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Common Coincidence Point in Fuzzy Metric Space

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Abstract
We prove common coincidence point in fuzzy metric space .We extend result of Sharma and others for multivalued mappings introduced by Kubiaczyk and Sharma . Servet and Sharma further extended this result to intuitionistic fuzzy metric space.
Key Words : Weakly compatible, multivalued mappings.

Introduction

In 1965, the concept of fuzzy sets was introduced initially by Zadeh [xxxvi] since then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [vi] , Erceg [vii] , Kaleva and Seikkala [xvii] , Kramosil and Michalek [xviii] have introduced the concept of fuzzy metric spaces in different ways. Many authors have also studied the fixed point theory in these fuzzy metric spaces are , Chang, Cho, Lee, Jung and Kang[iii] , Fang[viii] , Grabiec[x] , Hadzic[xi],[xii] , Jung, Cho and Kim [xiv] , Jung, Cho, Chang and Kang [xv], Sharma [xxiii],[xxiv],[xxv], Mishra [xxviii], Sharma and Singh , Sharma and Bamboria [xxvi] , Sharma and Deshpande [xxviii],[xxix],[xxx],[xxxi] , Sharma and Bagwan [xxvii], , Sharma and Tiwari [xxxiii],[xxxiv] , Sharma and Patidar [xxxii] and for fuzzy mappings are Bose and Sahani [i] , Butnariu [ii] , Chang[iv] Chang, Cho, Lee and Lee [v], Heilpern [xiii] , Sharma [xxiii] .

In this note we extend results of Sharma [xxiii] and others for multivalued mappings introduced by Kubiaczyk and Sharma [xix] . Servet and Sharma [xxii] further extended this result to intuitionistic fuzzy metric space.

Preliminaries

Definition 1.[21] . A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that $a*b \le c*d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$

Example of t-norm are $a*b = ab$ and $a*b = \min\{a,b\}$.

Definition 2.[18] . The 3-tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$,

(FM-1) $M(x, y, 0) = 0$,

(FM-2) $M(x, y, t) = 1$, for all $t > 0$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y x, t)$,

(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t+s)$,

(FM-5) $M(x ,y, .):[0,1)\to[0,1]$ is left continuous .

Note that M(x, y, t) can be thought of as the degree of nearness between x and y with respect to t . We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$ and $M(x, y, t) = 0$ with ∞ and we can find some topological properties and examples of fuzzy metric spaces in [9].

In the following example, we know that every metric induces a fuzzy metric.

Example 1.[9] Let (X,d) be a metric space . Define $a*b = ab$ (or $a*b = \min\{a,b\}$) and for all $x,y\in X$ and $t > 0$,

$$M(x,y,t) = \frac{t}{t + d(x,y)} \tag{1.a}$$

Then (X,M,*) is a fuzzy metric space . We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Lemma 1[10]. For all $x,y \in X$, $M(x,y,.)$ is non decreasing.

Definition 3**[10]**. Let (X,M,*) is a fuzzy metric space :

(1) A sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ (denoted by $\lim_{n\to\infty} x_n = x$), if $\lim_{n\to\infty} M(x_n,x,t) = 1$, for all $t > 0$.

(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if

$\lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$,
for all $t > 0$ and $p > 0$.

(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 1. Since * is continuous, it follows from (FM-4) that the limit of the sequence in FM-space is uniquely determined.

Let (X,M,*) is a fuzzy metric space with the following condition:

(FM-6) $\lim_{n\to\infty} M(x,y,t) = 1$ for all $x,y \in X$.

Lemma 2 [20] . Let $\{y_n\}$ be a sequence in a fuzzy metric space (X,M,*) with the condition (FM-6). If there exists a number $k\in(0,1)$ such that

$$M(y_{n+2},y_{n+1},kt) \ge M(y_{n+1},y_n,t) \tag{1.b}$$

for all $t > 0$ and $n = 1,2,...$ then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 3 [16] **.** If for all $x,y\in X$, $t > 0$ and for a number $k\in(0,1)$,

$$M(x,y,kt) \ge M(x,y,t)$$

then $x = y$.

Definition 4[19].Two maps A and B are said to be weakly compatible in fuzzy metric space if they commute at coincidence point.

Definition 5 [19]. Let (X, M, *) be a fuzzy metric space with $t*t \geq t$ for all $t \in [0,1]$. Consider $S : X \to X$ and $P : X \to CB(X)$. A point $z \in X$ is called a coincidence point of S and P if and only if $Sz \in Pz$.

Kubiaczyk and Sharma [xxxiii] introduced the following concept of multivalued mappings in the sense of Kramosil and Michalek [xviii].

We denote by CB(X) the set of all non-empty, bounded and closed subsets of X. We have

$M^{\nabla}(B, y, t) = \max\{ M(b, y, t) : b \in B \}$

$M_{\nabla}(A, B, t) = \min\{ \min_{a\in A} M^{\nabla}(a, B, t) , \min_{b\in B} M^{\nabla}(A, b, t) \}$

for all A, B in X and $t > 0$.

Main Results

Kubiaczyk and Sharma [xxxv] proved the following :

Theorem A. Let (X,M,) be a complete fuzzy metric space with $t*t \geq t$ for all $t \in [0,1]$ and condition (FM-6). Let $P,Q : X \to CB(X)$ be continuous and there exists mappings $S,T : X \to X$ satisfying :

(i) SP = PS, QT = TQ,

(ii) $P(X) \subset S(X)$ and $Q(X) \subset T(X)$,

(iii) the pairs {P,S} and {Q,T} are compatible,

(iv) there exists a number $k \in (0,1)$ such that

$M_{\nabla}(Px, Qy, kt) \geq \min\{M^{\nabla}(Sx, Tx, t), M^{\nabla}(Px, Sx, t), M^{\nabla}(Qy, Ty, t), M^{\nabla}(Px, Ty, (2-\alpha)t), M^{\nabla}(Qy, Sx, t)\}$

for all $x,y \in X$, $\alpha \in (0,2)$, $t > 0$.

Then P, Q, S and T have a common coincidence point, i.e. $Sz \in Pz$ and $Tz \in Qz$.

In this chapter we improve Theorem A, by removing condition (i) and continuity of the mappings. We prove the following:

Theorem 1: Let (X,M, *) be a complete fuzzy metric space with $t*t \geq t$ for all $t \in [0,1]$ and condition (FM-6).

Let $P,Q : X \to CB(X)$ be mappings and there exists mappings $S,T : X \to X$ satisfying :

(1.1) $P(X) \subset S(X)$ and $Q(X) \subset T(X)$,

(1.2) The pairs {P,S} and {Q,T} are weakly compatible,

(1.3) there exists a number $k \in (0,1)$ such that

$M_{\nabla}(Px, Qy, kt) \geq \min\{M^{\nabla}(Sx, Tx, t), M^{\nabla}(Px, Sx, t), M^{\nabla}(Qy, Ty, t), M^{\nabla}(Px, Ty, (2-\alpha)t), M^{\nabla}(Qy, Sx, t)\}$

for all $x,y \in X$, $\alpha \in (0,2)$, $t > 0$.

Then P, Q, S and T have a common coincidence point, i.e. $Sz \in Pz$ and $Tz \in Qz$.

Proof . Let x_0 be arbitrary point in X and $x_1 \in X$ is such that, $Sx_1 \in Px_0$ and $y_1 = Sx_1$, $k \in (0,1)$ and the inequality hold

$M(x_0, y_1, kt) = M(x_0, Sx_1, kt) \geq M^{\nabla}(x_0 , Px_0, kt) - \in$

$x_2 \in X$ is such that $Tx_2 \in Qx_1$, $y_2 = Tx_2$ and

$M(y_1,y_2, kt) = M(Sx_1, Tx_2, kt) \geq M^{\nabla}(y_1 , Qx_1, kt) - \in/2$.

Inductively

$M(y_{2n+1},y_{2n}, kt) = M(Sx_{2n+1}, Tx_{2n}, kt) \geq M^{\nabla}(y_{2n+1} ,Qx_{2n-1}, kt) - \in/2^{2n-1}$

and $M(y_{2n+1},y_{2n+2}, kt) = M(Sx_{2n+1}, Tx_{2n+2}, kt) \geq M^{\nabla}(y_{2n+1} ,Qx_{2n+1}, kt) - \in/2^{2n+1}$

Now we show that $\{y_n\}$ is a Cauchy sequence.

By (1.3), for all $t > 0$ and $\alpha = 1 - q$ with $q \in (0,1)$, we write

$M(y_{2n},y_{2n+1}, kt) \geq M^{\nabla}(y_{2n} , Px_{2n}, kt) - \in/2^{2n}$

$\geq M_{\nabla}(Px_{2n} , Qx_{2n-1} , kt) - \in/2^{2n}$

$\geq \min\{M^{\nabla}(Sx_{2n} , Tx_{2n-1}, t), M^{\nabla}(Px_{2n}, Sx_{2n}, t), M^{\nabla}(Qx_{2n-1}, Tx_{2n-1}, t), M^{\nabla}(Px_{2n}, Tx_{2n1}, (2-\alpha)t), M^{\nabla}(Qx_{2n-1}, Sx_{2n}, t)\} - \in/2^{2n}$

$\geq \min\{M(y_{2n} , y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t),M(y_{2n}, y_{2n-1}, t), M(y_{2n+1}, y_{2n-1}, (2-\alpha)t), M(y_{2n}, y_{2n}, t)\} - \in/2^{2n}$

$\geq \min\{M(y_{2n} , y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n-1}, (1+k)t),1\} - \in/2^{2n}$

Now using (FM-4), we have

$\geq \min\{M(y_{2n} , y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t)* M(y_{2n}, y_{2n-1}, kt),\} - \in/2^{2n}$ (1.4)

Since t-norm * is continuous and M(x,y, .) is left continuous, letting $k \to 1$ in (1.4), we have

(1.5) $M(y_{2n},y_{2n+1}, kt) \geq \min\{M(y_{2n-1} , y_{2n}, t), M(y_{2n,} y_{2n+1,} t)\} - \in/2^{2n}$

similarly we also have

(1.6) $M(y_{2n+1},y_{2n+2}, kt) \geq \min\{M(y_{2n}, y_{2n+1},t), M(y_{2n+1},y_{2n+2},t)\} - \in/2^{2n+1}$

Thus from (1.5) and (1.6) it follows that

$M(y_{n+1},y_{n+2}, kt) \geq \min\{M(y_n, y_{n+1},t), M(y_{n+1},y_{n+2},t)\} - \in/2^{n+1}$,

for n = 1,2,… and so for positive integers n,p,

$M(y_{n+1},y_{n+2}, kt) \geq \min\{M(y_n, y_{n+1},t), M(y_{n+1},y_{n+2}, t/k^p)\} - \in/2^{n+1}$

Thus, since $M(y_{n+1},y_{n+2}, t/k^p) \to 1$ as $n \to \infty$, we have

$M(y_{n+1},y_{n+2}, kt) \geq M(y_n, y_{n+1},t) - \in/2^{n+1}$.

Since $\in$ is arbitrary making $\in \to 0$, we obtain

$M(y_{n+1},y_{n+2}, kt) \geq M(y_n, y_{n+1},t)$

Therefore, by Lemma 2 , $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete , $\{y_n\}$ converges to a point z in X.

We observe that

$\lim_{n\to\infty} Sx_{2n+1} = z \in \lim_{n\to\infty} Px_{2n}$

and

$\lim_{n\to\infty} Tx_{2n+2} = z \in \lim_{n\to\infty} Qx_{2n+1}$

Hence by weak compatibility of S and P we have $Sz \in Pz$.

Similarly, $Tz \in Qz$.

Thus $z \in X$ is a coincidence point of P, Q, S and T.

This completes the proof of the theorem.

Yildiz, Sharma and Servet [xxxv] , further extended this definition of multivalued function for intuitionistic fuzzy metric space.

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