Control of Urban Traffic Network Based on Mixed Logical Dynamical Modeling and Constrained Predictive Control Approach

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***Abstract*— By increasing of urban population, the require to use vehicles has increased dramatically. Thus in large cities the importance of urban traffic control has greatly increased. In this paper mixed logical dynamical (MLD) model is used to model urban traffic control. Then predictive control approach based on MIQP optimization is used to control this MLD model. Simulation results in MATLAB software shows that MLD modeling with constrained predictive control strategy can effectively improve urban traffic control.**

Keywords— urban traffic control, mixed logical dynamical model, predictive control.

I. Introduction

By increasing trend of urbanization and use of private cars and also lack of transport infrastructure in urban areas, the time required to interurban trips has increased. This leads to the fact that people spend a lot of time in crowded streets and the queue length of cars in streets increase which turns to increasing fuel consumption. All these factors have caused urban air pollution, noise pollution and waste of time in the streets. To obviate this problem generally two approaches can be discussed:

- Developing urban transport infrastructure.
- Improving urban transport infrastructure.

In the first approach, development is critical but it should be noted that in heavily populated urban areas such as city centers, this approach is very expensive and difficult to implement. So the best approach to solve this problem is the effective use of the existing infrastructure, by improving the use of infrastructure (improved the way of using urban roads) to improve the quality of traffic flow.

Urban traffic system is a type of hybrid systems. In traffic control system there are continuous variables like density, queue length, velocity average and logical variables like lights switching. Conventional hydrodynamic models such as traffic wave model, vehicle-following model, lane changing model are based on analysis of traffic density characteristic which hardly can consider relation between continuous and logical variables in control of urban traffic network.

Methods of hybrid systems modeling divided into two groups, namely extended discrete event dynamic system modeling approach and extended continuous variable dynamic system modeling approach. Most researchers use petri nets or hybrid automata to control the intersection traffic signal [1-6]. It is based on extended discrete event dynamic system modeling method. This method needs to accurate design and analysis of the intersection phase switching. So in this method it is difficult for controller to consider relationship between discrete variables and continuous variables very well. In this paper, extended continuous variable dynamic system modeling approach is considered. The hybrid systems modeling based on this approach include MLD model. MLD model can consider relationship between discrete variables and continuous variables very well [7]. Predictive control method can stable MLD systems on reference paths [8]. In this paper MLD model for urban traffic network with five intersection is improved. Then proposed model is validate in MATLAB software to shows the effectiveness of MLD modeling with predictive control for urban traffic network.

II. Urban Traffic Network Modeling

In this paper, MLD systems are used for intersection signal control modelling in urban traffic networks.

A. Mixed Logical Dynamical Model

MLD systems are a class of the hybrid systems which are combination of logical components, dynamic and constraints. MLD model can be expressed as the following linear relationship [8]:

$$\begin{cases} x(\mathrm{k+1})=\mathrm{A}x(k)+B_1u(k)+B_2\delta(k)+B_3z(\mathrm{k}) \\ y(k)=Cx(k)+D_1u(k)+D_2\delta(k)+D_3z(k) \\ E_1x(k)+E_2u(k)+E_3\delta(k)+E_4z(k)\le E_5 \end{cases} \tag{1}$$

Wherein, A, B_1, B_2, B_3, C, D_1, D_2, D_3, E_1, E_2, E_3, E_4 and E_5 are matrixes with appropriate dimensions, x is the state variables vector, y is the output variables vector, u is the control variables vector, δ is the auxiliary logic variables vector and z is the auxiliary continuous variables vector. In this relationship, first equation is the state equation, second equation is the output equation and third equation is the inequality constraints which contains both the system state and the input output constraints. x, y and u has continuous parts and logic parts that shown in equation (2) [8]:

$$\begin{aligned} &x=\begin{bmatrix} x_c \\ x_l \end{bmatrix}, x_c\in \mathrm{R}^{n_c}, x_l\in\{0,1\}^{n_l}, n=n_c+n_l \\ &y=\begin{bmatrix} y_c \\ y_l \end{bmatrix}, y_c\in \mathrm{R}^{p_c}, y_l\in\{0,1\}^{p_l}, p=p_c+p_l \\ &u=\begin{bmatrix} u_c \\ u_l \end{bmatrix}, u_c\in \mathrm{R}^{m_c}, u_l\in\{0,1\}^{m_l}, m=m_c+m_l \end{aligned} \tag{2}$$

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B. MLD Modeling of Urban Traffic Network

Queue length model for one-lane road shown in equation (3) [9]:

$$L_d(t) = \frac{N_0 + N_u(t) - N_d(t) - K_m L}{K_j - K_m} \quad (3)$$

Wherein N_0 is the number of vehicles in the road at initial moment of time t=0, L is the length of road, $N_u(t)$ is the number of vehicles accumulated in input of the road until time t, $N_d(t)$ is the number of vehicles accumulated in output of the road until time t, K_j is the average blocking density of each lane of road, K_m is the average optimum density of each lane of road, $L_d(t)$ is the average queue length in each lane of the road at time t. Queue length model for M-lane road shown in equation (4) [9].

$$L_d(t) = \frac{N_0 + \sum_{i=1}^{M} N_u(i,t) - \sum_{i=1}^{M} N_d(i,t) - MK_m L}{M(K_j - K_m)} \quad (4)$$

Wherein N_0 is the number of vehicles in the road at initial moment of time t=0, L is the length of road, $N_u(i,t)$ is the number of vehicles accumulated in input of the i-th road lane until time t, $N_d(i,t)$ is the number of vehicles accumulated in output of the i-th road lane until time t, K_j is the average blocking density of each lane of road, K_m is the average optimum density of each lane of road, M is the number of road lanes, $L_d(t)$ is the average queue length in each lane of the road at time t. mixed logical dynamical model of the average queue length in each lane of road with M lane is shown in equation (5) [10].

$$L_d(k+1) = L_d(k) + \frac{T_s}{M(K_j - K_m)}(q_{in} - M\,m\,v) \quad (5)$$

Wherein m is the export discharge saturation flow rate at each lane of road, T_s is the sample rate, q_{in} is a road entrance flow rate in terms of the number of cars at hour and v is a logic variable (v=0 stands for red light and v=1 stands for green light) [10].
Network of urban traffic with five four-phase intersection is shown in Fig. 1.
In four-phase case in each intersection, each light is for straight movement and turn-left and turn-right movement for example in intersection U2, light v_{21} is for straight movement from intersection U3 to U4, turn-left movement from intersection U3 to U1 and turn right movement from intersection U3 to U5. This intersections consist of four phase as follows:

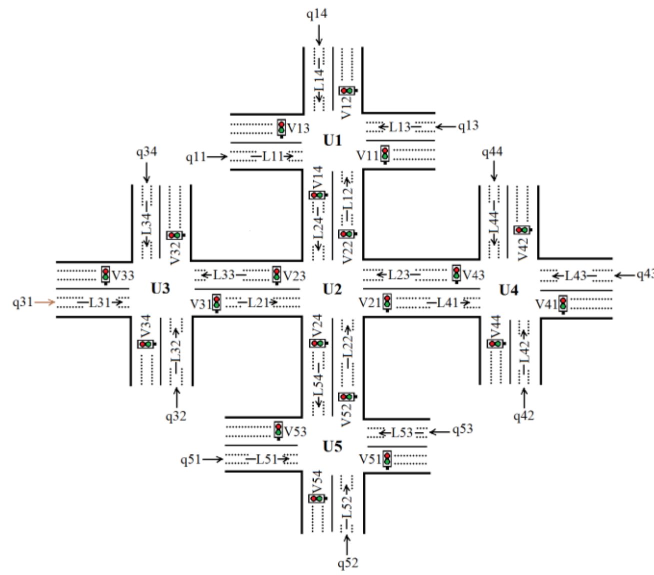

Fig. 1: Urban traffic network with five four-phase intersection.

- Light v_{21} is green and the other lights are red.
- Light v_{22} is green and the other lights are red.
- Light v_{23} is green and the other lights are red.
- Light v_{24} is green and the other lights are red.

Averaged queue length equations for each lane of roads leading to intersection U1 are shown in equation (6).

$$\begin{cases} L_{11}(k+1) = L_{11}(k) + \dfrac{T_s}{M_{11}(K_{j11} - K_{m11})}(q_{11}(k) - q_{out11}(k)) \\ L_{12}(k+1) = L_{12}(k) + \dfrac{T_s}{M_{12}(K_{j12} - K_{m12})}(q_{12}(k) - q_{out12}(k)) \\ L_{13}(k+1) = L_{13}(k) + \dfrac{T_s}{M_{13}(K_{j13} - K_{m13})}(q_{13}(k) - q_{out13}(k)) \\ L_{14}(k+1) = L_{14}(k) + \dfrac{T_s}{M_{14}(K_{j14} - K_{m14})}(q_{14}(k) - q_{out14}(k)) \end{cases} \quad (6)$$

$$\begin{cases} q_{out11}(k) = M_{11} \times m_{11} \times v_{11}(k) \\ q_{out12}(k) = M_{12} \times m_{12} \times v_{12}(k) \\ q_{out13}(k) = M_{13} \times m_{13} \times v_{13}(k) \\ q_{out14}(k) = M_{14} \times m_{14} \times v_{14}(k) \end{cases} \quad (7)$$

Wherein T_s is sampling time, L is averaged queue length in each lane of road, M is number of road lanes, q is road entrance flow rate, q_{out} is road output flow rate, K_j is the average blocking density of each lane of road, K_m is the average optimum density of each lane of road and m is the export discharge saturation flow rate at each lane of road. Parameter indexing in equations (6) and (7), according to number of queue length in Fig. 1 is considered. Averaged queue length equations for each lane of roads leading to

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intersections U2, U3, U4 and U5 would be achieve in a same way. So the average queue length state space equations for urban traffic network is shown in equation (8).

$$x(k+1) = Ax(k) + Bu(k) + E \tag{8}$$

Wherein A, B and E are matrixes with appropriate dimensions. x and u are state vector and control signal respectively.

$$\begin{aligned} x = [&L_{11}\ L_{12}\ L_{13}\ L_{14}\ L_{21}\ L_{22}\ L_{23}\ L_{24}\ L_{31}\ L_{32}\ L_{33}\ L_{34} \\ &L_{41}\ L_{42}\ L_{43}\ L_{44}\ L_{51}\ L_{52}\ L_{53}\ L_{54}]^T \\ u = [&v_{11}\ v_{12}\ v_{13}\ v_{14}\ v_{21}\ v_{22}\ v_{23}\ v_{24}\ v_{31}\ v_{32}\ v_{33}\ v_{34} \\ &v_{41}\ v_{42}\ v_{43}\ v_{44}\ v_{51}\ v_{52}\ v_{53}\ v_{54}]^T \end{aligned} \tag{9}$$

In urban traffic network which shown in Fig. 1 input flow rates q_{12}, q_{21}, q_{22}, q_{23}, q_{24}, q_{33}, q_{41}, q_{54} are related to output flow rates from surrounding intersection based on equation (10).

$$\begin{cases} q_{12}(k) = \beta_{l_{21},l} \times m_{21} \times v_{21}(k) + \beta_{l_{22},d} \times m_{22} \times v_{22}(k) + \beta_{l_{23},r} \times m_{23} \times v_{23}(k) \\ q_{21}(k) = \beta_{l_{31},d} \times m_{31} \times v_{31}(k) + \beta_{l_{34},l} \times m_{34} \times v_{34}(k) + \beta_{l_{32},r} \times m_{32} \times v_{32}(k) \\ q_{22}(k) = \beta_{l_{51},l} \times m_{51} \times v_{51}(k) + \beta_{l_{52},d} \times m_{52} \times v_{52}(k) + \beta_{l_{53},r} \times m_{53} \times v_{53}(k) \\ q_{23}(k) = \beta_{l_{44},r} \times m_{44} \times v_{44}(k) + \beta_{l_{43},d} \times m_{43} \times v_{43}(k) + \beta_{l_{42},l} \times m_{42} \times v_{42}(k) \\ q_{24}(k) = \beta_{l_{11},r} \times m_{11} \times v_{11}(k) + \beta_{l_{14},d} \times m_{14} \times v_{14}(k) + \beta_{l_{13},l} \times m_{13} \times v_{13}(k) \\ q_{33}(k) = \beta_{l_{24},r} \times m_{24} \times v_{24}(k) + \beta_{l_{23},d} \times m_{23} \times v_{23}(k) + \beta_{l_{22},l} \times m_{22} \times v_{22}(k) \\ q_{41}(k) = \beta_{l_{22},r} \times m_{22} \times v_{22}(k) + \beta_{l_{21},d} \times m_{21} \times v_{21}(k) + \beta_{l_{24},l} \times m_{24} \times v_{24}(k) \\ q_{54}(k) = \beta_{l_{21},r} \times m_{21} \times v_{21}(k) + \beta_{l_{24},d} \times m_{24} \times v_{24}(k) + \beta_{l_{23},l} \times m_{23} \times v_{23}(k) \end{cases} \tag{10}$$

Wherein β is relative percentage which is determined from past data, $\beta_{l_{21},l}$ determine relative percentage of cars in L_{21} queue which intend to turn left to intersection U1, $\beta_{l_{21},d}$ determine relative percentage of cars in L_{21} queue which intend to move straight to intersection U4 and $\beta_{l_{21},r}$ determine relative percentage of cars in L_{21} queue which intend to turn right to intersection U5.

For intersection with four phases based on MLD modeling, two modes for MPC controller can be considered. First, optimization problem in predictive control is not constrained to observe a specific phase sequence and second, the controller is constrained to observe a specific phase sequence to green the lights, for example in one cycle for intersection U1 lights should be green with phase sequence v_{11}, v_{12}, v_{13} and v_{14}. In first mode, controller determine phase sequence and green time of lights but in second mode controller constrained to consider a specific sequence and only green time of lights would be determine by controller. In both mentioned mode at any time in each intersection just one light most be green, so following constraints must be considered:

$$\begin{cases} v_{11}(k) + v_{12}(k) + v_{13}(k) + v_{14}(k) = 1 \\ v_{21}(k) + v_{22}(k) + v_{23}(k) + v_{24}(k) = 1 \\ v_{31}(k) + v_{32}(k) + v_{33}(k) + v_{34}(k) = 1 \\ v_{41}(k) + v_{42}(k) + v_{43}(k) + v_{44}(k) = 1 \\ v_{51}(k) + v_{52}(k) + v_{53}(k) + v_{54}(k) = 1 \end{cases} \tag{11}$$

In MLD modeling constraints should be expressed as linear inequalities, so equations (11) reformulated to linear inequalities (12).

$$\begin{cases} v_{11}(k) + v_{12}(k) + v_{13}(k) + v_{14}(k) \le 1 \\ -v_{11}(k) - v_{12}(k) - v_{13}(k) - v_{14}(k) \le -1 \\ v_{21}(k) + v_{22}(k) + v_{23}(k) + v_{24}(k) \le 1 \\ -v_{21}(k) - v_{22}(k) - v_{23}(k) - v_{24}(k) \le -1 \\ v_{31}(k) + v_{32}(k) + v_{33}(k) + v_{34}(k) \le 1 \\ -v_{31}(k) - v_{32}(k) - v_{33}(k) - v_{34}(k) \le -1 \\ v_{41}(k) + v_{42}(k) + v_{43}(k) + v_{44}(k) \le 1 \\ -v_{41}(k) - v_{42}(k) - v_{43}(k) - v_{44}(k) \le -1 \\ v_{51}(k) + v_{52}(k) + v_{53}(k) + v_{54}(k) \le 1 \\ -v_{51}(k) - v_{52}(k) - v_{53}(k) - v_{54}(k) \le -1 \end{cases} \tag{12}$$

In second mode, controller must consider specific sequence to green lights. So for considering phase sequence v_{11}, v_{12}, v_{13} and v_{14} in intersection U1 following conditions must be established. If in previous moment light v_{11} was green, now light v_{11} or v_{12} must be green and lights v_{13} and v_{14} shouldn't be green, i.e.

$$\text{if}\ \ v_{11}(k-1) = 1\ \ \wedge\ \ v_{11}(k) = 0\ \ \Rightarrow v_{12}(k) = 1 \tag{13}$$

Logic relation (13) is proportional with logic relation (14):

$$v_{11}(k-1)(1 - v_{11}(k)) \Rightarrow v_{12}(k) \tag{14}$$

Logic relation (14) can be expressed as inequality (15):

$$v_{11}(k-1)\big(1 - v_{11}(k)\big) - v_{12}(k) \le 0 \tag{15}$$

In order to observe phase sequence v_{11}, v_{12}, v_{13} and v_{14} in intersection U1, the following constraints should exert to controller.

$$\begin{cases} v_{11}(k-1)\big(1 - v_{11}(k)\big) - v_{12}(k) \le 0 \\ v_{12}(k-1)\big(1 - v_{12}(k)\big) - v_{13}(k) \le 0 \\ v_{13}(k-1)\big(1 - v_{13}(k)\big) - v_{14}(k) \le 0 \\ v_{14}(k-1)\big(1 - v_{14}(k)\big) - v_{11}(k) \le 0 \end{cases} \tag{16}$$

Also for considering phase sequence v_{21}, v_{22}, v_{23} and v_{24} in intersection U2, phase sequence v_{31}, v_{32}, v_{33} and v_{34} in intersection U3, phase sequence v_{41}, v_{42}, v_{43} and v_{44} in

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intersection U4 and phase sequence v_{51}, v_{52}, v_{53} and v_{54} in intersection U5, constraints in form of inequalities (16) with appropriate index should exert to controller.

C. Predictive Control of Urban Traffic Network

MPC controller can stabilize MLD systems on a desired reference paths while operating constraints are met. For MPC controller consider following cost function:

$$J\left(u_0^{N_p-1}, x_0\right) = \sum_{t=0}^{N_p-1} \left\|u(t) - u_f\right\|_{Q_1}^2 + \left\|x(t) - x_f\right\|_{Q_2}^2 \qquad (17)$$

Where in N_p is prediction horizon, Q_1 and Q_2 are appropriate weight matrixes for control signal and state variables respectively, x_f and u_f are desired states and desired control signal respectively. From equation (8), for time-invariant systems we have the solution formula

$$x(t) = A^t x(0) + \sum_{i=0}^{t-1} A^i (Bu(t-i-1) + E) \qquad (18)$$

By plugging equation (18) into cost function (17), the cost function is obtained in terms of control signal u. With simplification, cost function of the form (19) can be obtained.

$$J = U^T HU + F^T U \qquad (19)$$

The matrices H and F are obtained after simplification. By minimizing the cost function (19) in the presence of constraints that are posed, optimal control sequence U will be obtained.

$$U = \begin{bmatrix} u(0) \\ \vdots \\ u(N_p - 1) \end{bmatrix} \qquad (20)$$

According to the receding horizon philosophy, set

$$u(t) = u(0) \qquad (21)$$

Disregard the subsequent optimal inputs $u(1), \ldots, u(N_p - 1)$, and repeat the whole optimization procedure at time t+1.

III. **Results and Tables**

Results for urban traffic network which is shown in Fig. 1 with features such as all roads leading to intersections have three lanes, the export discharge saturation flow rate at each lane of road is 2400 vehicles per hour, relative percentage of vehicles in roads that intent to straight movement is 0.6, relative percentage of vehicles in roads that intent to left-turn movement is 0.3, relative percentage of vehicles in roads that intent to right-turn movement is 0.1, the average of a lane blocking density in all roads is 200 vehicles per kilometer and the average of a lane optimum density in all roads is 50 vehicles per kilometer.

Input flow rates q_{11}, q_{13}, q_{14}, q_{31}, q_{32}, q_{34}, q_{42}, q_{43}, q_{44}, q_{51}, q_{52} and q_{53} are considered 2000 cars per hour.

Averaged queue length in each lane of roads leading to intersection U1 and U2 in network which is shown in Fig. 1 for constrained phase sequence and arbitrary phase sequence are shown in Fig. 2 to Fig. 5.

Lights signals of intersection U2 for constrained phase sequence and arbitrary phase sequence are shown in Fig. 6 and Fig7.

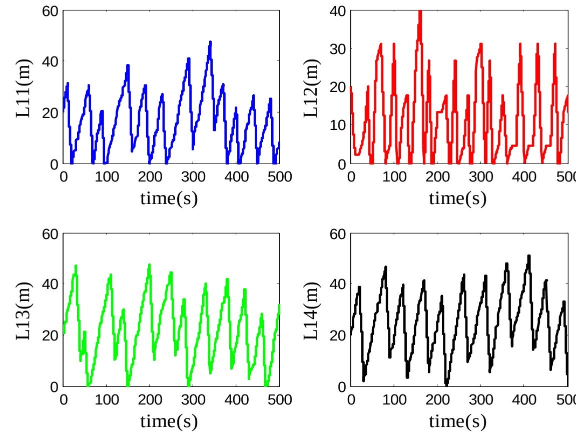

Fig. 2: The average queue length in each lane of the roads leading to intersection U1 with arbitrary sequence.

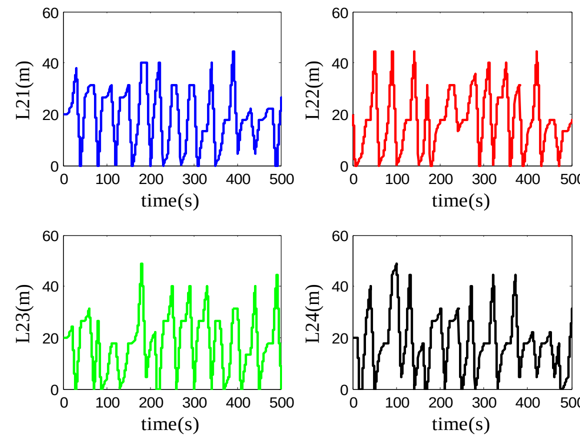

Fig. 3: The average queue length in each lane of the roads leading to intersection U2 with arbitrary sequence.

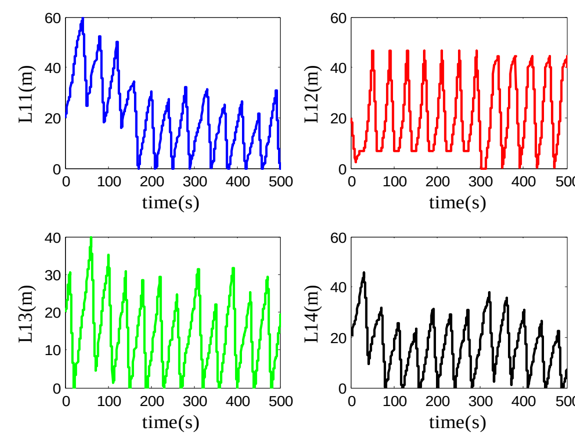

Fig. 4: The average queue length in each lane of the roads leading to intersection U1 with regular sequence.

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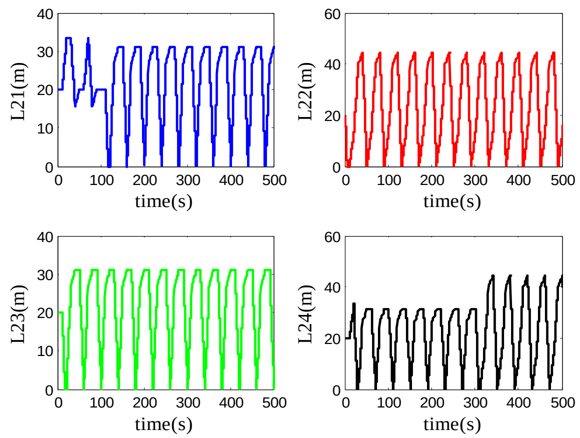

Fig. 5: The average queue length in each lane of the roads leading to intersection U2 with regular sequence.

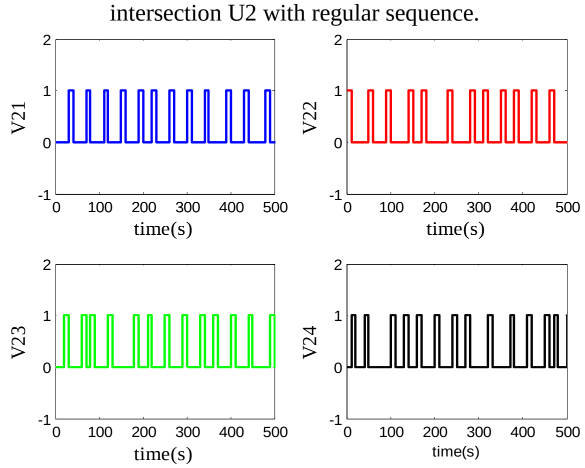

Fig. 6: Light signals for intersection U2 with arbitrary sequence.

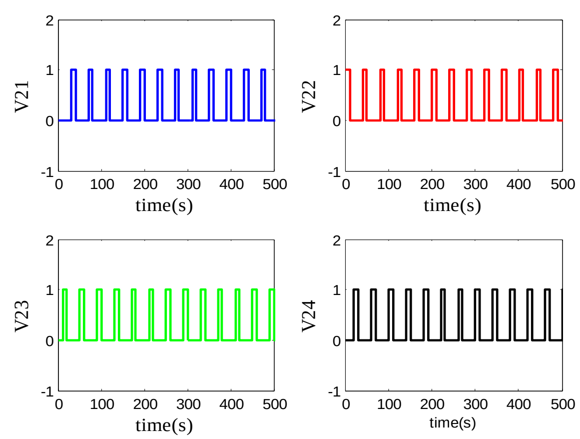

Fig. 7: Light signals for intersection U2 with regular sequence.

From Fig. 2 to Fig. 5, it can be observe that averaged queue length in each lane of roads, in mode without constraints on phase sequence is less than mode with constraints. The reason is more freedom of controller in without constraints mode. In without constraint mode, controller is capable to present less queue length by choosing appropriate phase sequence and green light time.

IV. Conclusion

Urban traffic system is a type of hybrid systems. In this paper MLD modeling approach with constrained predictive control strategy based on MIQP optimization for urban traffic network is presented. Two mode, constrained phase sequence and arbitrary phase sequence, for green lights are investigated. Simulation results shows MLD approach can effectively modeling urban traffic network and constrained predictive controller can efficiently control corresponding system.

References

i. *J. Jűlvez, R. K. Boel. "A continuous Petri net approach for model predictive control of traffic systems" IEEE Trans. Systems and Humans, vol. 40(4), pp. 686-697, 2010.*

ii. *M. Dotoli, M. P. Fanti, G. Iacobellis. "Validation of an urban traffic network model using colored timed Petri nets" IEEE International Conference on Systems, Man and Cybernetics, Vol. 2, 2005.*

iii. *M. A. Badamchizadeh, M. Joroughi. "Deterministic and stochastic Petri Net for urban traffic systems" The 2nd International Conference on Computer and Automation Engineering (ICCAE), Vol. 5, 2010.*

iv. *C. Yangzhou, L. Hongfeng, N. Jin. "Periodic linear differential automata modeling and analysis of T shaped intersection signal timing" Control Theory and Applications, vol. 28(12), pp. 1773-1778, 2011.*

v. *N. Jin, C. Yangzhou, Z. Liguo, L. Hongfeng. "Hybrid automata model and signal coordination control of two adjacent intersections" Traffic Information and Security, vol. 29(3), pp. 5-9, 2011.*

vi. *F. Liquan, C. Yangzhou, L. Zhenlong. "Freeway area-wide coordinated control research based on hybrid fuzzy switching " Traffic Information and Safety, pp. 44-48, 2009.*

vii. *D. Mignone, A. Bemporad, M. Morari. "A framework for control, fault detection, state estimation and verification of hybrid systems" American Control Conference, pp. 134-138, 1999.*

viii. *A. Bemporad, M. Morari. "Control of systems integrating logic, dynamics, and constraints" Automatica, vol. 35, pp. 407-427, 1999.*

ix. *Y. Ronghan, W. Dianhai. "The maximum equivalent queue length model and its time-space characteristics" Journal of Dalian University of Technology, vol. 50(5), pp. 699-70, 2010.*

x. *L. Wang, W. J. Xiu, Z. J. Li, L. B. Zhao. "Freeway Off-Ramp and Downstream Intersection Coordination Based on MLD Model and MPC Approach" Advanced Materials Research, Vol. 765, 2013.*