived as follow.

Normal force on the disc, Fy1=K (y1-y2)

Normal force on the beam, Fy2=-K (y1-y2)

Friction force on the disc, Fx1=µFy1=µK (y1-y2)

Friction force on the beam, Fx2=-µFy2=µK (z2-y1)

Friction force on the disc, Fz1=-µFy1=-µK (y1-y2)

Friction force on the beam, Fz2=µFy2=-µK (y2-y1)

The above equation can be arranged in matrix as follows.

$$\begin{bmatrix} Fx1 \\ Fy1 \\ Fz1 \\ Fx2 \\ Fy2 \\ Fz2 \end{bmatrix} = \begin{bmatrix} 0 & \mu K & 0 & 0 & -\mu K & 0 \\ 0 & K & 0 & 0 & -K & 0 \\ 0 & -\mu K & 0 & 0 & \mu K & 0 \\ 0 & -\mu K & 0 & 0 & \mu K & 0 \\ 0 & K & 0 & 0 & -K & 0 \\ 0 & -\mu K & 0 & 0 & \mu K & 0 \end{bmatrix} \begin{bmatrix} x1 \\ y1 \\ z1 \\ x2 \\ y2 \\ z2 \end{bmatrix}$$

The effect of the contact stiffness between the beam and the disc on the system stability has been investigated by simulation the model with different contact values. The analysis is carried out by changing the values of the contact stiffness while retaining the respective typical values for the others. The area of instability is conducted as in Figure 15 by applying a range of contact stiffness value 0.1, 50, 100 and 200 MN/m.

Figure 15 shows that the instability stated to increase when the contact stiffness reach to 0.1 MN/m. This increment was linearly until it reaches to 50 MN/m. The real part continues the increment until it reached to the higher value of the instability (1289.8Hz). The instability decreased until it diminishes with contact stiffness 200 MN/m. By comparing the result of the figures 15 and 9 (vertical and horizontal beam position), the system showed increase in the degree of the instability. The instability in the figure 9 is investigated with the sixth diametral mode (at frequency 960.19Hz) while in the figure 15 the instability is also investigated with the fifth diametral mode (at frequency 1289.8Hz). At the contact stiffness 200MN/m the vertical beam's result showed stable system while in the horizontal beam the system is unstable system. This different in the result is related to the position of the node or the anti node which is in contact with the beam at the squeal, this point illustrates in the section 8. This result showed that changing the pad position may lead to an increase in the unstable system frequency for certain contact stiffness while decreased it for other. The system in the figure 15 can vibrates until higher frequency range without any squeal because the mode which is responsible about the squeal is shafted to the higher frequency from 960.19Hz to 1289.8 Hz. The result in the figure 15 investigates a wide range of the area without any noise especially when the contact stiffness increases above 200MN/m or decreased less than 0.1 MN/m. The effect of friction coefficient on the system stability is studied by running the simulation with different coefficient of friction values. The instability is conducted as in Figure 16 by applying a range of friction coefficient value from 0.1 to 0.7 with increment of 0.1.

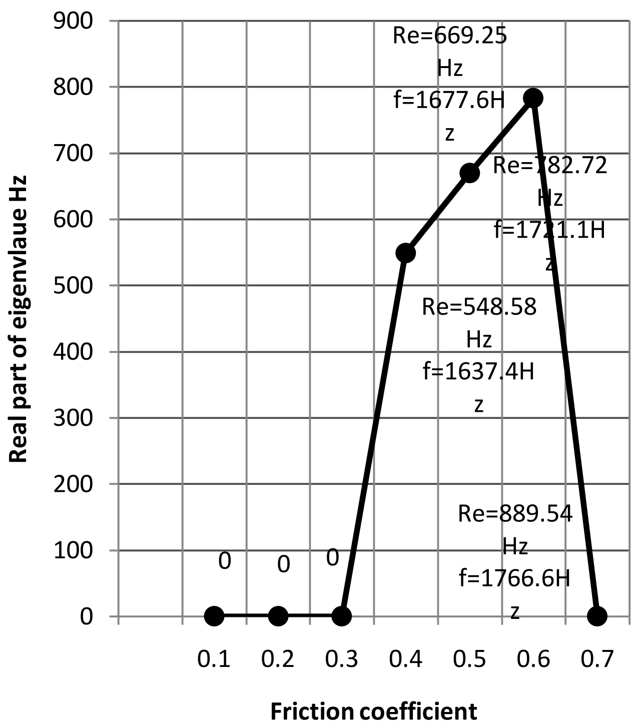


Figure 16 real part of eigenvalue as function for friction coefficient, K=50 MN/m

Figure 16 show that the squeal stated when the friction coefficient value becomes 0.3. The degree of the instability increased until the friction coefficient reach to 0.6 with frequency equal to 1721.1Hz. The result indicates that the degree of instability increased with increase the friction coefficient values and so on the frequency due to increase the instability. It is also clear that when the friction coefficient reach to 0.7 the system return to be stable system. The fourth diametral mode shape is the mode which is showed in the figure 16. By changing the friction coefficient most of the other modes did not show any effect for increase the friction coefficient, in simple word the others mode shape did not tend to be unstable mode by changing the friction coefficient. It can be said that the squeal generated due to the friction coefficient is related to the fourth diametral mode shape only. By comparing figures 16 for vertical beam and 13 for horizontal beam it is found that the squeal appear with the fifth, sixth and seventh for the vertical beam, while the squeal appear with the forth diametral mode only. This result showed that the pad position should be considered during the design in order to reduce the brake squeal. The complex eigenvalue analyses were carried out on each friction coefficient between 0.3 and 0.6 with increment 0.1 as shown in the figure 17.

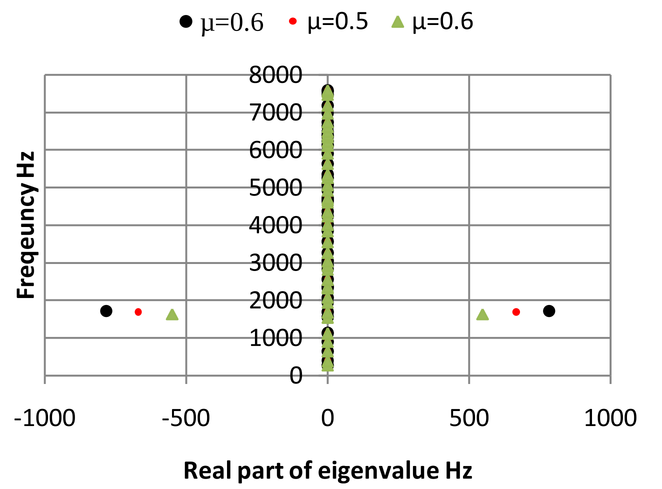


Figure 17 Real part of eigenvalue versus frequency as gauge to study the instability

The vertical axis is the imaginary part of eigenvalues, which is actually the natural frequency. The horizontal axis represents the real part of eigenvalues, which is measurement of damping or instability. System modes with positive real part of eigenvalues are modes with negative damping. Therefore, the right half-plane is the unstable region while the left half-plane is the stable region. It is seen that when the friction coefficient increased the degree of the instability (real part of eigenvalue) approach the vertical axis. The frequency decreased with decrease the real part of eigenvalue from 1721.1Hz (Re=782.72) to 1677.6Hz (Re=669.25Hz) then 1637.4Hz (548.58Hz). Figure 18 shows the effect of changing the vertical beam young's modulus on the system stability. The beam young's modulus is changed from 0.2 to 1 GN/m^2. The result does not show any instability with the young's modulus value above 1 GN/m^2 until 10 GN/m^2.

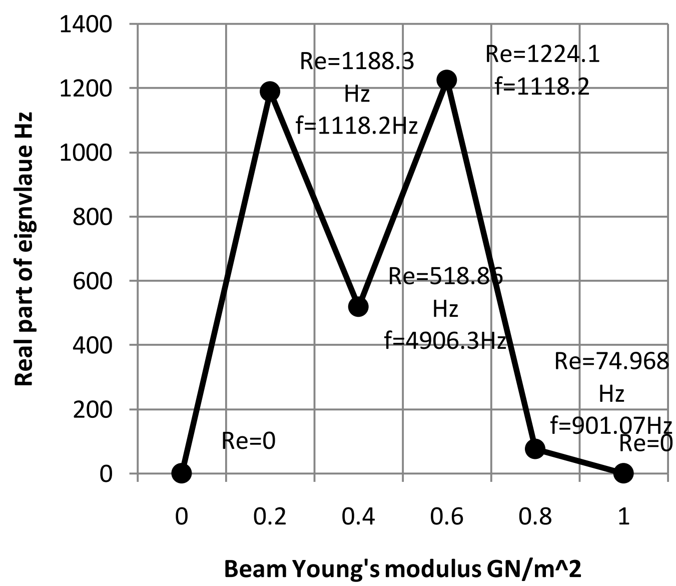
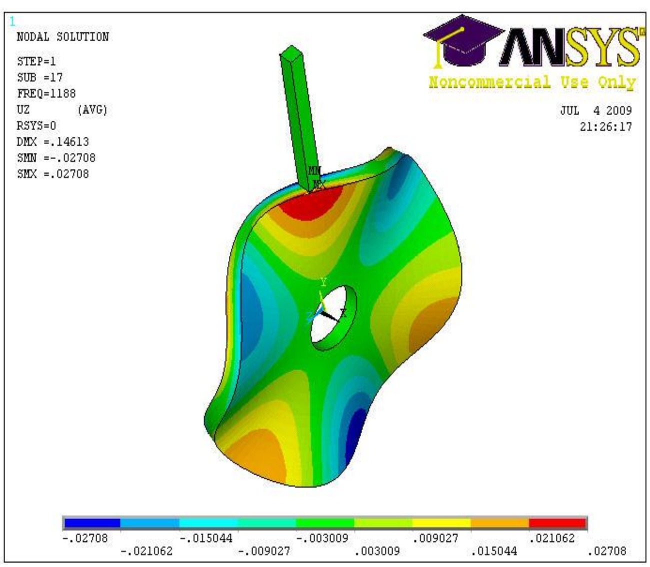


Figure 18 the relation between the instability and beam young's modulus, K=100MN/m, μ=0.01

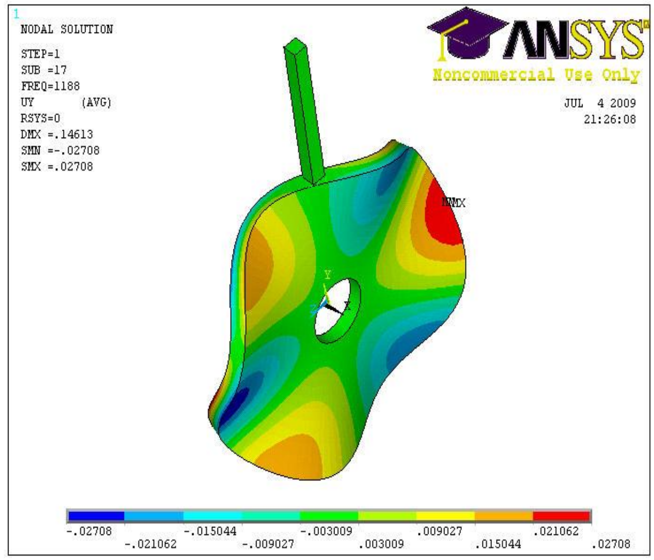
Figure 18 showed that with increase young's modulus the instability increased until the frequency reach to the 1118.2 Hz (third diametral mode). The degree of the instability is decreased when the young's modulus reached to 0.4 GN/m^2. The system at young's modulus 0.4 is squeal with the eight diametral mode for that the frequency is increase from 1118.2Hz at young's modulus 0.2GN/m^2 to 4906.3Hz at young's modulus 0.4GN/m^2. The system after that showed increase in the degree of instability until it reach to 0.6 GN/m^2. The results indicate that with increase young's modulus above 1GN/m^2 the system tends to be stable system. By comparing figures 20 and 14 the results indicate that with the horizontal beam position there are four diametral mode shape appeared (second, third, fifth and eight) while with the vertical beam position there are three diametral mode shape appeared (third, eight and second) for the same beam young' modulus from 0.2GN/m^2 to 1 GN/m^2. The result in the figure 12 showed that it is possible to appear two unstable modes at the same value of young's modulus while with the figure 18 only one unstable mode shape can appear at every young's modulus value. The instability can be achieved with the lower value of young's modulus 1 GN/m^2 for the vertical beam than the horizontal beam. The maximum value of the real part which indicates to the degree of the instability in the figure 12 appeared with the fourth diametral mode shape (1208.1Hz) while the maximum value of the real part in the figure 18 appeared with the third diametral mode shape (1224.1 Hz). This indicates that the beam in the vertical direction can shows higher degree of instability with lower mode shape than the beam in the horizontal direction.

8 SYSTEM MODE SHAPE

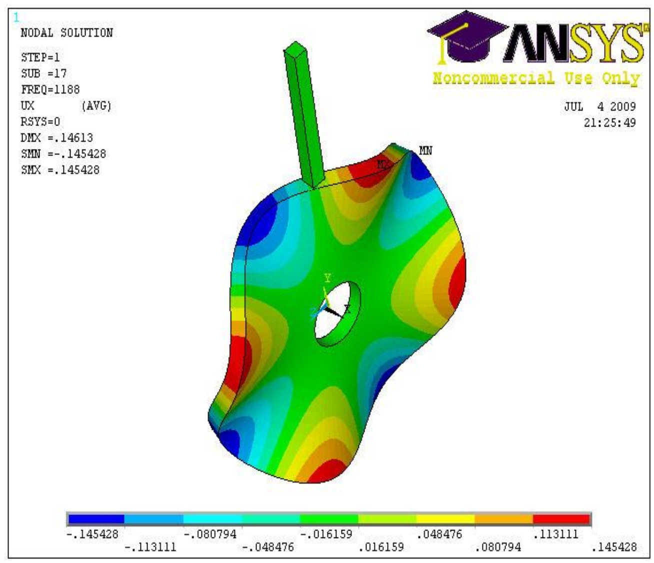
The mode shape at young's modulus 0.2 GN/m^2 is conducted to understand more about the relation between the instability and mode shape as in figure 19. The unstable mode in the figure 19 is conducted with imaginary value equal to zero while the two real parts have the same value with opposite signs. This indicated that the system damping ratio reached to its critical value.



Mode shape in z-direction



Mode shape in y-direction



Mode shape in x-direction

Figure 19 mode shape of the system at beam young's modulus 0.2 GN/m^2

The mode shape in the x-direction showed that the disc in the fourth diametral mode shape. In y-direction (normal to the beam) and z-direction the mode shape is a third diametral mode. Figure 19 indicates that the coupling between in-plane, transverse, and vertical displacements of the rotor generates dynamic instabilities and brake squeal. The mode shape of y-direction in the figure 19 which is already perpendicular to the beam showed that the beam during the squeal at the node position. The mode shape of the figure 10 which is already perpendicular to the beam showed that the beam in the antionode position. That different between the effect of the node and the antinode is the major reason for the result different.

9 CONCLUSIONS

The results showed that the disc beam (pad) is the source of generate the disc brake squeal. One easy method to reduce the brake squeal is by changing the pad density. As showed previously for the friction coefficient, young's modulus and density can change the elastic properties of the friction material, since it changes the pad stiffness which can alter the modal coupling between the pad and the rotor. The pad stiffness is very important parameter to determine the stability of the system. The center of the beam can be considered as the center of the pad which showed considerer in the brake pad design. The result showed that even the beam and disc at natural frequency, no squeal generated if the beam at node position.

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