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Exact Traveling Wave Solutions of the Nonlinear (2+1)-Dimensional Typical Breaking Soliton Equation via Exp$(-\phi(\xi))$-Expansion Method

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Abstract

In this paper, we present traveling wave solutions for the (2+1)-dimensional typical breaking soliton equation. The $\exp(-\phi(\xi))$-expansion method is applied to find exact solutions of that equation. The traveling wave solutions are expressed by the hyperbolic functions, the trigonometric functions solutions and the rational functions. It is shown that the method is really useful and can be used for many other nonlinear evolution equations (NLEEs) in mathematical science and engineering.

Keywords: ***The $\exp(-\phi(\xi))$-expansion method, the (2+1)-dimensional typical breaking soliton equation, traveling wave solutions, solitary wave solutions and graphical representation of the solutions.***

Mathematics Subject Classification: 35C07, 35C08, 35P99.

1. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. Most of the phenomena in real world can be described using non-linear equations. A nonlinear phenomenon plays a vital role in applied mathematics, physics and engineering branches. Many complex nonlinear phenomenons in plasma physics, fluid dynamics, chemistry, biology, mechanics, elastic media and optical fibers etc. can be explained by nonlinear evolution equations. There are a lot of NLEEs that are integrated using various mathematical techniques. In recent many powerful and effective methods have been presented such as the (G'/G)-expansion method [1-10], the homogeneous balance method [11-13], the complex hyperbolic function method [14,15],the ansatz method [16,17], the F-expansion method[18,19], the Backlund transformation method [20], the Darboux transformation method [21], the Adomian decomposition method [22, 23], the auxiliary equation method[24, 25], the $\exp(-\phi(\xi))$-expansion method [26], and so on.

In this paper, we apply $\exp(-\phi(\xi))$-expansion method to solve the (2+1)-dimensional typical breaking soliton equation . The paper is prepared as follows: In Section 2, the description of the $\exp(-\phi(\xi))$-expansion method. In Section 3, the application of this method to the(2+1)-dimensional typical breaking soliton equation, graphical representation of solutions. Conclusions are given in the end section.

2. Methodology

In this section, we describe $\exp(-\phi(\xi))$- expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in three independent variables x, y and t is given by

$$F(u,u_t,u_x,u_y,u_{tt},u_{xx},u_{xt},\ldots\ldots\ldots\ \ldots\ldots\ldots) = 0 \quad (1)$$

where $u(\xi)=u(x,y,t)$ is an unknown function, F is a polynomial of $u(x,y,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method:

Step 1: Combining the independent variables x, y and t into one variable $\xi = x+y\pm Vt$

we suppose that

$$u(x,t)=u(\xi),\ \xi = x+y\pm Vt \quad (2)$$

The travelling wave transformation Eq. (2) permits us to reduce Eq. (1) to the following ordinary differential equation (ODE):

$$\Re(u,u',u'',\ldots\ldots\ldots) = 0 \quad (3)$$

where $\Re$ is a polynomial in $u(\xi)$ and its derivatives, whereas $u'(\xi)=\frac{du}{d\xi}$, $u''(\xi)=\frac{du^2}{d\xi^2}$ and so on.

Step 2: We suppose that Eq.(3) has the formal solution

$$u(\xi)=\sum_{i=0}^{n} A_i(\exp(-\phi(\xi)))^i \quad (4)$$

Where $A_i\,(0 \le i \le n)$ are constants to be determined, such that $A_n \neq 0$ and $\phi = \phi(\xi)$ satisfies the following ODE:

$$\phi'(\xi) = \exp(-\phi(\xi)) + \mu \exp(\phi(\xi)) + \lambda \quad (5)$$

Eq. (5) gives the following solutions:

When
$\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\phi(\xi) = \ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right) \quad (6)$$

When $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$\phi(\xi) = \ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}{2\mu}\right) \quad (7)$$

When $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$,

$$\phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi+E))-1}\right) \quad (8)$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\phi(\xi) = \ln\left(-\frac{2(\lambda(\xi+E)+2)}{\lambda^2(\xi+E)-1}\right), \quad (9)$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$\phi(\xi) = \ln(\xi+E) \quad (10)$$

$A_n, \ldots\ldots V, \lambda, \mu$ are constants to be determined later, $A_n \neq 0$, the positive integer *n* can be

determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\phi(\xi))$. As a result of this substitution, we get a polynomial of $\exp(-\phi(\xi))$. We equate all the coefficients of same power of $\exp(-\phi(\xi))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_n, \cdots\cdots, V, \lambda, \mu$. Substituting the values of $A_n, \cdots\cdots, V, \lambda, \mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

3. Application of the method:

In this section, we will present the $\exp(-\phi(\xi))$ -expansion method to find the exact solutions of a nonlinear evolution equation.

First consider the (2+1)-dimensional typical breaking soliton equation [28] in the following form

$$u_{xt} - 4u_x u_{xy} - 2u_{xx}u_y + u_{xxxy} = 0 \quad (11)$$

which was first introduce by Calogero and Degasperis [27]. Tian et al. [28] have developed new families of solution via tanh method. Roshid et al [2] also obtained some new exact solutions using a new extended (G'/G) -expansion method which are important significance in explaining some physical phenomena.

Using the wave transformation
$u(x,t) = u(\xi), \xi = x + y - Vt$ Eq. (12) is carried to an ODE

$$-Vu'' - 6u'u'' + u'''' = 0 \quad (12)$$

Eq. (12) is integrable, therefore, integrating with respect to ξ once yields:

$$C - Vu' - 3(u')^2 + u''' = 0 \quad (13)$$

Now balancing the highest order derivative u''' and non-linear term $(u')^2$, we get $n = 1$ Therefore; the solution of Eq. (13) is of the form:

$$u(\xi) = A_0 + A_1(\exp(-\Phi(\xi))), \quad (14)$$

where A_0, A_1 are constants to be determined such that $A_n \neq 0$, while λ, μ are arbitrary constants.

Substituting Eq.(14) into Eq. (13) and then equating the coefficients of $\exp(-\phi(\xi))$ to zero, we get

$$-3A_1^2\mu^2 + VA_1\mu - 2A_1\mu^2 - A_1\lambda^2\mu + C = 0 \quad (15)$$

$$VA_1\lambda - 8A\mu\lambda_1 - 6A_1^2\mu\lambda - A_1\lambda^3 = 0 \quad (16)$$

$$VA_1 - 8A_1\mu - 6A_1^2\mu - 3A_1^2\lambda^2 - 7A_1\lambda^2 = 0 \quad (17)$$

$$-12A_1\lambda - 6A_1^2\lambda = 0 \quad (18)$$

$$-3A_1^2-6A_1=0 \qquad (19)$$

Solving the Eq. (15)-Eq. (19), yields

$$C=0,\ V=\lambda^2-4\mu,\ A_0=A_0,\ A_1=-2$$

Now substituting the values of V, A_0, A_1 into Eq. (14) yields

$$u(\xi)=A_0-2\exp(-\phi(\xi)), \qquad (20)$$

where $\xi=x+y-(\lambda^2-4\mu)t$

Now substituting Eq. (6)-Eq. (10) into Eq. (20) respectively, we get the following five traveling wave solutions of the (2+1)-dimensional typical breaking soliton equation

When $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$u_1(\xi)=A_0+\left(\frac{4\mu}{\sqrt{\lambda^2-4\mu}\tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}(\eta+E))+\lambda}\right)$$

where $\xi=x+y-(\lambda^2-4\mu)t$, E is an arbitrary constant

When $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$u_2(\xi)=A_0-\left(\frac{4\mu}{\sqrt{(4\mu-\lambda^2)}\tan\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\xi+E)\right)-\lambda}\right)$$

where $\xi=x+y-(\lambda^2-4\mu)t$, E is an arbitrary constant

When $\mu=0,\ \lambda\neq 0,$ and $\lambda^2-4\mu>0,$

$$u_3(\xi)=A_0-\frac{2\lambda}{\exp(\lambda(\xi+E))-1}$$

where $\xi=x+y-\lambda^2 t$, E is an arbitrary constant

When $\mu\neq 0,\ \lambda\neq 0,$ and $\lambda^2-4\mu=0,$

$$u_4(\xi)=A_0+\frac{\lambda^2(\xi+E)-1}{(\lambda(\xi+E)+2)}$$

where $\xi=x+y$, E is an arbitrary constant

When $\mu=0,\ \lambda=0,$ and $\lambda^2-4\mu=0,$

$$u_5(\xi)=A_0-\frac{2}{\xi+E}$$

where $\xi=x+y$, E is an arbitrary constant.

1 Graphical Representation: The graphical demonstrations of obtained solutions for particular values of the arbitrary constants are shown in Fig. 1 to Fig.5 in the following figures with the aid of commercial software Maple:

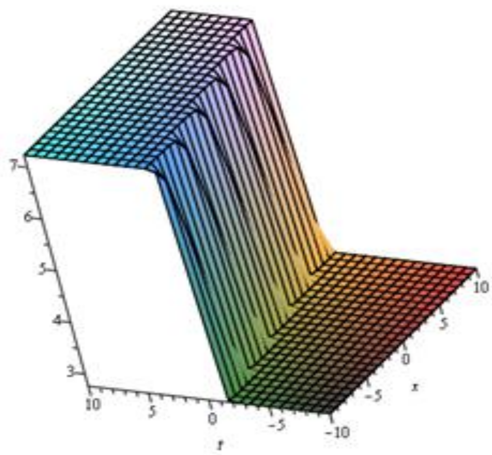

Fig. 1:The solution of u_1 with $A_0=2,\lambda=3,\mu=1,E=3,y=0$ and $-10\le x,t\le 10.$

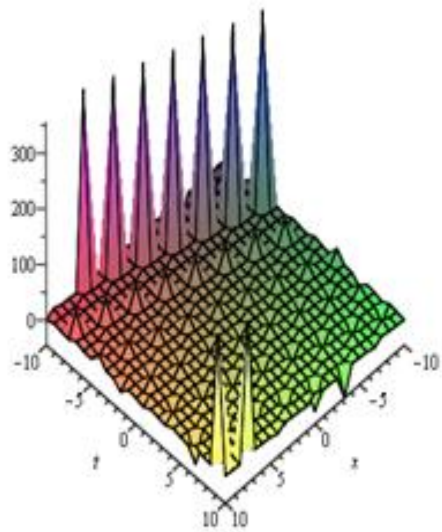

Fig. 2: The solution of u_2 with $A_0=2,\lambda=1,\mu=1,E=3,y=0$ and $-10\le x,t\le 10.$

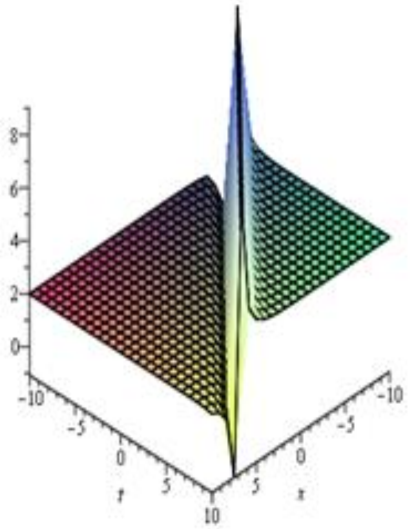

Fig. 3: The solution of u_3 with $A_0 = 2, \lambda = 1, \mu = 0, E = 3, y = 0$ and $-10 \le x,t \le 10.$

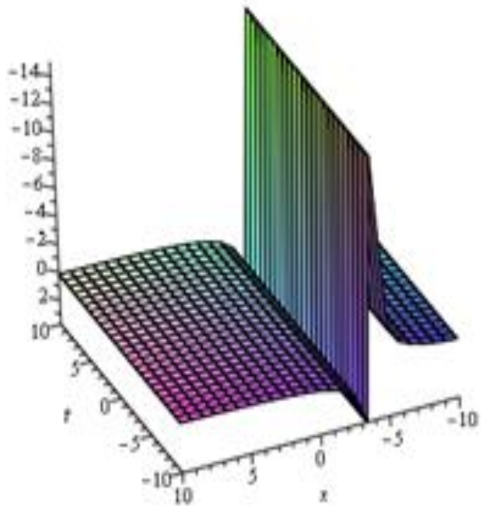

Fig: 4: The solution of u_4 with

$A_0 = 2, \lambda = 2, \mu = 1, E = 3, y = 0$ and

$-10 \le x,t \le 10.$

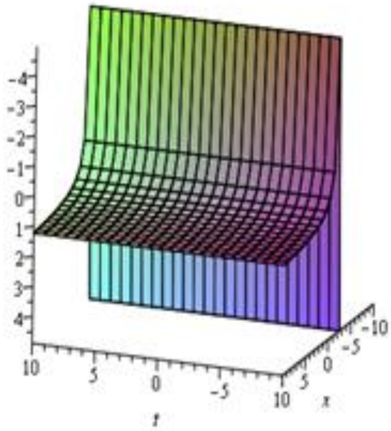

Fig: 5 The solution of u_5 when

$A_0 = 2, \lambda = 0, \mu = 0, E = 3, y = 0$ and

$-10 \le x,t \le 10.$

Conclusion

In this paper, we have applied the $\exp(-\phi(\xi))$-expansion method for the exact solution of the (2+1)-dimensional typical breaking soliton equation and observed some new exact traveling wave solutions. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions solutions and the rational functions. This paper shows that the $\exp(-\phi(\xi))$-expansion method is easy and helpful to find the exact solutions of NLEEs. Also this method is applicable to other NLEEs.

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