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A Mathematical Study of Two Phase Pulmonary Blood Flow in Artery with Special Reference to Tuberculosis

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ABSTRACT: *Our research deals with two phase pulmonary flow in lungs keeping in the view nature of pulmonary circulatory system in human body. Some researchers have considered the blood flow as two phased. One of which is that of red blood cells ad other is plasma. They have also applied Herschel Bulkley non Newtonian model in bio fluid mechanical set up. We have collected a clinical data in case of TB for hematocrit v/s blood pressure. The graphical presentation for particular parametric value is much closer to clinical observation. The overall presentation is tensorial form and solution technique adopted is analytical as well as numerical.*

I. INTRODUCTION

(1.1) Structure of the tissue

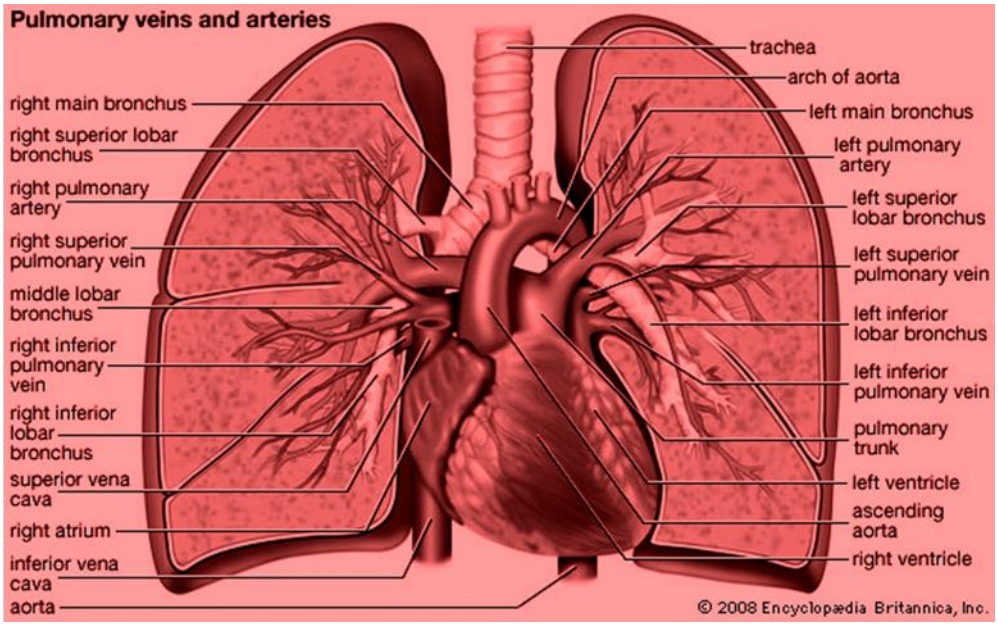
The lungs are located in the chest on either side of the heart in the ribcage. they are conical in shape with a narrow rounded apex at the top and a broad base that resets on the diaphragm[1].

The apex of the lung extends into the root of neck reaching shortly above the level of the sterna end of the first rib. The lungs stretch from close to the backbone into ribcage to the front of the chest and downwards from the lower part o trachea to the diaphragm [1]. The left lung share space with the heart with an impression in its medial surface called the cardiac impression [2]. The front and outer side of the lung face the rib. Which make light indentation on their surfaces. The bottom of the lung is smooth and resets on the diaphragm, matching its concavity. The medial surface of the lungs faces towards the centre of the chest and lies against the heart, great vessels and the carina where the two main bronchi branch off from the base of trachea [2].

Both lungs have a central recession called the hilum at the root of lung, where the blood vessels and airways pass into the lungs. There are also bronchopulmonary lymph nodes on the hilum [2]

The lungs are surrounded by the pulmonary pleurae. The pleurae are two serous membrance the outer parietal pleura lines the inner wall of the ribcage and the inner visceral pleura directly lines the surface of the lungs between the pleurae is a potential space called the pleural cavity containing pleural fluid. Each lung is divided into two lobes by invaginations of the pleura as fissures. The fissures are double folds of the pleura that section the lung and help in their expansion[3].

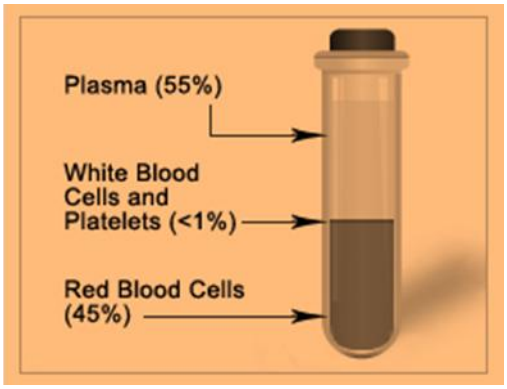
The lobes of the lungs are further divided into bronchopulmonary segments based on the locations of bronchioles [4].



(1.2) Function of Lungs

(1.3) Blood

Blood is a complex fluid consisting of particulate solids suspended in a non newtoian fluid. The particulate solids are red blood cells, white blood cells and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base. 50%of plasma and 45%of blood cells (RBCs) and there is a few parts of other cells, which are ignorable. Therefore one phase is plasma and second phase of the blood is RBCs [5].



(1.31) Blood Circulation

Arteries are musucular blood vessel that carry blood away from the heart oxygenated blood and deoxygenated blood. The pulmonary arteries will carry deoxygenated blood. The pulmonary arteries will carry deoxygenated blood to the lungs and the systematic arteries will carry oxygenated blood to the rest of the body[6].

In the pulmonary circulation, blood is pumped to the lungs from the right ventricle to the heart. It is carried to the lungs via pulmonary arteries. At lungs oxygen in the alveoli diffuses to the capillary surrounding the alveoli and carbon dioxide inside the blood diffuses to the alveoli. As a result blood is oxygenated which is then carried to the heart's left half to the left atrium via pulmonary veins. Oxygen rich blood is prepared for the whole organs and tissue of the body [6].

Two phase pulmonary blood flow is a study of measuring the blood pressure if hemoglobin is known. The percentage of volume covered by blood cells in the whole blood is called hematocrit. This work will focus on two phase pulmonary blood flow in artery.

(1.4) Lungs disease (T.B.)

Tuberculosis (TB) is an infectious disease caused by the bacterium mycobacterium tuberculosis (MTB) [7]. Tuberculosis generally affects the lungs, but can also affect other parts of the body. Most infections do not have symptoms, known as latent tuberculosis about 10% of latent infections progress to active disease which, if left untreated, kills about half of those infected. The classic symptoms of active TB are a chronic cough with blood containing sputum, fever, night sweats, and weight loss[1] The historical term "**consumption**" came about due to the weight loss[8]. Infection of other organs can cause a wide range of symptoms [9].

In the recent research the world health organization estimates that one third of the world's population is infected with the bacteria that cause tuberculosis. About 2 million people die from TB each year [10].

1. Almost 9 million new active TB cases occurred in 2011[11].
2. More than 80%of the people with TB live in 22 countries, most of them in Africa and Asia [11].
3. In the United States 11,162 people had active TB in 2010[12].

In 2014, TB killed 1.5 million people (1.1 million HIV-negative and 0.4 million HIV-positive). The toll comprised 890000 men, 480000 women and 140000 children [12].

II. REAL MODEL

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in a whole blood and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so one phase of the bloods plasma and 2nd phase of blood is RBCs.[13]

The first and foremost reason is that the blood is not an ideal fluid but it is a mixture of the two phases one is of plasma and other one is of blood cells. These blood cells, semi permeable packages of liquid of a density greater than that of plasma, are capable of changing their shape and size while flowing through different blood vessels [14]. Plasma is a liquid containing semi permeable packages of RBCs.

The behavior of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behavior [15]. We have selected generalized three dimensional orthogonal curvilinear co-ordinate system, briefly prescribed as E3 called as 3-dim Euclidean space. Here we have some quantities related to moving blood in cylindrical vessels: blood velocity $V^k = V^k(x^i, t)$, k=1,2,3 blood pressure $P = p(x^i,t)$ and density $\rho = \rho(x^i,t)$ where x^i be the co-ordinates of any point in space and i-1,2,3

If let us consider that the both phases- plasma and blood cells are equally distributed in whole blood. Then blood treated as homogeneous mixture.

(2.1) Equation of Continuity-

When there is absence of source and sink in any region of flowing fluid, the fluid mass is conserved in that region. As we observed that there is no source or sink in the whole circuit of the human blood circulatory system, the heart behaves merely like a pumping station, so the law of conservation of mass can well be applied to hemodynamic [16]. Since, whole blood flow circuit of the kidney is called a Renal Circulatory System. Hence renal circulatory system is a sub system of human circulatory system. Blood enter in kidney by arteries and out by veins and in a kidney no source or sink.

Mass of enter the blood = mass of outer the blood

Therefore law of conservation of mass can also be applied for renal circulatory system.

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.

(2.2) Equation of Motion- According to this principle, the total momentum of any fluid system is conserved in absence of external force. So the law of conservation of momentum can well apply to renal circulatory system. In other words, the rate of change of momentum of a fluid particle with respect to time equals to external force exerted on it. This is also called Newton's 2nd law of motion.

So, the rate of change of momentum is equal to sum of about two mentioned forces, which may be symbolically presented as follows.

$$\frac{dp}{dt} = -P + F$$

Where,

$\frac{dp}{dt}$ = Rate of change of momentum

P= internal pressures

F= viscous force

(2.3) Constitutive Equations –

Generally blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

1. Newtonian equation

$\tau = \eta e$

Where η is viscosity coefficient . This is found to hold good in the broad blood vessels where there is low hematocrit .

2. The non-Newtonian power law equation

$\tau = \eta e^n$

This is found to be conformable for strain rate between 5 and 200 sec-1,

$0.68 \leq n \leq 0.80$

The non-Newtonian Herschel – Bulkley equation .

$\tau = \eta e^n + \tau_0 \ (\tau \geq \tau_0)$

$e = 0 \ (\tau < \tau_0)$

It holds good when blood shows yield stress . We notice that the yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate

If ($\tau < \tau_0$) , If , no blood flow-takes place. It is found that yield stress is given by the following formula:

$$\tau_0^{\frac{1}{3}} = \frac{A(H - H_m)}{100}$$

Where $A = (0.008 \pm 0.002 \text{ dyne / c m}^2)^{1/3}$

Where, H is normal hematocrit and Hm is the hematocrit below which there is no yield stress.

(2.4) Boundary Conditions are as follows:

1. The velocity of blood flow on the axis of capillaries at r=0 will be maximum and finite, say V_0 = maximum velocity
2. The velocity of blood flow on the wall of blood vessels at r=R, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

(2.5) Hematocrit-

Hematocrit is the volume percentage (%) of red blood cells in blood. It is normally 45% for men and 40% for women. [17] Hematocrit is the most important determinant of whole blood viscosity. [18] Blood viscosity and vascular resistance affect total peripheral resistance to blood flow,[19] According to Berkow, Robert The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per deciliter).[20]

III. TWO PHASE NEWTONIAN POWER LAW FLOW

(3.1) Description of Bio physical Problem

The pattern of blood flow in the pulmonary arteries remote from the heart may be considered to be steady laminar due to negligible effect of pumping of heart. On the other hand the elasticity effect of blood vessels may also be taken to be negligible due to a far away from tissues. The renal arteries are cylindrical in shape. The blood viscosity may also be taken to be constant because the transverse section of vessels is uniform up to some extent as well as the blood is homogeneous mixture. Hence the blood flowing in renal arteries may be supposed to be Newtonian. As we know, the stress-tensor is proportional to rate of strain-tensor in Newtonian fluid. Hence the constitutive equation of blood flow in renal arteries is as follows [21]

$$T^{ij} = -p\,g^{ij} + \eta_m \left(g^{jk} V^{i}_{,k} + g^{ik} V^{j}_{,k} \right)$$

Where, symbols have their usual meaning

(3.2) Mathematical Modeling

Taking the above facts in view, we write the equation of continuity in tonsorial form as follows:

$$\frac{1}{\sqrt{g}} \left(\sqrt{g}\, v^{i} \right)_{,i} = 0$$

Again, we write down the equation of motion as follows [22]:

$$\rho_m \frac{\partial v^{i}}{\partial t} + \rho_m v^{j} v^{i}_{,j} = -p\,g^{ij} + \eta_m \left(g^{jk} V^{i}_{,k} \right)_{,j}$$

Where, $\rho_m = X\rho_c + (1-X)\rho_p$, is the density of blood as mixture of blood cells and plasma. While $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity of mixture of the blood .Other symbols have their usual meanings.

Now we have to transform the equations in cylindrical form. As we know. For cylindrical Co-ordinates, $X^1 = r,\ X^2 = \theta\ ,\ X^3 = z$

The determinant of metric tensor is $g = r^2$

The components of metric tensor are

$g_{11} = 1,\ g_{22} = r^2,\quad g_{33} = 1,$ Rests are zeros. Again the components of the conjugate metric tensor are

$g^{11} = 1,\ g^{22} = 1/r^2,\ g^{33} = 1$ rest are zeros.

The value of chistoffel's symbols of 2nd kind is as follows

$$\left\{ \begin{matrix} 1 \\ 2\ 2 \end{matrix} \right\} = -r,\quad \left\{ \begin{matrix} 2 \\ 2\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1\ 2 \end{matrix} \right\} = \frac{1}{r}, \text{rest are zero,}$$

The relation between physical component and covariant components of the velocity of the blood flow are as follows:

$\sqrt{g_{11}}v^1 = v_r \Rightarrow v_r = v^1$

$\sqrt{g_{22}}v^2 = v_0 \Rightarrow v_0 = r v^2$

And $\sqrt{g_{33}}v^3 = v_z \Rightarrow v_z = v^3$

Again the physical components of $p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}\, p_{,j}g^{ij}$

Now, we are in a position to write down the governing equation of blood flow in cylindrical form as follows:

(3.2.1)The equation of continuity-

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial(rv_\theta)}{\partial\theta}+\frac{\partial(v_z)}{\partial z}=0$$

(3.2.2)The equation of motion-

Components of equations of motion

r-component

$$\rho_m\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta_m\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$$

Θ-Component:

$$\rho_m\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}-\frac{v_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\eta_m\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

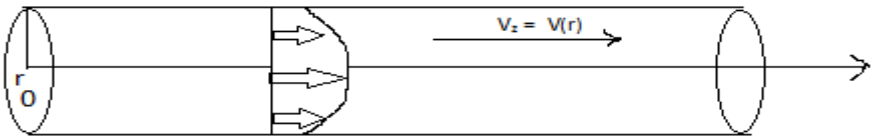
Z- Component:

$$\rho_m\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial t}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial(v_z)}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

The appropriate boundary conditions are as follows [23]

(1) The velocity of blood flow on the axis of artery i.e. r=0, will be maximum and finite, say V_0

(2) The velocity of the blood flow on the wall of the blood vessels, i.e. at r=R. Where R is radius of the traverse Section of the artery, will be zero .this condition is well known as no- slip condition.



Velocity profile

Fig.-3.4.1

(3.3) Solution

The blood flow in artery is symmetric w.r.t. axis . Hence $v_\theta = 0$ and also v_r, v_z and p donot depend upon θ.

Since only one component of the velocity which is along the axis is effective ,we have $v_r=0$, $v_\theta=0$ and $v_z=v$, =v say

The flow is steady , we have [24]

$$\frac{\partial p}{\partial t}=\frac{\partial v_r}{\partial t}=\frac{\partial v_\theta}{\partial t}=\frac{\partial v_z}{\partial t}=0$$

Keeping in view these facts , we obtain the following result

Equation of continuity reduces to

$$\frac{\partial v_z}{\partial z} = 0 \Rightarrow v_z = v(r)$$

The r^{th} component of equation of motion reduces to

$$\rho_m(0) = -\frac{\partial p}{\partial r} + \eta_m(0) \Rightarrow \frac{\partial p}{\partial r} = 0 \Rightarrow p = p(z)$$

Θ- component of equation of motion reduces to

$$\rho_m(0) = -(0) + \eta_m(0) \Rightarrow 0 = 0$$

Similarly , the z^{th} component of equation of motion reduces to

$$\rho_m v_z \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{\partial v_z}{\partial r} \right\} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

With the help of equations we get

$$0 = 0 = -\frac{\partial p}{\partial z} + \eta_m \left(\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{\partial v(r)}{\partial r} \right\} \right)$$

Whereas , the equation expresses the fact that the pressure p depends only on z .We also retain the fact that pressure gradiant $-\frac{\partial p}{\partial z}$ in the arteries remote from heart is constant ,say ,p then the equation (3.3.9) takes the Following form

$$0 = P + \eta_m \left(\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{\partial v(r)}{\partial r} \right\} \right)$$

Integrating the equation

$$\text{we get } r\frac{dv}{dr} = -\frac{Pr^2}{2\eta_m} + A$$

where A be the constant of integration.

Applying the first boundary condition we get $A = 0$

Hence equation reduces to

$$r\frac{dv}{dr} = -\frac{pr^2}{2\eta_m}$$

Again integrating the equation we get

$$v = -\frac{Pr^2}{4\eta_m} + B$$

Again using 2^{nd} boundary condition on the equation

we can evaluate the integraton constant B as follow :

$$B = \frac{PR^2}{4\eta_m}$$

Inserting the value of B in the equation

we obtain the velocity of the blood flow in the arteries remote

from the heart as follows

$$v = \frac{P}{4\eta_m}\left(R^2 - r^2\right)$$

(3.3.4) Bio physical interpretation:

This is obvious from (2.4.8) that the velocity profile in the arteries remote from heart is parboiled of the revolution .The flow flux of blood i.e. .the total volume per unit time passing through the transversal section of blood vessel is given by the following formula

$$Q = \int_0^R v.2\pi r dr = \int_0^R \frac{P}{4\eta_m}(R^2 - r^2)2\pi r dr$$

$$= \frac{P}{4\eta_m}\left[\pi R^2 r^2 - (\pi r^4)/2\right]_0^R$$

$$= \pi R^4 P/8\eta_m$$

$$Q = \frac{\pi R^4}{8\eta_m}\left(-\frac{dp}{dz}\right)$$

$$\int_{z_i}^{z_f} Q\, dz = -\int_{p_i}^{p_f} \frac{\pi R^4}{8\eta_m} dp$$

$$Q[z]_{z_i}^{z_f} = -\frac{\pi R^4}{8\eta_m}[p]_{p_i}^{p_f}$$

$$Q\left[z_f - z_i\right] = -\frac{\pi R^4}{8\eta_m}\left[p_f - p_i\right]$$

$$Q\left[\Delta Z\right] = \frac{\pi R^4}{8\eta_m}\left[\Delta P\right]$$

$$\left[\Delta P\right] = \frac{Q\left[\Delta Z\right]\times 8\eta_m}{\pi R^4}$$

(3.3.4.1) Observations: Hematocrit Vs Blood pressure from an authorized Organization
Clinical data-1

Sno.	HB	Hematocrit	BP mmgh	Pascal S
1	7.2	21.6	130/80	17331.86/10665.8
2	7.8	23.4	110/80	14665.42/10665.8
3	8.8	26.4	100/80	13332.2/10665.8
4	9.6	28.8	120/80	15998.64/10665.8
5	10.7	32.1	120/80	15998.64/10665.8

$\text{Average SystolicPressure} = 15465.35\,\text{Pa}$

$\text{Average DiastolicPressure} = 10665.76\,\text{pa}$

$P_i = \text{presure on artery} = 15465.35\,\text{pa}$

$P_f = \text{pressure on arteriols} = \frac{D+S}{2} = \frac{15465.35+10665.76}{2} = 13065.35$

$\text{Viscosity of mixture} = \eta_m = 0.027\ \text{P.S.}[25]$

$\text{Viscosity of plasma} = \eta_p = 0.0013\ \text{P.S.}[25]$

$\text{Length of Arteries} = 0.024\,\text{m}\ [26]$

$\text{Viscosity of cells} = \eta_c = ?$

$$\eta_m = \eta_c X + \eta_p + \left(1 - \frac{26.58}{100}\right)0.001$$

$$\eta_c = 0.009592\ \text{ps}$$

Using the relation (3.2)

$$\left[\Delta P\right] = \frac{Q\left[\Delta Z\right]\times 8\eta_m}{\pi R^4} \Rightarrow \frac{8\times 0.024\times 0.00708}{3.14\times(0.00416)^4}\left(\eta_c\times\frac{H}{100} + \left(1-\frac{H}{100}\right)0.0013\right)$$

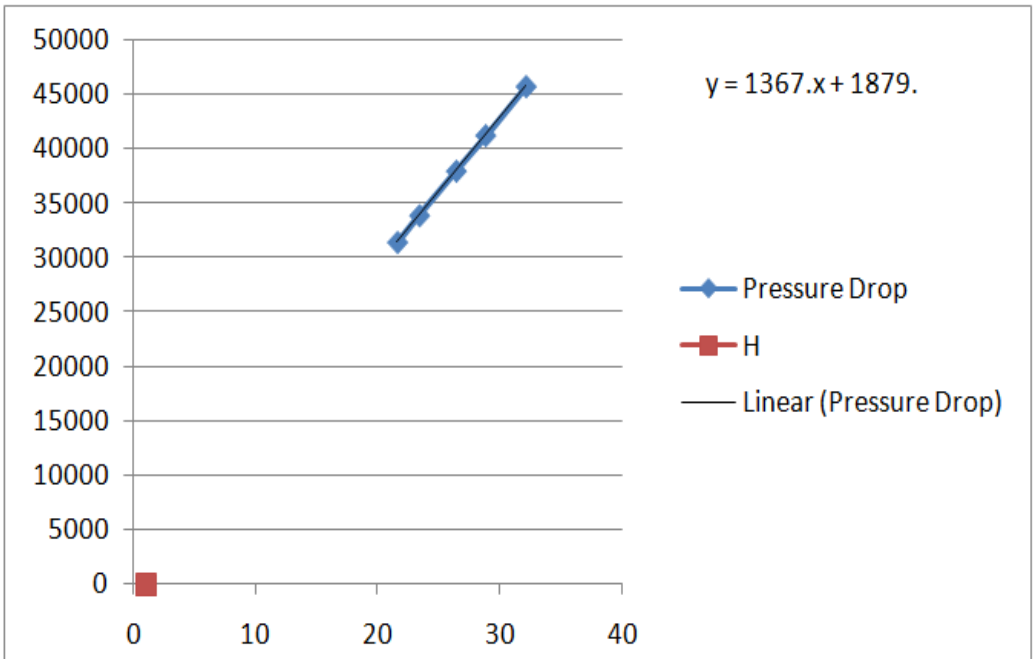
$$\Rightarrow \Delta P = 1445544.66\times\left(0.009592\times\frac{H}{100} + \left(1-\frac{H}{100}\right)0.0013\right)$$

$$\Rightarrow \Delta P = 1367.77H + 1879.2 \ldots\ldots\ldots\ldots (3.3.16)$$

Now using the above relation :-

H	21.6	23.4	26.4	28.8	32.1
ΔP	31423.03	33885.02	37988.328	41270.98	45784.617

Graphical representation :-



IV. RESULT & DISCUSSION

The graph shows that hematokrit proportional to pressure drop in the cases of tuberoculosis and the trend of the graph is a straight line $y=1367x+1879$ with slop 1367

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