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Designing a Software Application for Fibered Fano Planes

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ABSTRACT: *In this work, a computer program is designed to determine all fibered projective planes with base projective plane which is Fano plane. By applying this program we obtain the fibered points and lines of all these fibered projective planes using membership degrees of the points.*
Keywords*: fibered projective plane, fibered point, fibered line.*
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I. INTRODUCTION

Zadeh introduced the concept of fuzzy set in [13]. This concept is extended to vector spaces and projective spaces, [2,6,8,11]. Leen Kuijken introduced in [10,12] a new model of fuzzy projective geometries which is reacher than the model given in [11]. Leen Kuijken and H. Van Maldeghem defined *fibered points* and *fibered lines* in general incidence geometries , which they called *fibered geometries*.
Many notions and results in fibered projective planes are given in extensive study (see, for example, [3,4,5,10,11]).
Many of researchers worked to develop a computer package that will allow geometers to use the computer for research the points and lines of projective planes of small order in incidence geometry.
In this paper, we give a software application on ".Net platform" using C# codes, [9]. It determines points and lines of the fibered projective planes with the base projective plane of order 2, Fano plane, by using minimum operator when arbitrary membership degrees in $]0,1]$ are given the points. This application will be useful to construct the different fibered projective planes with the same base plane and also point to the comparison possible of the obtained fibered projective planes.

II. PRELIMINARIES

We denote by $\wedge$ a triangular norm on the (real) unit interval $[0,1]$, i.e., a symmetric and associative binary operator satisfying $(a \wedge b) \leq (c \wedge d)$ whenever $a \leq c$ and $b \leq d$, and $a \wedge 1 = a$, for all $a, b, c, d \in [1,2]$.
Let $\mathbf{P} = (P, B, \sim)$ be any projective plane with point set P and line set B, i.e., P and B are two disjoint sets endowed with a symmetric relation $\sim$ (called the *incidence relation*) such that the graph $(P \cup B, \sim)$ is a bipartite graph with classes P and B, and such that two distinct points p, q in $\mathbf{P}$ are incident with exactly one line (denoted by $\langle pq \rangle$), every two distinct lines L, M are incident with exactly one point (denoted by $L \cap M$), and every line is incident with at least three points. A set S of *collinear* points is a subset of P each member of which is incident with a common line L. Dually, one defines a set of *concurrent* lines. We now define fibered points and fibered lines, briefly called *f*-points and *f*-lines.
Definition (*see Zadeh* [13]) A fuzzy set λ on a set X is mapping $\lambda : X \to [0,1]: \quad x \to \lambda(x)$. The number $\lambda(x)$ is called the degree of membership of the point x in λ.
Definition (*see Kuijken and Maldeghem* [10]) *Suppose* $a \in P$ *and* $\alpha \in]0,1]$. *Then an f-point* (a, α) *is the following fuzzy set on the point set* P *of* $\mathbf{P}$ *:*

$$(a,\alpha): P \to [0,1]: \begin{cases} a \to \alpha \\ x \to 0 \quad \text{if} \quad x \in P \setminus \{a\}. \end{cases}$$

Dually, one defines in the same way the f-line (L, β) *for* $L \in B$ *and* $\beta \in]0,1]$.
The real number α *is called the membership degree of the f-point* (a, α), *while the point* a *is called the base point of it. Similarly for f-lines.*
Two f-lines (L, α) *and* (M, β), *with* $\alpha \wedge \beta > 0$, *intersect in the unique f-point* $(L \cap M, \alpha \wedge \beta)$.

Dually, the f *-points* (a,λ) *and* (b,μ) *, with* $\lambda \wedge \mu > 0$ *, span the unique f-line* $(\langle a,b\rangle, \lambda \wedge \mu)$ *.*

Definition *A (nontrivial) fibered projective plane* $\mathbf{FP}$ *consists of a set* FP *of f-points of* $\mathbf{P}$ *and a set* FB *of f-lines of* $\mathbf{P}$ *such that every point and every line of* $\mathbf{P}$ *is the base point and base line of at least one f-point and f-line, respectively (with at least one membership degree different from* 1 *), and such that* $\mathbf{FP} = (FP, FB)$ *is closed under taking intersections of f-lines and spans of f-points. Finally, a set of f-points are called collinear if each pair of them span the same f-line. Dually, a set of f-lines are called concurrent if each pair of them intersect in the same f-point.*

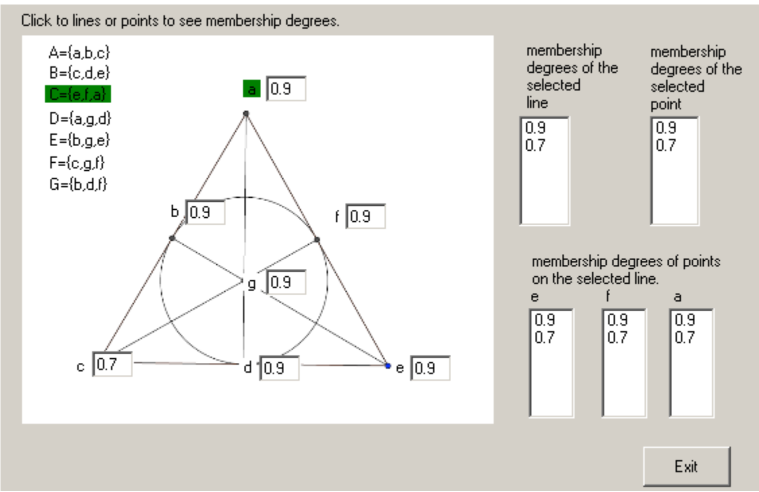
III. THE CONSTRUCTION OF FIBERED FANO PLANES

We consider the projective plane $\mathbf{P}$ of order 2, the Fano plane. We will construct a mono-point-generated fibered projective planes with base plane $\mathbf{P}$ by using minimum operator.
The fibered projective plane $\mathbf{FP}$ can be constructed as the following (see [12]).
Let $P' \subseteq P$ and $B' \subseteq B$ be such that the unique closed configuration containing $P' \cup B'$ is $P \cup B$. For each element x of $P' \cup B'$, a nonempty subset $\sum_x$ of $]0,1]$ is chosen arbitrarily, its elements are called the initial values of x. For each $x \in P' \cup B'$ and for each $\alpha \in \sum_x$, the element (x,α) belongs to $\mathbf{FP}$. This is step 1 of the construction. For step i, $i > 1$, any pair of f-points that we already obtained the f-line spanned by it also belongs to $\mathbf{FP}$ by definition and dually for any pair of f-lines the intersection f-point belongs to $\mathbf{FP}$.
The set of all f-points and of all f-lines of a fibered projective plane are constructed this way in a finite number of steps.
Now suppose $\sum_x$ is a singleton for ever $x \in P' \cup B'$. If $P' = P$ and $B' = \varnothing$, then we call the fibered projective plane mono point generated.
It is very difficult to determine all fibered points and lines of a fibered projective plane with base plane $\mathbf{P}$ by giving the initial values the points of $\mathbf{P}$. This program treats generating of f-points and f-lines of fibered projective planes with base plane $\mathbf{P}$ by using minimum operator. So one can generate different fibered projective planes by giving to every point (only to points) one (and only one) degree of membership from $]0,1]$ by using this program.

Some remarks
We get different fibered Fano planes have interesting properties when changing the initial conditions. Output results of this program consist of the tables containing the membership degrees of all fibered points and lines.

Fibered Fano planes with one values
If the initial values of all points in a Fano plane are same, all f-lines and f-points of the fibered Fano projective plane have one membership degree.

Fibered Fano planes with two different values
1) If the initial value α of one point is smaller than the initial value β of other six points of Fano plane, $\alpha \neq \beta$, all f-lines and f-points of the fibered Fano projective plane have two membership degrees α, β.



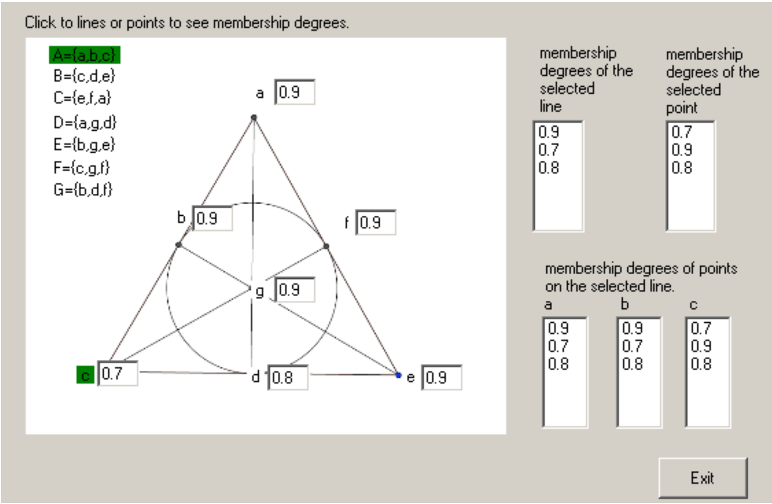
2) If the initial value α of two points is smaller than the initial value β of other five points of Fano plane,

$\alpha \neq \beta$, all *f*-lines and *f*-points of the fibered Fano projective plane have two membership degrees.

3) If the initial value α of collinear three points is smaller than the initial value β of other four points of Fano plane, $\alpha \neq \beta$, all *f*-lines and *f*-points of the fibered Fano projective plane have two membership degrees.

Fibered Fano planes with three different values

1) Let α, β, γ be three different the initial values and $\gamma > \alpha$, $\gamma > \beta$. If the initial values α, β of two different points and the initial value γ of other five points of Fano plane, all *f*-lines and *f*-points of the fibered Fano projective plane have three membership degrees α, β, γ .

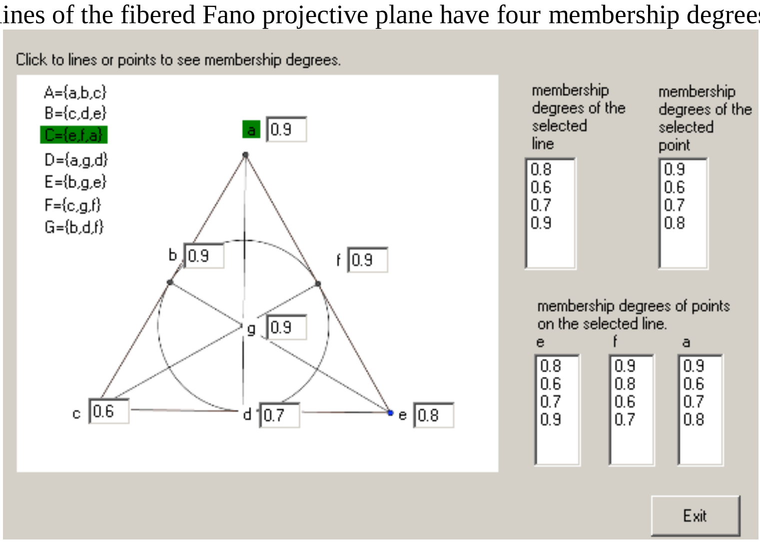


2) Let α, β, γ be three different the initial values and $\alpha < \gamma < \beta$. If the initial values α, β of two different points and the initial value γ of other five points of Fano plane, one point has membership degrees and all other *f*-lines and *f*-points of the fibered Fano projective plane have two membership degrees.

3) Let α, β, γ be three different the initial values and $\alpha < \beta < \gamma$. If the initial values α, β, β of collinear three points and the initial value γ of other four points of Fano plane, all *f*-lines and *f*-points of the fibered Fano projective plane have three membership degrees α, β, γ .

Fibered Fano planes with four different values

1) If different three initial values of collinear three points are smaller than the initial value of other four points of Fano plane, all *f*-lines of the fibered Fano projective plane have four membership degrees.

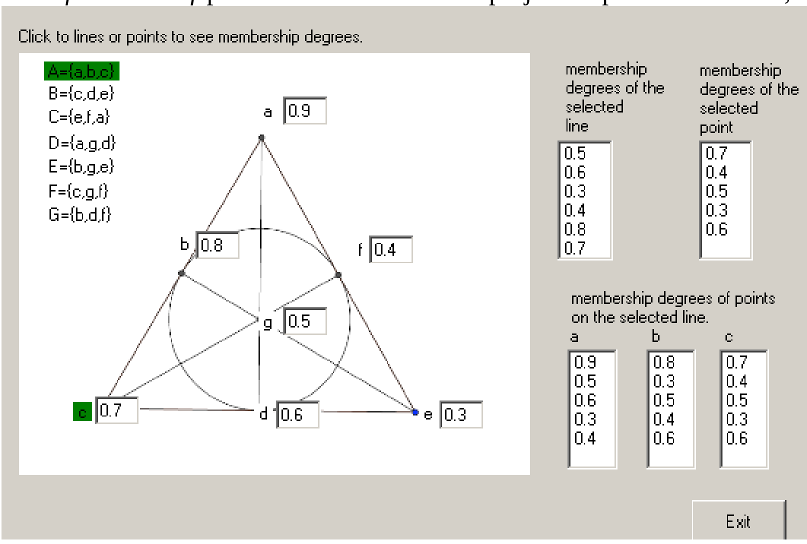


2) Let $\alpha, \beta, \gamma, \delta$ be four different the initial values and $\alpha < \beta < \delta < \gamma$ or $\alpha < \delta < \beta < \gamma$. If the initial values α, β, γ of three non collinear points and the initial value δ of other four points of Fano plane, two different points with the initial values β, γ and the *f*-line spanning these points have three membership degrees

and all other *f*-lines and *f*-points of the fibered Fano projective plane have two membership degrees.

Fibered Fano planes with seven different values

If the initial values of points of Fano plane are different, the maximum and minimum numbers of the membership degrees of *f*-lines and *f*-points of the fibered Fano projective plane are 6 and 3, respectively.



This program is a new practical application for finite geometry. So, this will be treated the interesting geometric and combinatorial problems.

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