

Optimal Digital Filter Design: A Survey

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Abstract

In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range. There are two types of filter analog and digital. FIR Filter is the kind of digital filter, which can be used to perform all kinds of filtering i.e. high pass, low pass, band pass and band reject etc. Many researchers have worked on performance improvement of these filters yet there is still space for that. In this paper we have reviewed the work for optimal filter design so that a new proposal overcoming the limitations can be proposed.

Keywords: : Filter, Signal processing, Noise

1. Introduction

In signal processing, there are many instances in which an input signal to a system contains extra unnecessary content or additional noise which can degrade the quality of the desired portion. In such cases we may remove or filter out the useless samples. For example, in the case of the telephone system, there is no reason to transmit very high frequencies since most speech falls within the band of 400 to 3,400 Hz. Therefore, in this case, all frequencies above and below that band are filtered out. The frequency band between 400 and 3,400 Hz, which isn't filtered out, is known as the pass band, and the frequency band that is blocked out is known as the stop band.

There are two basic types of digital filters, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters. The general form of the digital filter difference equation is

$$y(n) = \sum_{i=0}^N a_i x(n-i) - \sum_{i=0}^N b_i y(n-i) \quad (1)$$

where, $y(n)$ is the current filter output, the $y(n-i)$ are previous filter outputs, the $x(n-i)$ are current or previous filter inputs, the a_i are the filter's feed forward coefficients corresponding to the zeros of the filter, the b_i are the filter's feedback coefficients corresponding to the poles of the filter, and N is the filter's order.

FIR, Finite Impulse Response, filters are one of the primary types of filters used in Digital Signal Processing. FIR filters are said to be finite because they do not have any feedback. Therefore, if

we send an impulse through the system (a single spike) then the output will invariably become zero as soon as the impulse runs through the filter.

IIR filters have one or more nonzero feedback coefficients. That is, as a result of the feedback term, if the filter has one or more poles, once the filter has been excited with an impulse there is always an output. FIR filters have no non-zero feedback coefficient. That is, the filter has only zeros, and once it has been excited with an impulse, the output is present for only a finite (N) number of computational cycles [3]. Because an IIR filter uses both a feed-forward polynomial (zeros as the roots) and a feedback polynomial (poles as the roots), it has a much sharper transition characteristic for a given filter order. Like analog filters with poles, an IIR filter usually has nonlinear phase characteristics. Also, the feedback loop makes IIR filters difficult to use in adaptive filter applications.

Due to its all zero structure, the FIR filter has a linear phase response when the filter's coefficients are symmetric, as is the case in most standard filtering applications. A FIR's implementation noise characteristics are easy to model, especially if no intermediate truncation is used. In this common implementation, the noise floor is at $-6.02 B + 6.02 \log_2 N$ dB, where B is the number of actual bits used in the filter's coefficient quantization and N is again the filter order. That's why most Intersil filter ICs have more coefficient bits than data bits. An IIR filter's poles may be close to or outside the unit circle in the Z plane. This means an IIR filter may have stability problems, especially after quantization is applied. An FIR filter is always stable [3].

A digital filter is characterized in terms of difference equations. There are two types of digital filters, they are non-recursive, and recursive filters which are characterized based on their responses [1].

The response of a non-recursive filter at any instant depends on the present, past and future values of the input. At any specific instant nT . The response is of the form

$$y(nT) = f(\dots, x(nT - T), x(nT), x(nT + T) \dots) \tag{2}$$

Assuming linearity and time-invariance $y(nT)$ can be expressed as

$$y(nT) = \sum_{i=-\infty}^{\infty} a_i x(nT - iT) \tag{3}$$

where a_i represents constants.

Now assuming causality for the filter we have

$$a_{-1} = a_{-2} = \dots = 0$$

In addition, assuming $a_i = 0$ for $i > N$ the response can be written as Nth-order linear difference equation given as:

$$y(nT) = \sum_{i=0}^N a_i x(nT - iT) \tag{4}$$

Such a linear, time-invariant, causal, non-recursive filter represented as Nth-order linear difference equation is called the Finite Impulse Response (FIR) filter.

When a unit impulse defined as

$$\delta(nT) = \begin{cases} 0 & \text{for } n \neq 0 \\ 1 & \text{for } n = 0 \end{cases}$$

is applied to the system described by Equation (1.4), then the response, which is nothing but the impulse response $h(nT)$ is given as

$$y(nT) = \sum_{i=0}^N a_i \delta(nT - iT) \quad (5)$$

From the above equation it can be inferred that the impulse response is finite and from the property of the impulse function we can see that the constants a_i are nothing but the samples of the impulse response. That means

$$h(0) = a_0, h(T) = a_1, \dots, h(nT) = a_n \quad (6)$$

These constants are called the filter coefficients. They determine the type of the filter, whether it is Low-pass, or High-pass, etc. Thus in filter design it is always important to find the filter coefficients which mostly approximates the desired response. In general, one can view equation 1.3 as a computational procedure (an algorithm) to determine the output sequence $y(nT)$ from the input sequence $x(nT)$. In addition, in various ways, the computations in equation 1.3 can be arranged into equivalent sets of difference equations. Normally such a kind of re-arrangement of the basic difference equation is done, to gain benefits in terms of memory, time-delays, computational complexity, etc. before implementing the system in the computer. Each set of equations defines a computational procedure or an algorithm for implementing it in a digital computer system [1]. In the next session of this paper we will discuss the work done in recent years for optimum filter design which is followed by conclusion of survey.

2. Literature Review

Abderrahmane Ouadi et.al. [1] considered the multiobjective design of digital filters using the powerful Taguchi optimization technique. At start up, only magnitude response has been considered in the optimization task. The resulting filter was good in terms of this characteristic while it showed awful dynamic and phase performance. Next, the dynamic properties were included in the optimization algorithm to solve a multiobjective task. The Taguchi optimization method has succeeded in attaining the optimal design in terms of the previous requirements by achieving a compromise between them.

The optimized filter has been tested and it showed good performance with required practical characteristics. Same author also presented the work in [2] using spiral optimization technique. This new optimization tool is a metaheuristic technique inspired by the dynamics of spirals. It is characterized by its robustness, immunity to local optima trapping, relative fast convergence and ease of implementation. The objectives of filter design include matching some desired frequency response while having minimum linear phase; hence, reducing the time response. The results demonstrate that the proposed problem solving approach blended with the use of the spiral optimization technique produced filters which fulfill the desired characteristics and are of practical use. Balraj Singh et.al. [3] presented the methodology for the robust and stable design of infinite impulse response (IIR) digital filters using hybrid differential evolution method.

Differential Evolution (DE) is undertaken as a global search technique and exploratory search is exploited as a local search technique. Through simulation it has been shown that the DE method works well with an arbitrary random initialization and it satisfies prescribed amplitude specifications consistently. The proposed DE approach for the design of digital IIR filters allows each filter, whether it is LP, HP, BP, or BS filter, to be independently designed. The proposed DE is very much feasible to design the digital IIR filters, particularly with the complicated constraints. Parameters tuning still is the potential area for further research. The unique combination of exploration search and global search optimization method that is DE provided by the two types of algorithms yields a powerful option for the design of IIR filters. S. K. Saha [4] used a novel opposition-based harmony search (OHS) algorithm is applied to the solution of the constrained, multimodal FIR filter design problem, yielding optimal filter coefficients. Comparison of the results of PM, RGA, PSO, DE, and OHS algorithm is also presented. It is revealed that OHS has the ability to converge to the best quality near optimal solution and

possesses the best convergence characteristics in moderately less execution times among the algorithms.

The simulation results clearly indicate that OHS demonstrates better performance in terms of magnitude response, minimum stop band ripple, and maximum stop band attenuation with a very little deterioration in the transition width. Xiaobo Liu [5] improved the work of [3] by improving the DE algorithm with HS (harmony search) has been proposed, which not only increases the population's diversity, but also avoids the parents being selected from just two populations in one generation. Although the DE algorithm has reached an impressive state, the enhancements of the population is still one open problem. DEHS is a rotationally invariant way to generate more potential points without increasing the number N_p of population members. On one hand, DEHS can use the pitch-adjustment to improve the individuals that get better optimal results. On the other hand, DEHS makes a new vector after considering all existing vectors rather than considering only two (parents) as in DE. So, compared with standard DE, DEHS has similar capacity with DE on the usual test functions and the low-dimensional and multi-modal functions, but has better ability than DE on high-dimensional functions. Harpreet Kaur [6] proved that Artificial Neural Network is better and easy method of design of low Pass FIR Filter. Also, using Fourier series, Frequency sampling or Window methods the filter can be design but for each unknown parameter the filter coefficients have to calculated.

In comparison with ANN, the trained network can calculate the filter Coefficient for unknown parameter in that specified range. Using ANN if error graphs are drawn between ANN output and Kaiser Window method is almost zero if we use radial basis function. Following the line of action of [5] Ezgi Deniz Ulker [7] proposed the new hybrid algorithm, called HDPH, is proposed to achieve a robust algorithm with a good solution quality by combining the three well-known algorithms, DE, PSO and HS. The performances of chosen algorithms are based on the parameter selection. Therefore, all combination of parameter values are tested for each function for all three algorithms and the results that are tabulated are selected as the best values obtained through all possible trials. However, in the HDPH algorithm the parameters are chosen randomly in the given ranges which make the algorithm easier to implement. Even with this kind of simplification in HDPH algorithm, the good performance is verified. Also, the experimental results have shown that, when both solution quality and robustness of an algorithm are taken into consideration, in most of the test functions, HDPH is more robust than the other three algorithms. Shruti Singh [8] discussed the designing of FIR filter using particle search optimisation (PSO). This method is shown to be better in terms of magnitude and phase response than the conventional method. In addition, it is observed to produce maximum stop band attenuation with optimal solution. The design method and selection of parameters is also shown to be easier. Ankan Bhattacharya [9] modified window function for Finite Impulse Response (FIR) filter design with an improved frequency response.

The window function is generated and has been utilized to compute the frequency response of various types of FIR filter i.e. high pass, low pass, band pass and band stop types. Noticeable improvement is observed when the newly generated frequency response is compared with various other frequency responses using the other existing windows in perspective of stop band attenuation and presence of ripples, which is justified by relevant simulations and plots. In this paper the frequency response of the modified window is compared with that of Hamming window with quantized and generalized experimental results. Nuapett Sarasiri [10] compared the performance of adaptive bacterial foraging optimization (ABFO), the adaptive tabu search (ATS) and the cooperative algorithms thereof. Assessment scenario adopts some 3D surface optimization problems, one of which contains many global solutions. The computing results are elaborated, and indicate the superiority of the cooperative BF-TS algorithms. The article also presents an application of the proposed algorithms to the control of a hard disk head-stack. Vijay. P et. al [11] used PSO optimisation for IIR filter design as was in [8]. Sangeeta Mondal [12, 14]

improved the conventional particle swarm optimization (PSO) that proposes a new definition for the velocity vector and swarm updating and hence the solution quality is improved.

The inertia weight has been modified in the PSO to enhance its search capability that leads to a higher probability of obtaining the global optimal solution. The key feature of the proposed modified inertia weight mechanism is to monitor the weights of particles, which linearly decrease in general applications. In the design process, the filter length, pass band and stop band frequencies, feasible pass band and stop band ripple sizes are specified. FIR filter design is a multi-modal optimization problem. Evolutionary algorithms like real code genetic algorithm (RGA), particle swarm optimization (PSO), and the novel particle swarm optimization (NPSO) have been used in this work for the design of linear phase FIR low pass (LP) filter. A comparison of simulation results reveals the optimization efficacy of the algorithm over the prevailing optimization techniques for the solution of the multimodal, non-differentiable, highly non-linear, and constrained FIR filter design problems. Rajib Kar [13] combined the Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSO-CFIWA) for the design of linear phase digital low pass FIR filter using. FIR filter design is a multimodal optimization problem.

The conventional gradient based optimization techniques are not efficient for digital filter design. Given the filter specification to be realized, PSO algorithm generates a set of filter coefficients and tries to meet the ideal frequency characteristic. The simulation results have been compared with the well accepted evolutionary algorithm such as genetic algorithm (GA). The results justify that the proposed filter design approach using PSO-CFIWA outperforms to that of GA, not only in the accuracy of the designed filter but also in the convergence speed and solution quality. Yadwinder Kumar [15] combined Particle Swarm Optimization (PSO) and GA. The magnitude response and filter coefficients are demonstrated for different optimization techniques. Comparison the various optimization techniques has been done the hybrid optimization performs better in comparisons to genetic algorithm and particle swarm optimization. This in turn improves design efficiency as well as the algorithm's numerical stability which is of critical importance for the design filter. Design examples with comparisons are presented to illustrate the effectiveness of the hybrid optimization method. Pardeep Kaur [16] designed the FIR filter using Genetic Algorithm (GM) and its comparison is done with Blackman, Parks McClellan in MATLAB.

The comparisons are done to improve the magnitude response, phase variation and phase delay. The response is studied by keeping values of fixed order, crossover probability and mutation probability. Suman Kumar [17] used bacteria foraging optimization (BFO) for the design of linear phase positive symmetric FIR low pass, high pass, band pass and band stop filters, realising the respective ideal filter specifications. BFO is a population-based evolutionary optimisation concept used to solve nonlinear optimisation problem where each individual maintains the propagation of genes. BFO favours propagation of genes of those animals which have efficient foraging strategies and eliminate those animals that have weak foraging strategies i.e., method of finding, handling and taking in food. All animals with their own physiological and environmental constraints, try to maximise the consumption of energy per unit time interval. The performances of BFO-based FIR filter designs have proven to be superior as compared to those obtained by real coded genetic algorithm (RGA) and standard particle swarm optimisation (PSO) optimisation techniques. The simulation results of author' claims that BFO is the best optimiser among the other optimisation techniques, not only in the convergence speed but also in the accuracy and the optimal performances of the designed filters.

3. Conclusions

Digital filter design is always a field which seeks modification is present work as communication data is lost by noise or bandwidth consumed by these filters and these impose a limit to communication channel too. So we studied many papers of recent years so that we can be familiar

with latest work. It has been observed that optimal designing of digital filters is the field flooded with use of hybrid bio inspired algorithms. Because of their performance effectiveness and good convergence power to optimal point, these bio inspired algorithm's combinations are found in every engineering field, so is in our also. Many researchers tested the combination of algorithms with PSO as PSO is fast converging algorithm.

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