Exergoeconomic Analysis of Two-stage Thermoelectric Cooler with Genetic Algorithm

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Abstract

In this paper a two stage thermoelectric cooler which has total 50 elements out of which 30 elements are on hotter side and 20 elements are on colder side, analyzed on the basis of the principles of thermoeconomics. Exergy destruction has been used to calculate the second law efficiency. Then the exergy has been coupled with unit cost of exergy to calculate the cost of refrigeration produced. The model has been analyzed for a particular life cycle of thermoelectric cooler (TEC) where the operating conditions change with the variation in cold side and hot side temperatures. Two cases have been considered. In first case the cold side temperature is varied with hot side temperature maintained as constant. In second case hot side temperature is varied with cold side temperature maintained as constant. The analysis shows that on the basis of thermo-economics the first case is favorable but the comparison on the basis of coefficient of performance, rate of refrigeration and second law efficiency show that the second case is favorable.

1. Introduction

Thermoelectric devices are solid state devices. Semiconductor thermoelectric power generation based on the Seebeck effect and semiconductor thermoelectric cooling, based on the Peltier effect, have interesting capabilities compared to conventional power generation and cooling systems. The absence of moving components results in increase in reliability, reduction in maintenance, and increase of system life; the modularity allows for application in wide range without significant losses in performance; the absence of working fluid avoids environmental dangerous leakages; and the noise reduction appears also to be an important feature. With the rapid development of techniques to make excellent semiconductor materials the practical applications and theoretical investigations of the thermoelectric devices have come in to focus. Thermoelectric coolers have found applications in areas such as microelectronic system, laser diodes, telecommunication, and medical services. As refrigerators, they are friendly to the environment as CFC's or any other refrigerant are not. Due to these advantages, the thermoelectric devices have found a large range of applications. The basic knowledge of the thermoelectric devices and the prospects of the applications have been discussed (Riffat and Ma, 2003). In the beginning of the 21st century, the world is facing the major challenge of finding energy sources to satisfy the ever-increasing energy consumption while preserving the environment. In the race to search alternative energy sources, thermoelectric generators are called to play their role in the improvement of the efficiency of the actual energy system by harvesting wasted heat. The improvement in the efficiency of nano-engineering thermo-electrics mainly from the reduction in the thermal conductivity of various types of thermoelectric materials, nano-structured materials have shown the most promise for commercial use because of their extraordinary thermoelectric performances (Martín-González et al. 2013;Chen et al. 2012).

A review of research to improve the coefficient of performance of thermoelectric cooling systems was presented. This includes development of new materials for thermoelectric modules, optimization of module design and fabrication and system analysis (Riffat and Ma, 2004). Thermoelectric coolers were found suitable for such applications where a precise control of temperature is required. A model was developed to simulate the air dehumidification process using thermoelectrically cooled TEC channels. It was found that the model predicted well the variation in the air temperature along the channel with a relative error of less than 2.4% (Jradi et al. 2012; Riffat and Qiu, 2006). Various experimental and simulation studies of thermoelectric device have been carried out to examine their applications in different areas so that they can replace the conventional pollution creating power producing or power consuming devices (Belanger and Gosselin, 2012; Lertsatitthanakorn et al. 2008). An exergy-based thermoeconomic optimization has been applied to vapour compression and vapour absorption refrigeration systems. The application of thermoeconomic optimization design in these systems is important in achieving economical life cycle cost. Researchers have developed thermodynamic and thermoeconomic objective functions based on second law and thermoeconomic principles (Selbas et al. 2006; Zhang et al. 2004; Kizilkan et al. 2007; Ahmed et al. 2011;

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Vincent and Heun, 2006; Rezayan and Behbahaninia, 2010).

2. Mathematical Modeling

A mathematical model of a multi-stage thermoelectric cooler (TEC) has been developed to analyze its performance thermoeconomically. There are total 50 elements in this TEC, out of which 30 are kept at hotter side and 20 are kept on colder side. [17] Thermoeconomics is the branch of engineering that combines exergy analysis and economic principles to provide the designer and operator with information not available through conventional energy analysis and economic evaluations but crucial to the design and operation of a cost effective system. So thermoeconimics can be considered as exergy-aided cost minimization. Fig. 1 shows the schematic diagram of a two stage TEC in which two stages are connected thermally as well as electrically in series.

2.1 Exergy analysis of two stage TEC:

In this paper first the exergy analysis of the aforesaid multistage TEC is done.

A two stage thermoelectric TEC can be considered as a two combined single stage TEC's connected thermally in series. Each stage may have same number or different number of thermoelectric elements. The heat balance equation for a two stage thermoelectric TEC can be written as:

$$Q_{cc} = \left[\alpha I_c T_{cc} - \frac{I_c^2 R}{2} - k(T_{ch} - T_{cc})\right] N_c \quad (1)$$

$$Q_{ch} = \left[\alpha I_c T_{ch} + \frac{I_c^2 R}{2} - k(T_{ch} - T_{cc})\right] N_c \quad (2)$$

$$Q_{hc} = \left[\alpha I_h T_{hc} - \frac{I_h^2 R}{2} - k(T_{hh} - T_{hc})\right] N_h \quad (3)$$

$$Q_{hh} = \left[\alpha I_h T_{hh} + \frac{I_h^2 R}{2} - k(T_{hh} - T_{hc})\right] N_h \quad (4)$$

where Q_{cc} is the cooling capacity at the cold side of the colder stage; Q_{ch} is the rate of release of heat at the hot side of colder stage; Q_{hc} is the cooling capacity at the colder side of the hotter stage and Q_{hh} is the rate of release of heat at the hot side of the hotter stage. T_{cc} and T_{ch} represent the temperatures of the cold side and hot side of colder stage and T_{hc} and T_{hh} represent the cold side and hot side temperature of hotter side. It can be assumed that their exist a junction temperature T_m which can be calculated by equating Q_{ch} and Q_{hc}.

Hence $Q_{ch} = Q_{hc}$

$$T_m = \frac{k(T_{cc} + xT_{hh}) + (xj^2 + 1)\frac{1}{2} I_c^2 R}{k(x+1) + I_c \alpha (xj - 1)} \quad (5)$$

where x= N_h/N_c and j= I_h/I_c

Finally, the heat balance equations can be written as :

$$Q_{cc} = \left[\alpha I_c T_{cc} - \frac{I_c^2 R}{2} - k(T_m - T_{cc})\right] N_c \quad (6)$$

$$Q_{ch} = \left[\alpha I_c T_m + \frac{I_c^2 R}{2} - k(T_m - T_{cc})\right] N_c \quad (7)$$

$$Q_{hc} = \left[\alpha I_h T_m - \frac{I_h^2 R}{2} - k(T_{hh} - T_m)\right] N_h \quad (8)$$

$$Q_{hh} = \left[\alpha I_h T_{hh} + \frac{I_h^2 R}{2} - k(T_{hh} - T_m)\right] N_h \quad (9)$$

The thermoelectric material properties can be determined by:

$$\alpha = |\alpha_p - \alpha_n| \quad (10)$$

$$R = (\rho_p + \rho_n)\frac{L}{A} \quad (11)$$

$$k = (k_p + k_n)\frac{A}{L} \quad (12)$$

where subscripts p and n indicate the properties of p and n-type semiconductors, A and L are cross-sectional areas and the length of thermocouples.

The material properties are considered to be dependent on the average temperature, T_{ave}, given by $(T_{cc}+T_{hh})/2$, and can be obtained by applying the experiential formulae of MELCOR, USA. T_{hh} and T_{cc} have been taken 303 K and 263 K respectively in this analysis.

$$\alpha_p = -\alpha_n = (22224.0 + 930.6T_{ave} - 0.9905T_{ave}^2) \times 10^{-9} \quad (13)$$

$$\rho_p = \rho_n = (5112.0 + 163.4T_{ave} + 0.6279T_{ave}^2) \times 10^{-10} \quad (14)$$

$$k_n = k_p = (62605.0 - 277.7T_{ave} + 0.4131T_{ave}^2) \times 10^{-4} \quad (15)$$

COP of two stage thermoelectric TEC:

$$(COP)_{act} = \frac{Q_{cc}}{W_{hh} + W_{cc}} = \frac{Q_{cc}}{(Q_{hh} - Q_{hc}) + (Q_{ch} - Q_{cc})} = \frac{Q_{cc}}{(Q_{hh} - Q_{cc})} \quad (16)$$

W_{hh} and W_{cc} are the power inputs in hotter and colder sides respectively.

Exergy analysis of double-stage TEC is as follows:

$$X_{in,cc} - X_{out,cc} - X_{destroyed,cc} = \frac{dX_{cc}}{dt} \quad (17)$$

$\frac{dX_{cc}}{dt} = 0$ as the exergy does not change anywhere within the TEC.

So equation (32) can be written as:

$$Q_{cc}\left(1-\frac{T_o}{T_{cc}}\right)+W_{cc} - Q_{ch}\left(1-\frac{T_o}{T_m}\right) = X_{destroyed,cc} \quad (18)$$

After solving this equation, the exergy destruction is:

$$X_{destroyed,cc} = T_o\left(\frac{Q_{ch}}{T_m} - \frac{Q_{cc}}{T_{cc}}\right) \quad (19)$$

But $\left(\frac{Q_{ch}}{T_m} - \frac{Q_{cc}}{T_{cc}}\right)$ is entroy generation, $S_{gen,cc}$ in colder satge of multistage TEC

So $X_{destroyed,cc} = T_o S_{gen,cc}$ (20)

Similarly for hotter stage:

$$X_{in,hh} - X_{out,hh} - X_{destroyed,hh} = \frac{dX_{hh}}{dt} \quad (21)$$

$\frac{dX_{hh}}{dt} = 0$ as the exergy does not change anywhere within the TEC.

So equation (36) can be written as:

$$Q_{hc}\left(1-\frac{T_o}{T_m}\right)+W_{hh} - Q_{hh}\left(1-\frac{T_o}{T_{hh}}\right) = X_{destroyed,hh} \quad (22)$$

After solving this equation, the exergy destruction is:

$$X_{destroyed,cc} = T_o\left(\frac{Q_{hh}}{T_{hh}} - \frac{Q_{hc}}{T_m}\right) \quad (23)$$

But $\left(\frac{Q_{hh}}{T_{hh}} - \frac{Q_{hc}}{T_m}\right)$ is entroy generation, $S_{gen,hh}$ in hotter satge of multistage TEC

So $X_{destroyed,hh} = T_o S_{gen,hh}$

Total exergy destroyed in multistage TEC will be:

$$X_{destroyed,total} = X_{destroyed,cc} + X_{destroyed,hh}$$

$$X_{destroyed,total} = T_o\left(S_{gen,cc} + S_{gen,hh}\right) \quad (24)$$

Taking $T_o = T_{hh}$ equation (39) can be written as:

$$X_{destroyed} = T_{hh}\left(S_{gen,cc} + S_{gen,hh}\right) \quad (25)$$

So the Second Law Efficiency of double stage TEC on the basis of exergy destruction can be calculated as:

$$\eta_{II} = 1 - \frac{X_{destroyed}}{X_{\sup plied}} \quad (26)$$

2.2 Thermoeconomic analysis of Two stage TEC:

Thermoeconomics is an orderly method which combines concept of exergy method with those belonging to economic analysis. The pupose of thermoeconomic analysis is to reach a trade-off between capital costs and costs of the input exergy of the system. In other words, in this method, the objective is either to minimize the unit cost of the product of the system for a fixed product or maximize the output product for a fixed total cost of the system. Therefore the thermoeconomic objective function includes cost involving exergy input and capital cost in monetary units. One can write the relationship between the product cost and the total cost of the system as follows. (Bejan et al. 1996)

$$C_{cc} + C_w + Z = C_{hh} \quad (27)$$

$$c_{cc} X_{cc} + c_w X_w + Z = c_{hh} X_{hh} \quad (28)$$

Where C_{cc}, C_w and C_{hh} are the total annual cost of the system associated with cold side, power input and hot side exergy transfers respectively and c_{cc}, c_w and c_{hh} are the unit cost of exergy associated with cold side, power input and hot side exergy transfers. Z is cost rate associated with capital investment and operating and maintenance of TEC. Equation (28) can be written as

$$c_{cc}Q_{cc}\left(1-\frac{T_o}{T_{cc}}\right)+c_w\left(Q_{hh}-Q_{cc}\right)+Z=c_{hh}Q_{hh}\left(1-\frac{T_o}{T_{hh}}\right) \quad (29)$$

c_{cc} and c_{hh} both costs are associated with same nature of exergy, they can be taken equal.

$$c_{cc}=c_{hh}$$

Equation (29) will be reduced as

$$c_{cc}=\frac{c_w\left(Q_{hh}-Q_{cc}\right)+Z}{Q_{hh}\left(1-\frac{T_o}{T_{hh}}\right)-Q_{cc}\left(1-\frac{T_o}{T_{cc}}\right)} \quad (30)$$

In engineering economics, the unit of time interval chosen for capital cost and operating and maintenance cost is usually taken as a year. This cost in a year is obtained using the capital recovery factor (CRF).

The system is presumed to work with following details.
T_{cc} = 263 K, T_{hh} = 303 K and T_o = 298 K
Operating period, n = 5 years
Let us assume that TEC works for 8 months in a year and there are 30 days in a month.
Period of operation per year = 24*30*8 = 5760 hrs

Annual interest rate, i = 8%

Electricity cost, c_w = 4.25 Rs per kWh

Current supplied to TEC = 8 amp (Sharma et al. 2014)

Z = CRF* Cost of the TEC

$$CRF=\frac{i(1+i)^n}{(1+i)^n-1}*C$$

C is cost of the system.

The cost of the system can be calculated as per the following details.
Total number of thermoelectric elements = 50
Dimension of a thermoelectric element taken = 15 mm*15 mm*4.3 mm (Cheng et al. 2005)
Volume of one element = 15*15*4.3 = 967.5 mm^3 = 967.5 *10^{-9} m^3
Density of Bismuth Telluride = 7.7*10^3 kg/m^3 (Abdul-Wahab et al. 2009)
Mass of total Bismuth Telluride required = 50*967.5*10^{-9}*7.7*10^3 = 0.372 kg
Cost of Bismuth Telluride per kg = 110000 Rs
Cost of Bismuth Telluride required = 0.372*110000 = 40920 Rs
Cost of battery = 4000 Rs
Miscellaneous cost (connection wires, plates etc.) = 10% of the material cost = 4092 Rs
Total cost = 40920+4000+4092 = 49012 Rs ≈ 50000 Rs.

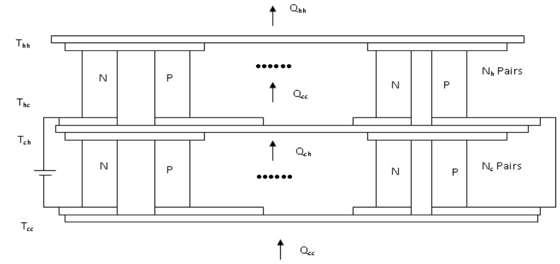

Fig. 1 Schematic diagram of the double stage TEC

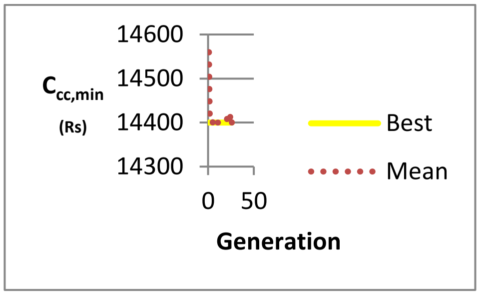

Fig. 2 Variation in cost of refrigeration produced wrt. cold side temperature

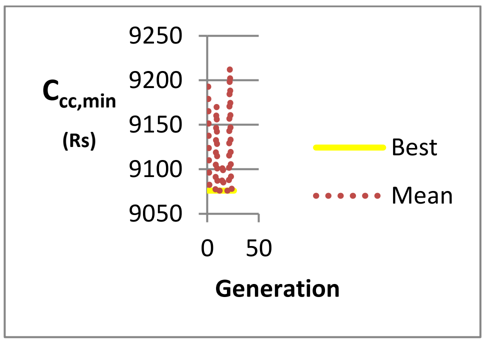

Fig.3 Variation in cost of refrigeration produced wrt. hot side temperature

3. Results and Discussion

Let us take two cases for operating TEC: Case 1: T_{hh} remains constant at 303 K and T_{cc} varies from 253 K to 273 K. This may be considered as a case when the heat is rejected at constant temperature at 30^0C and TEC is being used for various applications where the refrigeration temperature T_{cc} varies from -20^0C to 0^0C.
Case 2: T_{cc} remains constant at 270 K (obtained from case one at which the cost is minimum) and T_{hh} varies from 293 K to 345 K. This may be considered as a case when the refrigeration temperature is constant temperature at -3^0C and the temperature of hotter plate varies from 20^0C to 72^0C.

Table.1 Comparision of the two cases considered

Case 1.	Thh (K) taken	Tcc (K) varied	Tcc (K) obtained	Ccc (Rs)	COP	ηII	ROR (W)
	303	253-273	270	14400.48	0.6903	0.0843	6.0413
Case 2.	Tcc (K) taken	Thh (K) varied	Thh (K) obtained	Ccc (Rs)	COP	ηII	ROR (W)
	270	293-345	339	9075.923	0.0403	0.0104	4.014

Fig. 2 shows that the cost of refrigeration produced is Rs. 14400.48 (minimum) at T_{cc} = 270 K. Now the performance measuring parameters, coefficient of performance, second law efficiency and rate of refrigeration can also be caculated at this point. The values are mentioned in Table 1.

Fig. 3 shows that the cost of refrigeration produced is Rs. 9075.92 (minimum) at T_{hh} = 339 K. Now the performance measuring papameters, coefficient of performance, second law efficiency and rate of refrigeration can also be caculated at this point. The values are mentioned in Table 1.

Comparing the results obtained in case 1 and case 2, it can be observed that the cost of refrigeration produced is minimum when the TEC works between temperature limits T_{cc} = 270 K and T_{hh} = 339 K. Therefore comparing the results on the basis of cost of refrigeration produced, it can be noted that case 2 is more favorable. But the values of COP, η_{II} and ROR are higher in case 1, when the TEC works between temperaure limits T_{cc} = 270 and T_{hh} = 303 K.

4. Conclusion

Analysis of a two stage thermoelectric cooler on the basis of thermoeconomic principles which has 30 elements on hotter side and 20 elements on colder side show that minimum cost of refrigeration produced is obtained when the TEC works T_{cc} = 270 K and T_{hh} = 303 K. This result has been obtained by varying T_{cc} and maintaining T_{hh} as constant. But when T_{hh} is varied and T_{cc} is maintained constant, the minimum cost of refrigeration produced is obtained at T_{cc} =270 K and T_{hh} = 339 K. So on the basis of thermoeconomics the case 2 is the favourable case. But when the two cases are compared on the basis of coefficient of performance, rate of refrigeration and second law efficiency, it is observed that case 1 is favourable.

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