

# Transforming the Concept of Double Bridge Experiment

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## Abstract

**Double Bridge Experiment (DBE), which is based on an analyzing behavior of ants, was introduced by Deneubourg and colleagues. In this paper I am going to merge this very concept with the concept of Analytical Hierarchy Process (AHP), which is a modern day decision making technique. Till now, DBE is performed as a single criterion decision making problem, but introduction of AHP enhances this concept to multiple criteria decision making problem (MCDM). This is going to be a step ahead work in the field of Ant Colony Optimization.**

**Keywords: DBE, AHP, Ant Colony Optimization, MCDM.**

## 1. Introduction:

Deneubourg, a great contributor in *Ant Colony Optimization* algorithms, introduced a concept of Double Bridge Experiment in 1990, according to which whenever ants are travelling from their nest to food and vice-versa, their tendency is to gradually shift towards the shortest path between the source and destination. This happens because of pheromone laying and following skills. Traditionally while analyzing the behavior of ants, he has practiced this experiment as a single criterion based technique. In this paper I am going to enhance the concept and re-perform the experiment using Analytical Hierarchy Process.

## 2. Double Bridge Experiment:

This experiment was performed by varying the ratio  $r = l_1/l_2$  between the length of the two branches of the double bridge, where  $l_1$  was the length of the longer branch and  $l_2$  the length of the shorter one. I am specifically picking up a case, where the length ratio between the two branches was set to  $r = 2$ , so that the longer branch was twice as long as the shorter one. In this case, in most of the trials, after some time all the ants chose to use only the shorter branch. As in the first experiment, ants

leave the nest to explore the environment and arrive at a

decision point where they have to choose one of the two branches. Because the two branches initially appear identical to the ants, they choose randomly. Therefore, it can be expected that, on average, half of the ants choose the shorter branch and the other half chooses longer one, although stochastic oscillations may occasionally favor one branch over the other. However, this experimental setup presents a remarkable difference with respect to the previous one: because one branch is shorter than the other, the ants choosing the short branch are the first to reach the food and to start their return to the nest. But then, when they must make a decision between the short and the longer branch, the higher level of pheromone on the short branch will bias their decision in its favor. Therefore, pheromone starts to accumulate faster on the short branch. Finally, it results in convergence of most of the ants towards the short branch.

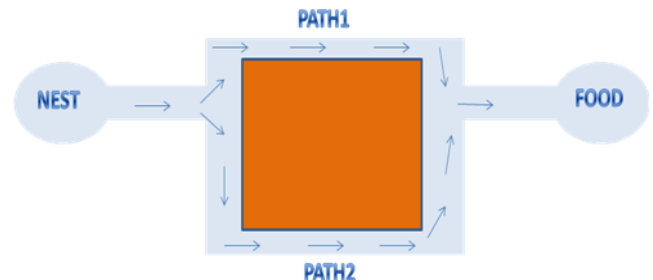


Figure 1(a) Double Bridge Experiment in which both the branches have different length( $r=2$ ).

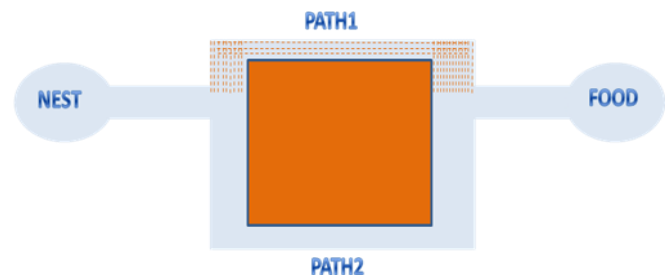


Figure 1(b) Double Bridge Experiment Result, Ants converging towards one path( $r=2$ ) and that is the shorter one. Dotted path is the one, where ants finally converged.

### 3. Analytical Hierarchy Process:

Suppose we have m alternatives and n criteria corresponding to the problem, then in the first step we have to construct a pair wise comparison matrix for the criteria.

	Criteria 1	Criteria 2	-----	Criteria n
Criteria 1	$a_{11}$	$a_{12}$	-----	$a_{1n}$
Criteria 2	$a_{21}$	$a_{22}$	-----	$a_{2n}$
.	.	.		.
.	.	.		.
.	.	.		.
Criteria n	$a_{n1}$	$a_{n2}$	-----	$a_{nn}$

Figure 2 Comparison Matrix

This matrix indicates the priorities of the user as far as criteria have been concerned. For example  $a_{12} = 2$  indicates that 1 is twice prior for the user than criteria 2. In comparison matrix  $a_{ij} = 1/a_{ji}$ , for every  $i = j$  and  $a_{ij} = 1/a_{ji}$ . After pair wise comparison, we need to determine weights of all the criteria. For determining weights, first normalize the pair wise matrix by dividing entries in the column by the sum of the elements in the corresponding column.

	Criteria 1	Criteria 2	-----	Criteria n
Criteria 1	$b_{11}$	$b_{12}$	-----	$b_{1n}$
Criteria 2	$b_{21}$	$b_{22}$	-----	$b_{2n}$
.	.	.		.
.	.	.		.
.	.	.		.
Criteria n	$b_{n1}$	$b_{n2}$	-----	$b_{nn}$

Figure 3 Normalized Matrix

Where

$$b_{ij} = a_{ij} / \sum_{i=1}^n (a_{ij}) \quad , \quad \text{for } j=1,2,\dots,n$$

Using this normalized matrix calculate the weights of all the criteria using the relation

$$w_i = \frac{\sum_{j=1}^n (b_{ij})}{n} \quad , \quad \text{for } i=1,2,\dots,n$$

Then we find n different comparison matrices for alternatives (on the basis of each criterion), which gives information about the alternatives.

Criteria i	Alternative 1	Alternative 2	-----	Alternative m
Alternative 1	$c_{11}$	$c_{12}$	-----	$c_{1n}$
Alternative 2	$c_{21}$	$c_{22}$	-----	$c_{2n}$
.	.	.		.
.	.	.		.
.	.	.		.
Alternative m	$c_{m1}$	$c_{m2}$	-----	$c_{mn}$

Figure 4 n different Comparison Matrix

Here,  $c_{ij} = 1$  when  $i = j$ , and  $c_{ij} = 1/c_{ji}$ . For example:  $c_{12} = 2$  and  $i=1$  indicates that alternative 1 is twice beneficial than alternative 2, as far as criteria 1 is concerned. Next we normalize all these n matrices and calculate the weight corresponding to each matrix. Let  $S_1, S_2, \dots, S_n$  are the vectors (column vectors) carrying weights corresponding to criteria 1, 2, 3, ..., n respectively.

$$S_j = \begin{bmatrix} V_{1j} \\ V_{2j} \\ V_{3j} \\ \vdots \\ V_{mj} \end{bmatrix}$$

Figure 5 Vector Matrix

Construct a matrix using these vectors  $S_1, S_2, \dots, S_n$

	Criteria 1	Criteria 2	.....	Criteria n
Alternative 1				
Alternative 2	S <sub>1</sub>	S <sub>2</sub>	.....	S <sub>n</sub>
⋮				
Alternative m				

Figure 6 Final Matrix for Weight Calculation

Now obtain the final results using this matrix and the weights  $w_i$  ( $i = 1, 2, 3, \dots, n$ )  
Score of alternative i:

$$A_i = \sum_{j=1}^n V_{ij} \times w_j$$

Whichever alternative has highest score, will be the most suitable choice to the user (according to his requirement).

#### 4. Double Bridge Experiment re-performed using Analytical Hierarchy process:

Now, we are going to re perform Double Bridge Experiment using AHP, specifically second step. Consider a setup in which again there are two paths, with unequal length ( $r = 2$ ). But this time an enhancement in consideration has been made. Refer Figure 2 for understanding the changes we made in the traditional approach:

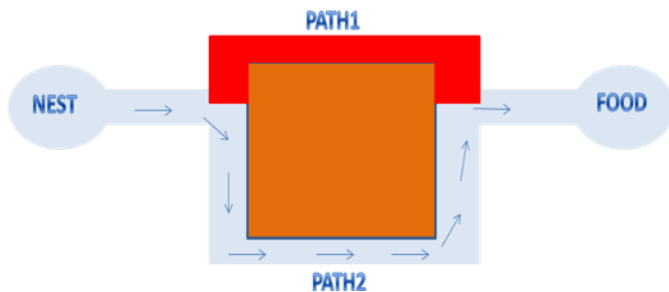


Figure 7 Double Bridge Experiment in which Path 1 is metallic, while Path 2 is wooden

In this experiment, we are assuming that Path 1, which is the shorter path, is actually made up of an insulated metallic material. On the other hand, Path 2, the longer one, is built up of non insulated wooden material. Along with this, before the start of experiment the lab temperature is set to a

high point, so that Path 1 is carrying respectively more temperature than Path 2.

When ants coming from nest are at the decision-making point to choose a path between the available two, then this time problem would be a multi-criterion decision making problem, and it can be solved using AHP method. Two criteria involved in the problem are now length and temperature, unlike the traditional approach of DBE, in which length was the only criterion. Let comparison matrix between the criteria is:

	Length	Temperature
Length	$a_{11} = 1$	$a_{12} = 0.5$
Temperature	$a_{21} = 2$	$a_{22} = 1$

Figure 8 Comparison Matrix

Value  $a_{21}=2$  indicates that while choosing a path, decision making would be more dependent on temperature rather than length. This is because of the natural compatibility of the ants with the environment.

Length	Path1	Path2
Path1	1	2
Path2	0.5	1

Figure 9 Comparison Matrix on the basis of length

Temperature	Path1	Path2
Path1	1	0.25
Path2	4	1

### Figure 10 Comparison Matrix on the basis of Temperature

In the matrix for *Length*, value corresponding to row1 and column2 indicates that, path 1 is more optimum on the basis of length. Similarly in the matrix for *Temperature*, value corresponding to row2 and column1 indicates that, path 2 is more optimum on the basis of temperature.

Finally after using all these matrices and applying concept of AHP which we've discussed earlier, following results comes out,

**Score for Path 1 = 0.354, and Score for Path 2 = 0.646**

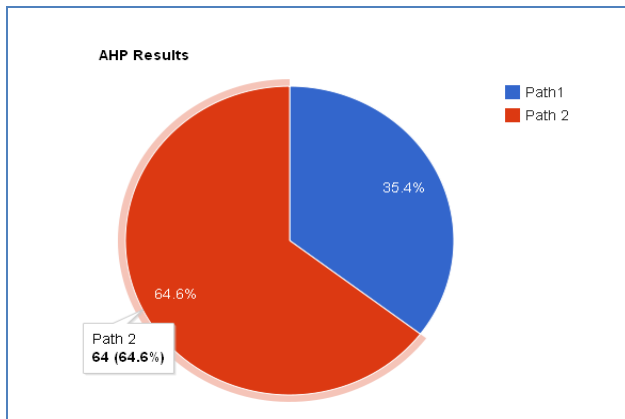


Figure 11 Result for Double Bridge Experiment

### 5. Conclusion:

Scores for both the paths using AHP indicates that Path 2 comparatively is more optimum than Path 1, as its score is on higher side. So, in this paper we have considered DBE as a Multi-Criterion Decision Making problem, and got those results that are more practically competent. In spite of the fact that Path1 was the shorter one, ants have chosen the longer path for travelling.

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