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The Growth and Decay Behavior of Sonic Waves in Non-Ideal Gases

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Abstract
In this paper, using compatibility conditions of Thomas, growth and decay behavior of sonic waves in non-ideal gases are discussed. Effect of curvature and non-ideal gas parameter on growth and decay behavior of waves are discussed and is concluded that in case of non-ideal gas critical times for shock formation increases.

Keyword: Sonic-Waves, Non-ideal gas

Introduction

The assumption that the medium is an ideal gas is no more valid when the flow takes place in extreme conditions. Anisimov and Spiner [1] studied a problem of point explosion in low density non-ideal gas by taking the equation of state in a simplified from which describes the behavior of medium satisfactorily. Robert and Wu[6] have studied the gas that obeys a simplified Vander Waal's equation of state.

Vishwakarama, Chaube and Patel[15] have investigated the one dimensional unsteady self-similar flow behind a strong shock, driven out by a cylindrical or spherical piston in a medium which is assumed to be non-ideal and which obey the simplified Vander Waal's equation of state as considered by Robert and Wu[8] However they have assumed that the piston is moving with time according to law given by Steiner and Hirschler[9], Madhumita and Sharma[3] have considered the model equation for a low density gas, which describes the behavior of the medium satisfactorily for implosion problems where the temperature attained by the gas motion in the strong shock limit is very high.

Thomas[13] has considered the growth and decay of sonic discontinuities in ideal gases. Applying compatibility conditions given by the Thomas[14] several investigators[5,6,10,11,12] have obtained growth and decay of sonic waves in Radiating, Relaxing, Magnetogasdynamics and dusty gases for moderate particle loading.

All above investigators[3,6,7,8,15] have considered the case of strong shocks and have not investigated the problem of weak discontinuities in non-ideal gases. The aim of present paper is to discuss the growth and decay of sonic discontinuities in non-ideal gases, it is concluded that compressive wave terminates into a shock wave (fig.2) and for low density case critical time of shock formation increases. Figure 1 shows that there is a decay for the case of expansion waves. For negligible parameter $b\rho \ll 1$, waves behave in a manner similar to ideal gas.

Basic Equations

Equations governing the unsteady motion of non-ideal gas when dissipative effects are neglected are given by[8]

$$\rho_t + \rho u_{i,i} + u_i \rho_{,i} = 0 \quad (1)$$

$$\rho u_{i,t} + \rho u_j u_{i,j} + p_{,i} = 0 \quad (2)$$

and

$$E_{,t} + u_j (E+p)_{,j} + (E+p) u_{i,i} = 0 \quad (3)$$

where ρ, p, u_i are density, pressure, components of fluid velocity and a comma followed by an index implies partial derivative with respect to that index and E is the total energy density given by

$$E = \rho e + \frac{1}{2} u^2 \rho \quad (4)$$

where e is internal energy given by

$e = C_v T = \frac{(v-b)}{(\gamma-1)} p,$ T being temperature.

Following Robert and Wu[8], we consider the equation of states as

$$p = \frac{\rho RT}{1-b\rho} = \frac{RT}{(v-b)}, \quad v = \frac{1}{\rho} \quad (5)$$

R universal gas constant, b being the internal volume of the gas molecules, which is known in term of the molecular interaction potential in high temperature gases and γ being ratio of specific heat.

With help of equation of state, equations (2) and equation (3) reduces to

$$\rho u_{i,t} + \rho u_j u_{i,j} + \frac{\rho RT_{,i}}{1-b\rho} + \frac{RT}{(1-b\rho)^2} \rho_{,i} = 0 \quad (6)$$

and

$$\rho RT_{,t} + \frac{RT_{\rho,t}}{(1-b\rho)} - \rho u_j u_{j,t}(1-b\rho) - (1-b\rho)\rho u_i u_j u_{i,j} + \frac{\gamma RT_{\rho} u_{i,i}}{(1-b\rho)} = 0 \quad (7)$$

where second and higher powers of $b\rho$ are neglected. Taking jump in equation (l), (6) and (7) and using geometrical and Kinematical compatibility conditions given by Thomas[14], we have

$$\xi(u_n - G) + \rho \lambda_i n_i = 0 \quad (8)$$

$$\rho \lambda_i (u_n - G) + \frac{R\rho\mu n_i}{(1-b\rho)} + \frac{RT}{(1-b\rho)^2} \xi n_i = 0 \quad (9)$$

$$-R\rho G\mu - \frac{GRT\xi}{(1-b\rho)} + \rho u_n G(1-b\rho) - \rho(1-b\rho) u_i u_j \lambda_i n_j + \frac{\gamma RT \rho \lambda_i n_i}{(1-b\rho)} = 0 \quad (10)$$

Where

$$\lambda_i = [u_{i,j}]\, n_j \qquad -G\lambda_i = \left[\frac{\partial u_i}{\partial t}\right]$$

$$\mu = [T_{,j}]\, n_j \qquad -G\xi = \left[\frac{\partial \rho}{\partial t}\right]$$

$$\xi = [\rho_{,j}]\, n_j \qquad -G\mu = \left[\frac{\partial T}{\partial t}\right]$$

and [ ] denotes the jump in the quantities enclosed.

If $\lambda = \lambda_i n_i$ from equation (8) to (10) we have

$$\xi = \frac{\rho\lambda}{G - u_n}, \quad (11)$$

and

$$\xi = \frac{(1-b\rho)R\rho\mu}{(u_n - G)^2 (1-b\rho)^2 - RT} \quad (12)$$

From equation (11) and (12) we have

$$\xi = \frac{\rho\lambda}{G-u_n} = \frac{(1-b\rho)R\rho\mu}{(u_n-G)^2(1-b\rho)^2-RT} \qquad (13)$$

Applying relation (13) into equation (9) and equation (10) and after certain manipulation we have

$$\rho\lambda\left[(G-u_n)^2 - \frac{\gamma RT}{(1-b\rho)^2} = 0\right]$$

as $\rho \neq 0$, $\lambda \neq 0$, we have

$$(G-u_n)^2 = \frac{\gamma RT}{(1-b\rho)^2} = a^2. \qquad (14)$$

we have

If medium ahead of $\sum(t)$ is uniform and at rest, u_i vanishes on $\sum(t)$ and for this case, thus

$$G^2 = a^2 \qquad (15)$$

Consequently relation (13) reduce to

$$\xi = \frac{\rho\lambda}{G} = \frac{(1-b\rho)R\rho\mu}{G^2(1-b\rho)^2-RT} \qquad (16)$$

Growth and Decay Equations

Differentiating equation (l), (6) and (7) with respect to x_k and taking jump across $\sum(t)$ and using second order compatibility conditions of Thomas[14] and fact that $u_i = 0$ on $\sum(t)$ we have

$$\frac{\delta\xi}{\delta t} = G\bar{\xi} + 2\rho\lambda\Omega - \rho\bar{\lambda}_i n_i - 2\xi\lambda, \qquad (17)$$

$$\rho\frac{\delta\lambda}{\delta t} = G\rho\bar{\lambda}_i n_i + G\lambda\xi - \rho\lambda^2 - \frac{R\rho\bar{\mu}}{(1-b\rho)} - \frac{2R\mu\xi}{(1-b\rho)} - \frac{RT\bar{\xi}}{(1-b\rho)^2} - \frac{2bRT}{(1-b\rho)^3}\xi^2 \qquad (18)$$

,

$$R\rho\frac{\delta\mu}{\delta t} = GR\rho\bar{\mu} - \frac{RT\rho\bar{\lambda}_i n_i(\gamma-1)}{(1-b\rho)} - \frac{RT\lambda\xi(\gamma-2)}{(1-b\rho)} + \frac{2RT\rho\lambda\Omega(\gamma-1)}{(1-b\rho)} - \rho\lambda^2 G(1-b\rho) - \frac{\gamma R\rho\lambda\mu}{(1-b\rho)} + \frac{GR\xi\mu(2-b\rho)}{(1-b\rho)^2} - \frac{\gamma RTb\rho\xi}{(1-b\rho)^2} + \frac{RTGb\xi^2}{(1-b\rho)^2} \qquad (19)$$

where

$$\bar{\xi} = [\rho_{,ij}]n_i n_j, \qquad \bar{\mu} = [T_{,ij}]n_i n_j,$$
$$\bar{\lambda}_i = [u_{i,jk}]\, n_j n_k, \qquad \bar{\lambda}_i n_i - 2\lambda\Omega = [u_{i,ij}], \qquad (20)$$

Ω being mean curvature of $\sum(t)$.

δ time derivative of equation (13) will given by

$$\frac{\delta\xi}{\delta t} = \frac{\rho}{G}\frac{\delta\lambda}{\delta t} = \frac{R\rho\frac{\delta\mu}{\delta t}}{\{G^2(1-b\rho)^2-RT\}} \qquad (21)$$

Applying equation (21) in equation (17) to (19) and after certain manipulation and taking into consideration that square and higher power of $b\rho$ are negligible, we have

$$G\bar{\xi} - \rho\bar{\lambda}_i n_i = 2\lambda\xi + \frac{R\mu}{2G}\xi b\rho - \frac{\rho\lambda^2}{G} - \rho\lambda\Omega - \frac{\gamma R\rho\lambda\mu(1+2b\rho)}{2G^2} - \frac{RT\xi^2 b}{2G} - \frac{\gamma RT\xi b\rho}{2G^2} \qquad (22)$$

With help of equation (17) and (22), we have

$$\frac{\delta\xi}{\delta t} = \xi\left(G\Omega - \frac{b\rho}{2}\right) - \xi^2\left(\frac{\gamma+1}{2\rho}\right)G - \xi^2 b\left(\frac{RT}{G} - \frac{G}{2}\right) \qquad (23)$$

Equation (23) is the fundamental differential equation for the variation of ξ along the normal trajectories of family of sonic surface $\sum(t)$ and governs the growth and decay of sonic discontinuities in non-ideal gas. In similar way fundamental equation for variation of λ and μ along the normal trajectories of family of sonic surface $\sum(t)$ can be obtained.

Let $\xi(t_0)$ represent the sonic wave surface at time t_0 and let σ represents the distance measured from $\sum(t_0)$ along the normal trajectories to family of surface $\sum(t)$ in the direction of propagation then

$$\sigma = G(t - t_0)$$

and scalar function λ, ξ and μ can be regarded as function of σ hence, we have

$$\frac{\delta\lambda}{\delta t} = G\frac{d\lambda}{d\sigma}, \quad \frac{\delta\xi}{\delta t} = G\frac{d\xi}{d\sigma} \quad \text{and} \quad \frac{\delta\mu}{\delta t} = G\frac{d\mu}{d\sigma} \qquad (24)$$

With help of equation (24) equation (23) reduces to

$$\frac{d\xi}{d\sigma} = \xi\left(\Omega - \frac{b\rho}{2G}\right) - \xi^2 A \qquad (25)$$

Where $A = \frac{\gamma+1}{2\rho} + \frac{b\rho}{G}\left(\frac{RT}{G} - \frac{G}{2}\right)$ is quantity defined on the discontinuity surface $\sum(t)$.

Solution of equation (25) is given by

$$\xi = \frac{\xi_0}{e^{\frac{b\rho\sigma}{2G}} + \frac{2A}{b\rho}\left(e^{\frac{b\rho\sigma}{2G}} - 1\right)\xi_0} \qquad (26)$$

By Lane[2]

$$\Omega = \frac{\Omega_0 - \nu_0\sigma}{1-2\Omega_0\sigma+\nu_0\sigma^2}$$
$$= \frac{\Omega_0 - \nu_0 c_f t}{1-2\Omega_0 c_f t+\nu_0 c_f^2 t^2}$$

where $\Omega_0 = \frac{k_1+k_2}{2}$, the mean curvature and $\nu_0 = k_1 k_2$ is the Gaussian curvature of $\sum(t_0)$ with k_1 and k_2 being the principal curvatures. When k_1 and k_2 are both non positive, the wave is divergent. On the other hand if one or both the principal curvatures are positive, then it corresponds to the case of a convergent wave.

For the situation envisaged here, the quantities with subscript 0 appearing in (26) are constants and therefore, it can be integrated to yield

$$\xi = \frac{\xi_0 e^{-\frac{b\rho}{2G}\sigma}(1-2\Omega_0\sigma+\nu_0\sigma^2)^{-1/2}}{\left\{1+\xi_0\frac{Ac_f}{\rho}\int_0^\sigma \frac{e^{-\frac{b\rho}{2}\sigma}}{\sqrt{1-2\Omega_0\sigma+\nu_0\sigma^2}}d\sigma\right\}}, \qquad (27)$$

where ξ_0 is the value of ξ at the wave front at $t = 0$. If we put $\sigma = Gt = c_f t$ at $t = 0$, equation (27) reduce to

$$\xi = \frac{\xi_0 e^{-\frac{b\rho}{2}t} I_1}{\left\{1+\xi_0\frac{Ac_f}{\rho}I_2\right\}}, \qquad (28)$$

where $I_1(t) = \left(1-k_1 c_f t\right)^{-1/2}\left(1-k_2 c_f t\right)^{-1/2}$ (29)

and $I_2(t) = \int_0^t \frac{e^{-\frac{b\rho}{2G}\tau}}{\sqrt{(1-k_1 c_f\tau)(1-k_2 c_f\tau)}}d\tau$ (30)

Result and Discussion

Case 1: Diverging waves

Let $\frac{b\rho}{2} > 0$ from which I_2 converges as $t \to \infty$. If $\xi_0 > 0$ (expansion wave), then $\xi \to 0$ as $t \to \infty$ (i.e., the wave decays). If $\xi_0 < 0$ (compressive wave), there is a critical values ξ_c is given by

$$\xi_c = +\frac{1}{\frac{A}{c_f t}\int_0^\infty \frac{e^{-\frac{b\rho}{2G}\tau}}{\sqrt{(1-k_1 c_f\tau)(1-k_2 c_f\tau)}}d\tau}$$

such that if $|\xi_0| < \xi_c$ then $\xi \to 0$ as $t \to \infty$; if $|\xi_0| > \xi_c$ the wave grows into a shock in a finite time t_c given by

$$\int_0^{t_c} \frac{e^{\left|-\frac{b\rho}{2}\right|t}}{\sqrt{(1-k_1 c_f t)(1-k_2 c_f t)}} d\tau = +\frac{\rho}{A c_f \xi_0}$$

such that

$$I_2(t_c) = \frac{\rho}{\xi_0 A\, c_f}$$

If $|\xi_0| = \xi_c$ then $\xi \to \frac{c_f t}{A}$ as $t \to \infty$. i.e. the wave take a stable form. The derivatives of t_c with respect to $\frac{b\rho}{2}$, $|k_1|$ and $|k_2|$ are positive. Therefore an increase in $\frac{b\rho}{2}$, $|k_1|$ or $|k_2|$ delays the onset of the shock waves.

Let $\frac{b\rho}{2} < 0$, from which $\lim_{t\to\infty} I_2(t) = \infty$. If $\xi_0 > 0$, then $\xi \to \left(\frac{\left|\frac{b\rho}{2}\right|}{A\, c_f t}\right)$ as $t \to \infty$. If $\xi_0 < 0$, all compressive waves, no matter how small their initial values, always terminate into shock waves in a finite time t_c (i.e. $|\xi| \to \infty$ as $\to t_c$) given by

$$\int_0^{t_c} \frac{e^{\left|-\frac{b\rho}{2}\right|t}}{\sqrt{(1-k_1 c_f t)(1-k_2 c_f t)}} d\tau = +\frac{\rho}{A c_f \xi_0}$$

Case 2: Equation (26) can be written as

$$\delta = \frac{1}{e^{\eta} + A_0(e^{\eta}-1)} \qquad (31)$$

where $\delta = \frac{\xi}{\xi_0}$, $\eta = \frac{b\rho\sigma}{2G}$, $A_0 = \frac{2A}{b\rho}\xi_0$

The equation (31) suggested that if A_0 is negative the discontinuity ξ grows continuously till it tends to infinity as

$$\sigma \to \frac{2G}{b\rho}\log\left\{\frac{2A|\xi_0|}{2A|\xi_0| - b\rho}\right\}$$

In such a situation the continuity of density across $\sum(t)$ will break down and the consequently the sonic-waves will terminate into a shock wave after a critical time t_c given by

$$t_c = t_0 + \frac{2}{b\rho}\log\left\{\frac{2A|\xi_0|}{2A|\xi_0| - b\rho}\right\} \qquad (32)$$

For typical cases the growth of sonic discontinuity and its termination into shock wave have been shown in fig (2) and is concluded that for low density case critical time of shock formation increases.

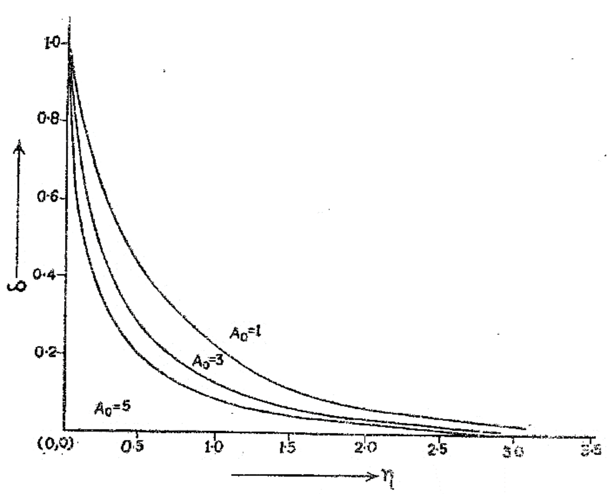

Figure 1: Variation of δ versus η for positive value of A_0

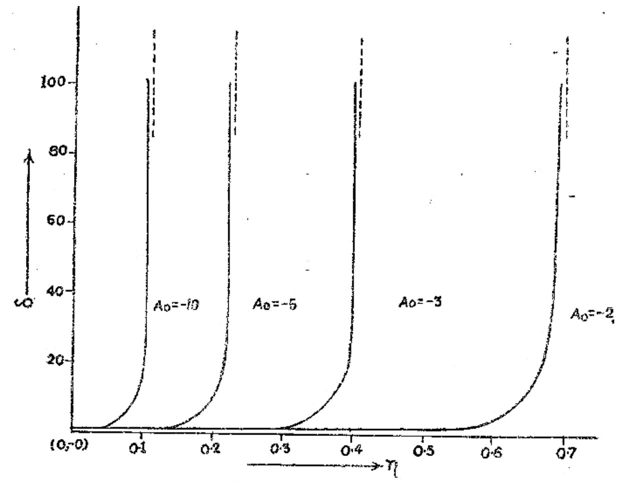

Figure 2: Variation of δ versus η for negative value of A_0

References

i. *Anisinov S.I. and Spiner O.M. (1972): Motion of an almost ideal gas in the presence of a strong point explosion, J. Applied Maths, Vol.36 (5), pp.883-887.*

ii. *Lane E.P. (1940): Metric Differential Geometry of curves and surfaces, University of Chicago, Press Chicago, pp.209.*

iii. *Madhumita G. and Sharma V.D. (2004): Imploding cylindrical and spherical shock waves in a non-ideal medium Jour. of Hyperbolic Diff. Eq. Vol. l (3), pp.521-530.*

iv. *Nariboli G.A. and Secrest, B.G. (1967): Weak discontinuities in Magnetogasdynamics in the presence of dissipative mechanism Tensor N.S. Vol. 18 (1), pp. 22-25.*

v. *Pandey K. and Saxena M. (1989): Grwoth and decay of sonic waves in two phase flows, Prg. of Math, Vol. 23, pp.125-135.*

vi. *Robert P.H. and Wu C.C. (1993): Shock wave propagation in a sonoluminescing gas bubble, Phy. Rev. lell, Vol. 70 (22), pp. 3424-3427.*

vii. *Robert P.H. and Wu C.C. (1996): Structure and stability of a spherical implosion, Phy. Lett. A. Vol., 213 (1-2), pp.59-64.*

viii. *Robert P.H. and Wu C.C. (2003): Shock focusing effect in medical science and sonoluminescence (eds, R.C. Srivastava et. al.) Berlin, Springer Verlog pp. 1-27.*

ix. *Steiner H. and Hirschler T. (2002): A self similar solution of a shock propagation in a dusty gas. Eur. J. Mech. B/Fluids Vol. 21 (3), pp.371-380.*

x. *Sharma V.D. and Shyam R. (1980): The onset of converging and diverging shock waves in non-equilibrium gas flows. Acta Astronautica. vol. 7, pp 269-275.*

xi. *Sharma V.D., Sharma J. and Shyam R. (1981): Growth and decay of weak waves in radiative magnetogasdynamics AIAA Vol. 19 (2), pp.1246-1248.*

xii. *Srinivasan S. and Ram Rishi, (1977): Propagation of Sonic waves in radiating gases, ZAMM, Vol. 57, pp. 191-193.*

xiii. *Thomas T.Y. (1957): The growth and decay of sonic discontinuities in ideal gases, J. Math. Mech. Vol. 6, pp.445-469.*

xiv. *Thomas T.Y. (1957): Extended compatibility conditions for the study of surface of discontinuity in continuum mechanics, J. Math. Mech. Vol. 6, pp.311-312.*

xv. *Vishwakarma J.P., Chaube V. and Patel, A. (2007): Self-similar solution of a shock propagation in a non-ideal gases, Int. J. of Applied mech. and Engineering, Vol. 12 (3), pp.813-829.*