

# Study of residual stress mechanism using three elasto-plastic bars model

Djarot B. Darmadi

**Abstract** — Detrimental effects of residual stress in weldments have been well known. However, the mechanism of residual stress formation is not well established. The new mechanism of residual stress development in welding is proposed in this paper. The proposed mechanism is evaluated using three elasto-plastic bars model. Following the proposed mechanism, the residual stress at those three bars model can be determined quantitatively. The residual stress can also be obtained using numerical approach. One of the prominent numerical approaches is finite element method (FEM). Results from analytic solution following the proposed mechanism showed good agreement with FEM which mean the proposed mechanism is correct.

**Index Terms** — residual stress, FEM, three bars model

## I. INTRODUCTION

Since the first time welding simulation is developed in 1970s, the scope is obtaining residual stresses and corresponding deformation [1]. Non-uniform temperature history in welded structure is a major factor that affects significantly the residual stress which can lead to premature fatigue damage, stress corrosion and fracture [2 - 6]. The first analysis of welding was established by Rosenthal [7] for a quasi-steady state moving point heat source which can be considered as the first analytic foundation for welding. Following this, the analytic thermal solutions for moving heat sources have attracted further attention [8-13] to provide a good understanding of the welding process. In recent years the high speed computational resources needed for numerical analysis have become more readily available, and the FEM has become a popular tool for weld modeling.

There different mechanisms are proposed to describe residual stress development in welding. First proposed mechanism describes residual stress is developed as a result of contraction of hot region closed to the weld line which is constraint (externally or internally) by the cold region surrounds it. The contraction happens when the hot region cools down to the room temperature [14 - 17]. Secondly, Sindo Kou [18] used three bars with equal cross section area model to observed residual stress development in welding. The middle bar represents hot region closed to the weld line and the side bars represent cool region far enough from the weld line. The stress development is a total result since the middle

bar is heated and the detailed mechanism is well explained as follow. When the middle bar is heated the expansion is constrained by the side bars results in compressive stress in the middle bar. This compressive stress increase with temperature until the compressive yield stress is achieved. When the middle bar is cooled down, the contraction is restrained as a consequent the compressive stress drop rapidly and change to tensile stress. The tensile stress continuing increase until tensile yield stress is achieved and the final results are tensile stress at the middle bar equal to the yield stress and compressive stress at the side bars equal to a half of tensile yield stress. E.M. Vander A.A [19] used five bars model to explain the third stress development in welding mechanism with trailing heat sink. The mechanism of residual stress is described as a result of pushing and pulling action between the bars. This pushing and pulling action is started since the middle bar is heated. The pulling and pushing forces is counted based on elastic Hooke's law as a result of thermal expansion and compression.

Some different variations also proposed, but basically they can be categorized in the above three mechanisms. Satoh [20] and Masubuchi [21] used three bars model to describe stress development in conventional welding which can be categorized in the third mechanism. Some following papers also can be categorized in the second mechanism. Canas *et.all* [22] used FORTRAN code to describe longitudinal residual stress and angular distortion of welded plate. The welded plate was represented by three bars model. The bars have elasto-plastic behaviors and mechanical properties vary with temperature. Michaleris *et.all.* [23,24] used the same mechanism with equivalent temperature field as a thermal load. Baroso *et.all.* [25] evaluated the effect of simplification in welding modeling to the residual stress results using FEM. The residual stress development follows the second mechanism. Simplification comprise of two categories, material properties and temperature field. Simplifying materials properties considers it has a constant value with temperature variations. Temperature field is simplified using thermal envelope in which the temperature load is represented by the maximum experienced temperature.

This paper discusses different mechanism with the above presented three mechanisms. The mechanism evaluated using three bars model. Finite Element Method (FEM) was carried out to the three bars model and thermal stresses are evaluated when the middle bar is heated. Thermal stress also evaluated at

Djarot B. Darmadi is a Brawijaya University - Indonesia lecturer, he is studying at University of Wollongong - Australia (e-mail : [b\\_darmadi\\_djarot@yahoo.co.id](mailto:b_darmadi_djarot@yahoo.co.id) or [dbd991@uowmail.edu.au](mailto:dbd991@uowmail.edu.au) )

those three bar when the middle bar is cooled down to the initial temperature which can be considered as the residual stress. Analytic solution for thermal stress following the proposed mechanism is demonstrated and compared to the FEM. In both methods multi linear kinematic hardened material model is used to describe stress-strain relation. Comparing FEM results and analytic method that follows proposed mechanism showed good agreement each other which mean the new proposed mechanism is correct.

## II. FINITE ELEMENT SIMULATION

Subsequent Thermo-mechanical FEM was carried-out using ANSYS Parametric Design Language (APDL) mode to obtain thermal stress. In this paper is used ANSYS 12 software package to run FEM analysis. The thermal load is simulated at thermal stage. In this stage solid thermal element SOLID70 are used. Basically the element model describes the relationship between geometry of a model with its degree of freedom. Solid70 element is suitable for 3D thermal analysis model. Typically the element is comprised of eight nodes with temperature as degree of freedom at each node. However, if a node is formed as a coalescence of nodes the element can be comprised of four, five or six nodes and tetrahedral element, pyramid element or prismatic element are formed as it is shown at figure 1.

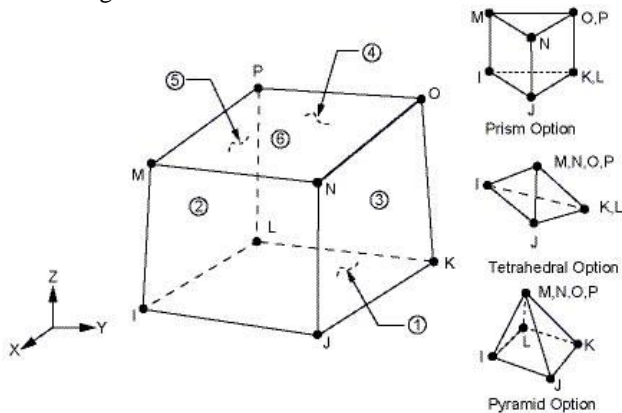


Figure 1. Solid70 3D-thermal element (Release 12.0 Documentation for ANSYS)

In thermal stage, first it is modeled three bar model that comprised of SOLID70 elements as shown at figure 2. Those three bar model are connected at their ends with two other tough bars. The geometry of the three bars is 1mm x 1mm cross section with 10mm length. The typical properties for the bars are: density  $\rho = 8000$  (kg/m<sup>3</sup>), thermal conductivity  $k = 20$  (watt/m.°C), specific heat  $c = 500$  (J/kg.°C), Young modulus  $E = 15$ GPa, poisson ratio  $\mu = 0.3$ , thermal expansion  $\alpha = 20 \cdot 10^{-6}$  /C°. The two clamping bars at the ends of the three bars have same properties except for  $k = 20 \cdot 10^{-6}$ ,  $\alpha = 20 \cdot 10^{-20}$  GPa and  $E = 10 \cdot 10^{10}$  GPa. With the much lower  $k$  it is expected the elevated temperature will be only localized at the middle bar. Very low  $\alpha$  makes no thermal expansion is existed and high  $E$  to model perfect toughness of the two bars.

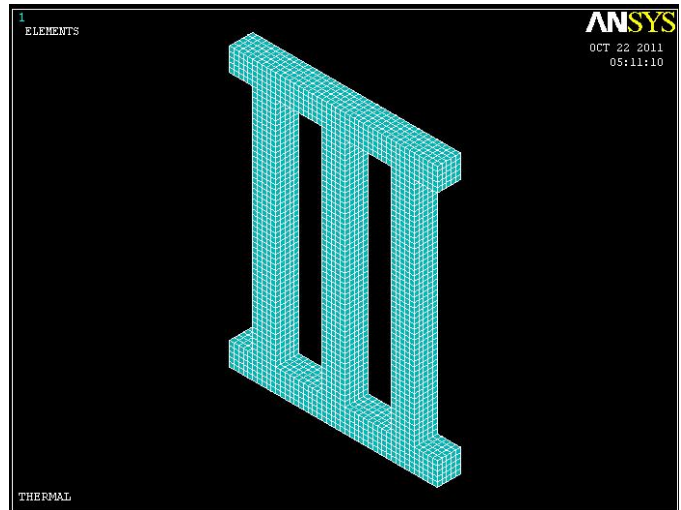


Figure 2. Built three bars model

Elasto-plastic material model is used and to avoid convergence problem at the plastic state, very low  $E$  is applied to represent perfectly plastic state. Stress-strain relation of the elasto-plastic material is shown at figure 3. The typical yield stress 150MPa is used. Since Young's modulus of the material is 15GPa, the yield point is defined at  $\epsilon_y = 0.01$ .

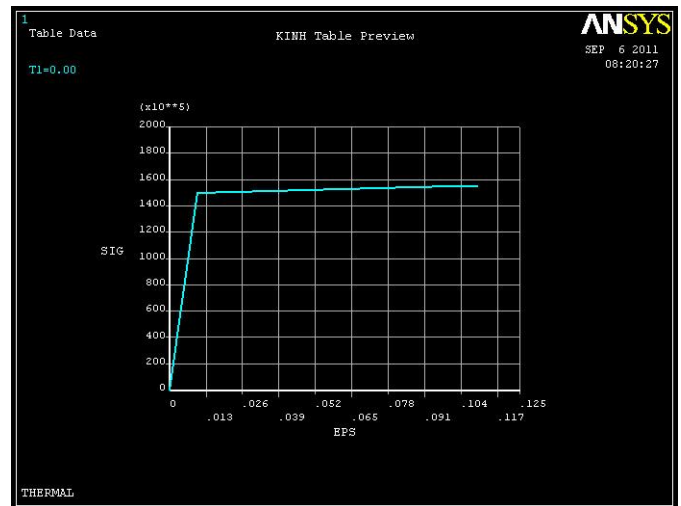


Figure 3. Strain – stress diagram material model

After modeling, the next step (the second) is defining temperature load. The first thermal load is heating the middle bar with certain temperature. The second thermal load is omitting the previous thermal load at the middle bar. Temperature histories at elements are store in \*.rst file which can be retrieved in the next mechanical stage. The next step is calculation, which is done after SOLVE command is inputted.

As in thermal stage, the first step in mechanical stage is geometry modeling. Since the thermal load will be adopted from thermal stage, the model should be exactly same as in thermal analysis. The individual element geometry should be exactly same with SOLID70 accept for the degree of freedom. Observing the available element model in ANSYS, SOLID45 as shown at figure 4 is chosen. It can be seen that as in

SOLID70, the typical element comprise of eight node which can coalesce each other. The degree of freedom of the SOLID45 structural element is displacement at nodes. To build a geometry which has exactly same elements structure, the geometry from thermal analysis is imported to mechanical analysis using RESUME command. To change element model, EMODIF command was used. It should be noted that SOLID45 element cannot in the tetrahedral form, thus in the thermal analysis this element form should be avoided.

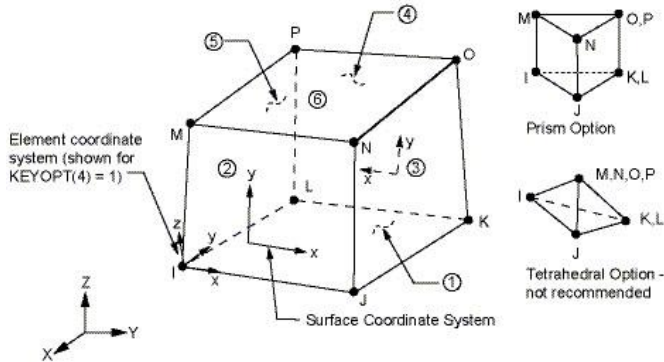


Figure 4. Solid70 3D-thermal element (Release 12.0 Documentation for ANSYS)

III. ANALYTIC SOLUTION

The proposed residual stress mechanism is as follows:

- The thermal expansion existed at the heated region (middle bar).
- The expansion is constrained by the surround cooler region (side bars).
- The constrained may caused plastic compressive stress at the heater region.
- When the temperature return back to the room temperature the plastic compressive stress still left at the heating region, which causes misfit.
- The misfit develops residual stress.

Following the above mechanism will be discussed stress distributions when the middle bar is heated and also when the middle bar is cooled down to the initial temperature. There will be two cases, first when the temperature load causes elastic stress – strain state and the second when the temperature load causes plastic stress – strain state. Based on the aforementioned mechanism only temperature load which causes plastic stress – strain state produces residual stress. In all condition equilibrium forces condition is put as governing condition. Since the area of the three bars is equal, the stress in the middle bar should be twice and in the opposite direction to the side bars stresses. For temperature load in elastic state follows flow-chart as shown at figure 5. No stress is developed in all bars when mid bar is cooling down to initial temperature.

When temperature load exceeds elastic state the mechanism follows flow-chart at figure 6 when the middle bar is heated whilst when the middle bar is cooled down to initial

temperature follows flow-chart at figure 7.

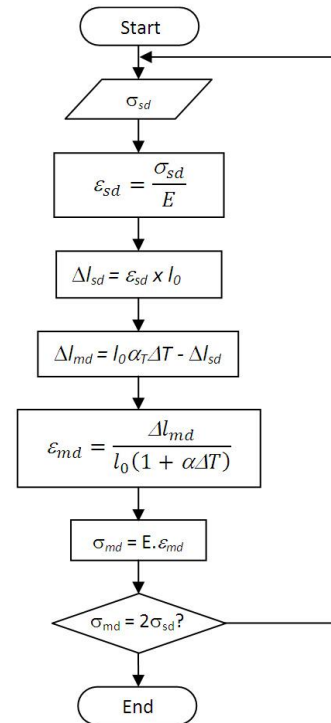


Figure 5. Flow-chart for evaluating thermal stress in elastic strain state.

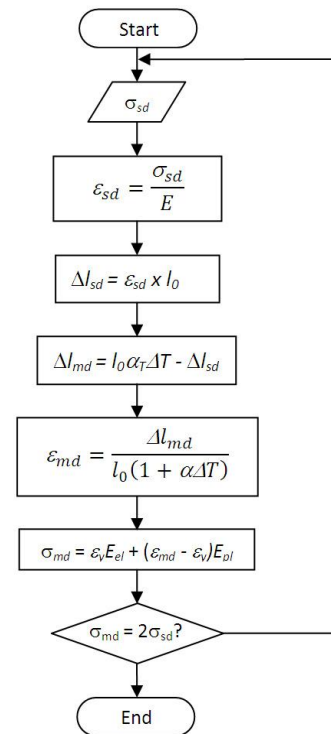


Figure 6. Flow-chart for evaluating thermal stress in plastic state.

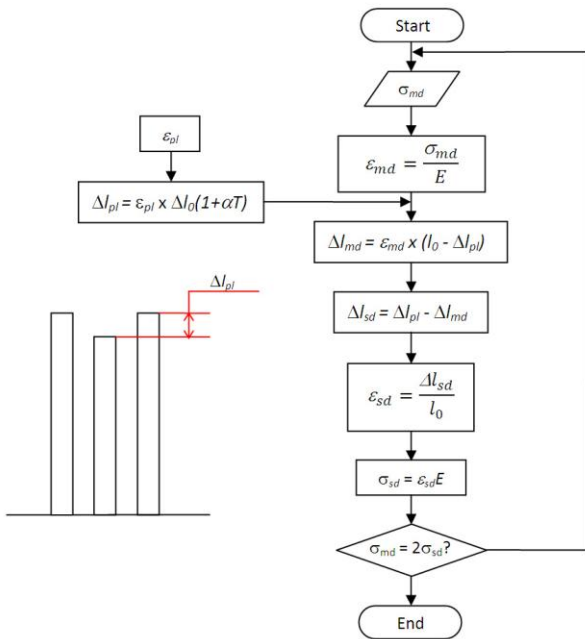


Figure 7. Flow-chart for evaluating residual stress.

IV. RESULTS AND DISCUSSIONS

The results from ANSYS simulation can be retrieved using post processing commands. The results can be presented graphically and numerically. First is evaluated thermal stress when thermal load caused elastic strain – stress state on the three bars. The stress distribution when the middle bar is heated to 400°C is shown at figure 8. At figure 8a is shown stress distribution at full size whilst at figure 8b is at half model. In figure 8b can be clearly seen stress distribution at cross section in the middle part of all three bars which will be compared to analytical method results. Evaluation on the cross section of figure 8b, the thermal stress at the middle bar is -79.487 MPa (compression) and at the side bar is 39.877 MPa (tensile). The analytical results following the proposed mechanism for elastic stress- strain state can be obtained using flow chart at figure 5. The result for the thermal stress at the middle bar is -79.576 MPa (compression) whilst at the side

bars are 39.788 MPa (tensile). The difference with the FEM results is may caused by the expansion as a results of the temperature load as well as stress load at the lateral direction which is not taken into account in the analytic solution. Also the analytic solution does not accommodate stress distribution at the ends of the bars which causes localized high strain in the region. From FEM side, perfect tough material for the clamping bar is modeled by high Young modulus which is still not perfectly tough. However, the difference between analytic solutions with FEM simulation is low (0.11%). When temperature load is omitted, there are no stresses left on the three bars since no plastic strain existed when the system is heated.

The next analysis demonstrates the three bars model where the plastic deformation is occurred, it is when the middle bar is heated to 1000°C. The analytic solution for thermal stress when the middle bar is heated follows the flow chart at figure 6. Following the flow-chart when the three bars are heated the stress distribution at the middle bar is -150.261 MPa (compression) and at the side bars is 75.131 MPa (tension). The FEM simulation produces stress at the middle bar, as shown at figure 9a, -150.005 MPa (compression) which show insignificant difference with analytical model (0.17%).

When the temperature load is removed, it should be a stress distribution as a result of previous compressive plastic strain when the system is heated. The FEM simulation for the left stress (residual stress) shows that the residual stresses are 48.834 MPa (tensile) at the middle bar and -24.417 MPa (compression) at the side bars. The analytic solution for the residual stress follows the flow chart at figure 7. Following the flow chart the residual stress at the middle bar is 48.067 MPa (tensile) and at the side bars are -24.033 MPa (compression) which again shows small difference with the FEM simulation (1.57%).

Overall it can be said that FEM simulation and analytic solution show good agreement. The tensile residual stress at the middle bar and the side bars can be demonstrated by both methods. The confirmations from FEM results have endorsed the proposed mechanism of residual stress formation.

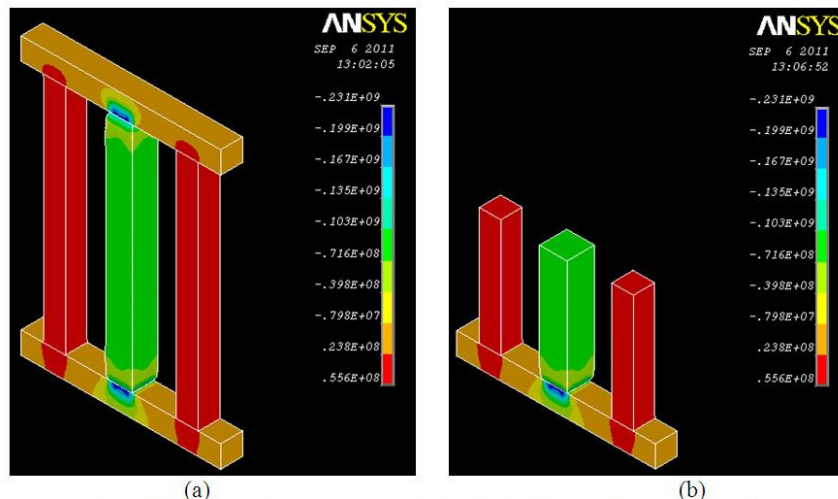


Figure 8. Stress distribution when middle bar is heated to 400°C (a) full model and (b) half model.

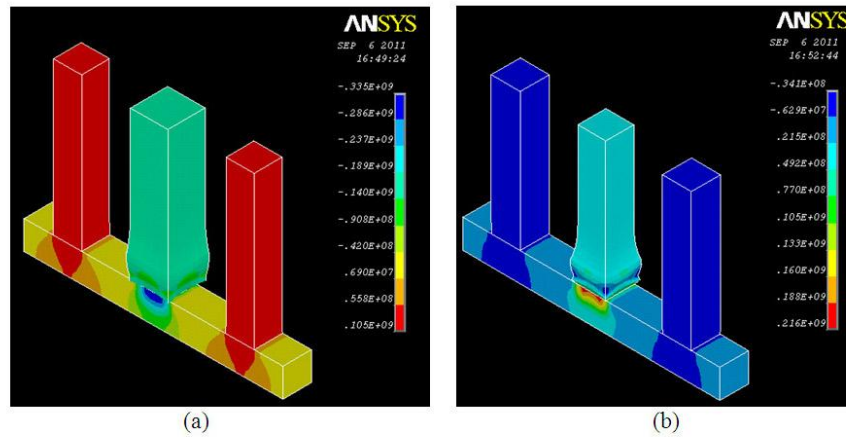


Figure 9. Stress distribution at the three bars model heated to 1000°C. (a) when the middle bar is heated, (b) when the temperature load is omitted.

## V. CONCLUSION

The proposed mechanism can give insights how the tensile residual stress at weld line developed in the welding process. Furthermore, comparing to the FEM simulation the results obtained from analytical method that follows the proposed mechanism showed very well agreements not only qualitatively but also quantitatively. Finally, it can be said the proposed mechanism can be considered as the correct mechanism of residual stress development in the welding process.

Material properties in this paper are considered temperature independence. Future discussion on temperature dependence material properties will be worthy full.

## REFERENCES

- [1] Lars-Erik Lindgren (2001), *Finite element modeling and simulation of welding part 1: increased complexity*, Journal of Thermal Stresses, vol.24, pp.141 – 192.
- [2] Lei Yu-cheng, Yu Wen-xia, Li Chai-hui and Cheng Xiao-nong, *Simulation on temperature field of TIG Welding of cooper without preheating*, Transaction of Nonferrous Metals Society of China, Vol.16, pp. 838-843, 2006.
- [3] Jinjin Xu, Ligong Chen and Chunzhen Ni, *Effect of vibratory weld conditioning on the residual stresses and distortion in multi-pass girth-butt welded pipes*, Pressure Vessels and Piping, no.84: pp. 298-303, 2007.
- [4] M. Muruganath, HKDH Bhadesia, E. Keehan, HO Andren and L. Karlson, *Strong and tough steels welds*, Mathematical Modelling of Weld Phenomena – 6. Institute of Material, pp.205 – 229, Graz University of Technology, 2002.
- [5] E. Keehan, HO Andrea, L. Karlsson, M. Muruganath and HKPH Bhadesia, *Mechanical effects of nickel and manganese on high strength steel weld metal*, Recent Trends in Welding Research, Atlanta, 2002.
- [6] Mochammad Noer Ilman and Triyono, *Fatigue crack growth behavior of friction stir welded aluminium alloy 2024-T3 under local preheating*, International Journal of Material Science, vol.5, no.6, pp.791 – 800, 2010.
- [7] D. Rosenthal, *The theory of moving source of heat and its application to metal transfer*, Trans. ASME, Vol.43 no.11; 1946.
- [8] H.S. Carslaw and J.C. Jaeger, *Conduction of heat in solid*, Clarendon Press, Oxford, 1959.
- [9] Z. Paley, J.N. Lync and Adam C.M. Jr, *Heat flow in welding heavy steel plate*, Welding Research Supplement, pp.71-79, 1964.
- [10] N. Christensen, V. Davies and K. Gjermundsen, *Distribution of temperature in arc welding*, British Welding Journal 12(2), pp. 54-75, 1965.
- [11] C.L. Tsai, *Heat flow in fusion welding*, Proceeding of the conference on trends in welding research in the united states, ASM International, New Orleans, pp.91 – 108, 1982.
- [12] R. Komanduri and Z.B. Hou, *Thermal analysis of the arc welding process: part I. General solutions*, Metallurgical and Materials Transactions, vol. 31B, pp. 1353 – 1370, 2000.
- [13] Djarot B. Darmadi, John Norrish and Anh Kiet Tieu (2011), *Analytic and finite element solutions for temperature profiles in welding using varied heat source models*, World Academy of Science, Engineering and Technology, vol. 81, pp. 154 – 162, 2011.
- [14] S. Vaidyanathan, A.F. Todaro and I. Finnie, *Residual stresses due to circumferential welds*, Journal of Engineering Materials and Technology, vol.95, no. 4, pp.233 – 237, 1973.
- [15] A.J.A. Parlene, *Residual stresses in welded construction and their effects*, Proc. The Welding Institute, Abington, Cambridge, 1977.
- [16] C.P. Chou and Y.C. Lin, *Reduction of residual stress by parallel heat welding in small specimens of type 304 stainless steel*, Materials Science and Technology, vol.8, pp. 179 – 183, 1992.
- [17] Y.C. Lin and K.H. Lee, *Effect of welding parameters on the residual stress by the parallel heat welding*, International Journal of Pressure Vessel and Piping, vol.71, pp. 197- 202, 1997.
- [18] Sindo Kou, *Welding metallurgy*, A John Wiley & Sons, Inc., Publication, pp. 122 – 141, 2003.
- [19] E.M. Van der Aa, M.J.M. Hermans and I.M. Richardson, *Conceptual model for stress and strain development during welding with trailing heat sink*, Science and Technology of Welding and Joining, vol.11 no.4, pp.488 - 495, 2006.
- [20] K. Satoh & M. Toyoda, *Static Tensile and Brittle Fracture Strengths of Soft welded joints*, Trans. JWRI, 73 – 80, 1973.
- [21] K. Masubuchi, *Analysis of welded structures*, Part 1, 642; Pergamon Press, New York, 1980.
- [22] J. Canas, R. Picon, F. Paris and J.L. Rio, *One dimensional model for the prediction of residual stress and its relief in welded plate*, International Journal of Mechanical Science, vol.38, pp. 735 – 751, 1996.
- [23] P. Michaleris and A. De Biccari, *Prediction of welding distortions*, Welding Journal, vol.76, pp. 172s – 181s, 1997.
- [24] M.V. Deo, P. Michaleris and J. Sun, *Prediction of buckling distortion of welded structures*, Science and Technology of Weld Join, vol.8, pp. 49 – 54, 2003.
- [25] A. Barroso, J. Canas, R. Picon, F. Paris, C. Mendez and I. Unanue, *Prediction of welding residual stresses and displacements by simplified models. Experiment validation*, Materials and Design, vol. 31, pp. 1338 – 1349, 2010.