

**APPLICATION OF LINEAR PROGRAMMING FOR
DETERMINING THE OPTIMUM REPRODUCTIVE HEALTH
COMMODITY DISTRIBUTION IN GEZIRA STATE, SUDAN
(2013).**

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Abstract

Optimization is the art and science of allocating scarce resources in an optimal way. Linear programming (LP) is the one of the statistical optimization models used in allocating scarce resources in an optimal way to maximize the profit or to minimize the cost. This study aimed to construct an LP model for distributing sources of Sudan Family Planning Association (SFPA) to estimate the optimum commodities of Reproductive Health (RH). It also aimed to investigate which distributing source of SFPA was more relatively efficient in distribution of RH commodities and to measure the level attitude of women aged 15-49 years towards use of RH commodities in Gezira state. The secondary data were obtained from SFPA 2013 Annual Report which provided data on RH commodities distributed by quantities, women who used RH commodities for the first time and women who ever used RH commodities. The data were analyzed by Statistical Package for Social Science (SPSS) and Quantitative System for Business (QSB) computer software's through the use of Linear Programming models. The findings of the study showed that integrated health center achieved the greatest return of SDG 9350 by distributed 40 implants and 132 Intra Uterine Devices (IUDs) compared with other sources, also, the study presented that midwives were more relatively efficient in distributing RH commodities compared with other sources. The study recommended the need to encourage institutions to use LP to allocate scarce resources in an optimal way. Governmental and private health institutions should provide a variety of RH commodities so that women users can chose a commodity that is suitable to their body.

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1. Introduction

Operational Research is a science designed to provide quantitative tools to decision-making processes. It comprises a set of mathematical optimization and simulation methods and models, such as Linear Programming, Non-linear Programming, Combinatory Optimization, Theory of Queues, Dynamic Programming, Theory of Decisions, etc. Today, implementing optimized solutions by linear programming has reduced costs by hundreds or even thousands dollars in many middle to large-sized companies in several industrialized countries (Hillier and Lieberman 1995).

Nearing and Tucker, (1993) noted operational research is a mathematical method developed to solve problems related to tactical and strategic operations. Its origins show its application in the decision process of business analysis funds. This shortage of funds is a characteristic of hyper environments. Although the practical application of a mathematical model is wide and complex, it will provide a set of results that enable the elimination of a part of the subjectivism that exists in the decision-making process as to the choice of action alternatives.

Linear Programming (LP) is a subset of Mathematical Programming that is concerned with efficient allocation of limited resources to known activities with the objective of meeting a desired goal of maximization of profit or minimization of cost. In Statistics and Mathematics, LP is a technique for optimization of linear objective function, subject to linear equality and linear inequality constraint. Informally, LP determines the way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model and given some list of requirement as linear equation. In LP, all of mathematical expressions for the objective function and the constraints are linear. The programming in LP is an archaic use of the word programming to mean planning. So you might think of LP as planning with linear models (Bierman et al, 1973).

LP can be applied to various fields of study. Most extensively, it is used in business and economic situation, but can also be utilized in some engineering problems. Some industries that use LP model include transportation, energy, telecommunications and production or manufacturing companies. To this extent, LP has proved useful in modeling diverse types of problems in planning, routing, scheduling assignment and design (Bierman et al, 1973).

The elements of LP model are: decision variables, objective function, and model constraints. The objective function and constraints of LP model are the decision

variables and parameters respectively. The decision variables include mathematical symbols which represent the level of activity of any organization. The objective function is a mathematical linear relationship which expresses the objective of the organization in terms of decision variables. The objective function is always set as - maximizing or - minimizing. The model constraints also express the linear relation among the decision variables. The constraints are imposed on the organization by the operational environment, and are often due to limited resources or the organization's internal policies (Azar, 1999).

2. Study Questions:

1. What are the optimum quantities of RH commodities that will maximize the return of the distribution sources of RH commodities for SFPA?
2. How SFPA use the limited resources in an optimal way among the distribution activities of the organization?
3. What are the procedures that will be used to maximize the return of the distribution of RH commodities?

3. Study Objectives:

1. To investigate the optimum quantities of RH commodities that will maximize the return of the distribution sources of RH commodities for SFPA.
2. To attempt the use of SFPA limited resources in an optimal way among the distribution activities.
3. To determine the procedure that will be used to maximize the return of the distribution of RH commodities.

4. Importance of the Study:

RH commodities refer to those medical supplies and equipment that are used in the promotion of RH, prevention, diagnosis and management of RH conditions. The importance of this study stems from the importance of the distribution of RH commodities in an optimal way and use certain criteria so as to increase the profit. The application of the simplex method to obtain optimum distribution of RH commodities is used for the first time in Sudan. This will help health visitor for proper counseling and better distribution.

5. Data Sources and Data Analysis:

The study depends mainly on secondary data which were collected from SFPA in Gezira state was utilized as need arises. The data were collected from five sources of RH commodity distribution points which are:

(Integrated health center , Midwives, Clinics of Ministry of Health (CMoH) (Central points), Mobile Clinics, and SFPA 2013).

The data were analyzed using computer software's Quantitative System for Business (QSB) in order to obtain an optimum quantity of RH commodities distribution to maximize the profit. Also the data is manipulated by Statistical Packages for Social Science (SPSS).

6. The Models

6.1 Linear Programming (LP):

LP is the name of a branch of applied mathematics that deals with solving optimization problems of a particular form. Linear programming problems consist of a linear cost function (consisting of a certain number of variables) which is to be minimized or maximized subject to a certain number of constraints. The constraints are linear inequalities of the variables used in the cost function. The cost function is also sometimes called the objective function. LP is closely related to linear algebra; the most noticeable difference is that LP often uses inequalities in the problem statement rather than equalities (Mark A, 2010).

Characteristics of LP:

1. A problem for solution by LP methods must have a definite, identified, numerical goal. The goal (objective function) has to be stated as the sum of a series of terms, each consisting of the product of the activity level and a constant multiplier which is the cost or profit per unit of that activity. The goal can only maximize the total profit or minimize the total cost, whichever is appropriate. ("Profit" and "cost" are generic terms, however, "Profit" includes total yield from a process, the total income including expenses, and so on; "cost" includes waste, amount of material used, total inventory, and so on.
2. There must be separate and identifiable activities, and the level of each activity must be measurable in numerical terms.
3. The activities must be interrelated.
4. The restrictions must be identified and stated in numerical terms.

5. All activities must appear only in linear combinations, both in the constraints and in the objective function (Garvin, 2011).

Assumptions of LP:

1. Certainty: does not change during the period being studied.
2. Proportionality: exists in the objectives and the constraints, this means that, if production of one unit of a product uses two of a particular scarce resource, then making five units of that product uses ten resources.
3. Additivity: which means that the total of all activities equals the sum of each individual activity.
4. Divisibility: which means that solution may take fractional values and needs not be in whole numbers (integers). If a fraction of a product cannot be produced, then an integer programming problem exists.
5. Non-negativity which assumes that all answers or variables are non-negative. Negative values of physical quantities are an impossible solution.

6.2 LP Models:

Model Components: A model consists of linear relationships representing a firm's objectives and resource constraints. Model components are:

1. Decision variables: mathematical symbols representing levels of activity of an operation.
2. Objective function: a linear mathematical relationship describing the objective of the firm, in terms of decision variables, that are to be maximized or minimized.
3. Constraints: restrictions placed on the firm by the operating environment stated in linear relationships of the decision variables.
4. Parameters / cost, coefficients: numerical coefficients and constants used in the objective function and constraint equations (Mahendra, 2009).

Standard Form of LP Model: Data for standard LP in Table (1)

Table (1) Standard Form of LP

Resources	Activity			Amount of Resource Available
	1	2	3	
1	a ₁₁	a ₁₂ a ₁₃	b ₁	
.	.	a ₂₂ a ₂₃	b ₂	
..	.	.	.	
.	.	.	.	
m	a _{m1}	a _{m2} a _{mn}	b _n	
Z / Unit of Activity	c ₁	c ₂ ...c _n		
Level of Activity	x ₁	x ₂ ...x _n		

Source :Hiller and Liberman (2001)

6.3 Linear Programming Problems (LPPs):

LPP is a special case of a mathematical programming problem. From an analytical perspective, a mathematical program tries to identify an extreme (i.e., minimum or maximum) point of a function $f(x_1, x_2, \dots, x_n)$, which furthermore satisfies a set of Version of a mathematical programming to the case where both, function f - to be called the objective function and the problem constraints are linear. (Mahendra, 2009). Every LPP falls into one of three categories:

1. Infeasible: LPP is infeasible if a feasible solution to the problem does not exist; that is, there is no vector x for which all the constraints of the problem are satisfied.
2. Unbounded: LPP is unbounded if the constraints do not sufficiently restrain the cost function so that for any given feasible solution, another feasible solution can be found that makes a further improvement to the cost function.
3. Has an optimal solution: LPPs that are not infeasible or unbounded have an optimal solution; that is, the cost function has a unique minimum (or maximum) cost function value. (San Diego 2013).

6.4 Methods for Solving the LPPs:

Graphical Method: To solve a LPP for each product (or variable), two axes (dimensions) are required. Therefore, a graph for a 2-dimensional model is easy. But for solving a LPP with more dimensions (variables), we have to use the simplex method and often a computer software. Then, it would be better to solve such problems by graphical method. Although drawing lines for model constraints limits the region of possible solutions, but still there are many more solutions left. The final optimal value

(solution) for the LP model occurs at one of the vertices of the region determined by the constraints. Combinations of x_1 and x_2 which lie in one of these vertices are called the basic solution. There are only two more steps left for a graphical method as follows:

1. Find the vertices of the region.
2. Test the objective function at each of the vertices (profit obtained from selling a product). The vertex that has the maximum value is the optimal solution.

LPP with two variables (dimensions) can be easily solved graphically, but solving more complicated models (with three or more variables) could be done by the Simplex method (manually or using a computer). (Hadawy, 2006).

Simplex Method: The simplex method is an iterative method that traverses the faces of the feasible region and proceeds stepwise towards the optimal solution increasing the value of the objective function at each step. If LP solutions exist, they lie on vertices of the feasible region. (Bergstorm, 2006).

Simplex Algorithm : (Maximization Case): Simplex Steps

Step 1: Transform the LP into canonical slack form. to construct the first simplex tableau. The first simplex tableau is the canonical slack form where the 0th row represents the objective function as a 0th constraint as:

$$z - c_x = 0 \quad (1)$$

The variables that have unique row with 1 as coefficient, and 0 as coefficient in all other rows, will be chosen to be the Basic Variable (BV). Typically, the slacks are chosen to be the Basic Variables (BVs). In that case the decisions are set to be zero, and thus the first Simplex tableau will be solved for the slacks. So, most typically, the slack form LP is:

$$\text{maximize } Z = c_1 x_1 + \dots + c_n x_n \quad (2)$$

Subject: to

$$a_{11}x_1 + \dots + a_{1n}x_n + s_1 < b_1 \quad (3)$$

$$a_{21}x_1 + \dots + a_{2n}x_n + s_2 < b_2 \quad (4)$$

$$a_{m1}x_1 + \dots + a_{mn}x_n + s_m < b_m \quad (5)$$

$$x_1, \dots, x_n, s_1, \dots, s_m > 0$$

becomes:-

$$\text{maximize } z$$

Subject :to

$$z - c_1x_1 - \dots - c_nx_n = 0 \quad (6)$$

$$a_{11}x_1 + \dots + a_{1n}x_n + s_1 = b_1 \quad (7)$$

$$a_{21}x_1 + \dots + a_{2n}x_n + s_2 = b_2 \quad (8)$$

$$a_{m1}x_1 + \dots + a_{mn}x_n + s_m = b_m \quad (9)$$

$$x_1, \dots, x_n, \dots, s_1, \dots, s_m > 0$$

Since one has to keep track of the Basic Variables (BVs), this form is then represented as the Simplex tableau.

Table (2) the Simplex Tableau

Row	Z	x_1	x_n	s_1	s_2	s_m	BV	RHS
0	1	$-c_1$	$-c_n$	0	0	0	$z =$	0
1	0	a_{11}	a_{1n}	1	0	0	$s_1 =$	b_1
2	0	a_{21}	a_{2n}	0	1	0	$s_2 =$	b_2
.
.
.
M	0	a_{m1}	a_{mn}	0	0	1	$s_m =$	b_m

Source :Hiller and Liberman (2001)

Step 2: Check if the current Basic Feasible Solutions, (BFS) is optimal In the first Simplex tableau where the BVs are s_1, \dots, s_m , and a (BFS) related to this solution is:- $x_1 = 0, \dots, x_n = 0, s_1 = b_1, \dots, s_m = b_m$. The value of the objective function can be read from the 0th row:

Row	Z	x_1	x_n	s_1	s_2	s_m	BV	RHS
0	1	$-c_1$	$-c_n$	0	0	0	$Z =$	0

This solution is hardly optimal. Indeed, suppose that the coefficients c_i are non-negative (as is usually the case). But now all the decisions x_i related to the coefficients c_i are zero, as they are Non Basic Variables (NBVs). But then, obviously increasing the value of any x_i will increase the value of the objective z .

One then considers the general case. Suppose that, after some steps, one has come up with a simplex tableau with the 0th row.

Row	Z	x_1	x_n	s_1	s_2	s_m	BV	RHS
-----	---	-------	------	-------	-------	-------	------	-------	----	-----

0	1	d_1	d_n	d_{n+1}	d_{n+2}	d_{n+m}	Z=	Z^*
---	---	-------	------	-------	-----------	-----------	------	-----------	----	-------

Where all the coefficients d_i are non-negative for all the (NBVs). Then the simplex tableau is optimal, if in the 0th row there are non-negative coefficients in any (NBV). If the tableau is optimal, the algorithm terminates and the optimal value and decision can be read from the (BVs) and Right Hand Side (RHS) columns.

Step 3: Determine the entering variable if the BFS is not optimal, then one has to change the BVs. One of the NBVs will become a BV (entering), and one of the old (BVs) will become (NBV) leaving. The entering variable will be the one with the smallest coefficient in the 0th row. Indeed, in this way we increase the value of the objective z the most.

Step 4: By determining the leaving variable in step 3, we chose some variable to be a (NBV). Now each BV in a Simplex tableau is associated to some row. The leaving (BV) will be the one associated to the row that wins the ratio test (the smallest value is the winner).

(RHS of the row \ Coefficient of entering variable in the row). The idea of the ratio test is that it shall increase the entering variable as much as possible. At some point the increasing of the entering variable will force one of the BVs to become zero. This BV will then leave. The ratio test picks up the row associated with the leaving variable.

Step 5: To find a new BFS, now emerges a new system of BVs. Next solve the Simplex tableau in terms of the new BVs. Then arrive to a new simplex tableau, and go back to Step 2 (Sottinen, 2009).

7. Analysis of Data:

7.1 Maximization of the Return of RH Commodities Distributed by

Midwives: Midwives distribute two kinds of RH commodities: Mono hormonal pills and Bi hormonal pills. Table (3) below shows the RH commodities distributed by midwives and the sale prices of RH commodities sold by SFPA and midwives.

Table (3): RH Commodities and Sale Prices by SFPA and Midwives

	RH Commodities	
	Mono Hormonal Pills	Bi Hormonal Pills

Denote	x_1	x_2
The Sale Prices by SPFA	2	2
The Sale Prices by Midwives	10	10
The Profit	8	8

Source: SFPA 2013 annual report

Table (4): Constraints, RH Commodities Distributed by Midwives and Commodities Targeted Annually

Constraints	RH Commodities		Target
	Mono Hormonal Pills (x_1)	Bi Hormonal Pills (x_2)	
Quantities of RH Commodities.	1619	1673	4000
Number of Women Who Use RH Commodities for the First Time.	307	450	920
Number of Women Who Ever Use RH Commodities.	1082	1135	3080

Source: SFPA 2013 annual report.

Table (4) shows the activities (constraints), quantities of RH commodities, number of women who use RH commodities for the first time and number of women who ever use RH commodities. Also the table shows the number of RH commodities distributed by means of constraints and the RH commodities targeted annually.

For presenting the problem Tables (3) and (4) can be used.

$$\text{Maximize } Z = 8x_1 + 8x_2$$

Subject: to

$$1619x_1 + 1673x_2 \leq 4000$$

$$307x_1 + 450x_2 \leq 920$$

$$1082x_1 + 1135x_2 \leq 3080$$

$$\text{Further; } x_1 \text{ and } x_2 \geq 0$$

(Since if $x_1, x_2 < 0$ it means that negative quantities)

7.2 Solving the Problem:

1. Graphical solution:

The decision variable values (x_1 & x_2) are placed on x and y axes respectively of a graph; the non-negativity constraints together require that x_1 and x_2 be positive. Consider now the first major constraint (quantity availability). If x_1 and x_2 are such that all available quantity is used up then mathematically:

$$1619x_1 + 1673x_2 = 4000$$

This is the equation for line C_1 in Figure (1). Geometrically, any combination of x_1 and x_2 which requires exactly 4000 (quantity of pills) will be plotted on this line. A combination requiring less than 4000 quantity of pills will be plotted below it and one requiring more than 4000 quantity of pill will be plotted above. Similar treatment of the other two constraints (No. of women who use RH commodities for the first time and No of women who ever use RH commodities) yields the lines.

$$307x_1 + 450x_2 = 9204$$

$$1082x_1 + 1135x_2 = 30804$$

C_2 and C_3 in Figure (1) on or below which the optimal solution must also lie. To require that all the constraints (including $x_1, x_2 \geq 0$) hold simultaneously is equivalent to require that the optimal solution lie below c_1, c_2 and c_3 with x_1 and x_2 non-negative. The area, which comprises the set of all possible solutions is called the solution space. We can substitute all possible solutions into the objective function to find the optimal profit.

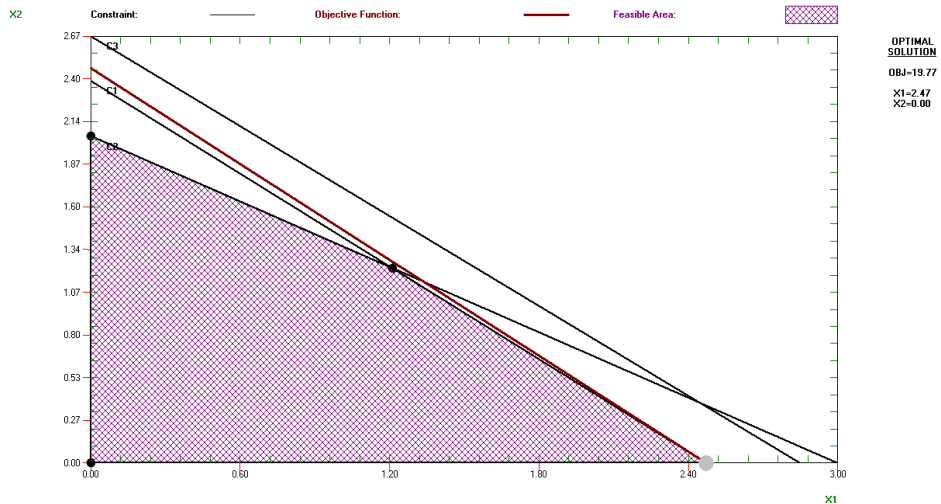
From the Figure (1) points of solution space are A (0,0), B ((2.47), (0)), C ((0), (2.1)) D ((1.20), (1.25)). To substitute these points in the objective function equation (4-1) to find the optimal profit.

Substitute point B (2.47,0) in objective function: $8*(2.47) + 8*(0) = 19.77$

Substitute point C (0,2.1) in objective function: $8*(0) + 8*(2.1) = 16.8$

Substitute point D (1.20,1.25) in objective function: $8*(1.20) + 8*(1.22) = 19.38$

Point B yield the greatest profit (19.77). Then the maximum return achieved by midwives was SDG (19.77), when they distributed (2.47) tape of Mono hormonal pills.



Source: QSB Graphical Solution Based on Tables (1) and (2)

Figures (1): Graphical Solution of LPP

7.3 Maximization of the Return of RH Commodities Distributed by Mobile Clinics:

Mobile clinics distribute three methods of RH commodities: Mono hormonal pills, Bi hormonal pills and Injection.

Table (5) RH Commodities and Sale Prices by SFPA and Mobile Clinics

	RH Commodities		
	Mono Hormonal Pills	Bi Hormonal Pills	Injection
Denote	X_1	X_2	X_3
The Sale Prices by SFPA	2	2	10
The Sale Prices by Mobile Clinics	10	10	25
The Profit	8	8	15

Source: SFPA 2013 annual report

Table (5) shows the RH commodities distributed by Mobile clinics and the sale prices of the RH commodities sold by SFPA and moving clinics.

Table (6) Constraints, RH Commodities Distributed by Mobile Clinics and RH Commodities Targeted Annually

Constraints	RH Commodities			Target
	Mono Hormonal Pills	Bi Hormonal Pills	Injection	
Quantities of RH Commodities.	617	840	82	1800
Number of Women Who Use RH Commodities for the First Time.	171	229	18	489
Number of Women Who ever Use RH Commodities.	350	519	64	1311

Source: SFPA 2013 annual report

Table (6) shows the activities (constraints), quantities of RH commodities, number of women who use RH commodities for the first time and number of women who ever use RH commodities. Also the table shows the number of the RH commodities distributed by means of constraints and the RH commodities targeted annually.

For presenting the problem Tables (5) and (6) can be used.

$$\text{Maximize } Z = 8x_1 + 8x_2 + 15x_3$$

Subject: to

$$617x_1 + 840x_2 + 82x_3 \leq 1800$$

$$171x_1 + 229x_2 + 18x_3 \leq 489$$

$$350x_1 + 519x_2 + 64x_3 \leq 1311$$

$$\text{Further; } x_1, x_2 \geq 0$$

(Since if x_1, x_2 and $x_3 < 0$ it means that negative quantities).

2. Solving the problem by simplex method manually:

Steps of simplex methods:

Step (1): put the LPP in standard form and adding slack variables

$$Z = 8x_1 + 8x_2 + 15x_3 + (0)s_1 + (0)s_2 + (0)s_3$$

$$617x_1 + 840x_2 + 82x_3 + s_1 = 1800$$

$$171x_1 + 229x_2 + 18x_3 + s_2 = 489$$

$$350x_1 + 519x_2 + 64x_3 + s_3 = 1311$$

Step (2) : translate objective function to zero equation

$$Z - 8x_1 - 8x_2 - 15x_3 - (0)s_1 - (0)s_2 - (0)s_3$$

Step (3) : construct simplex tableau

Table (5) shows the first simplex tableau include the non basic variables and basic variables.

Table (7) First Simplex Tableau

Non. Basic variable								
Basic variables	Z	x ₁	x ₂	x ₃	s ₁	s ₂	s ₃	RHS
Z	1	-8	-8	-15	0	0	0	
S ₁	0	617	840	82	1	0	0	1800
S ₂	0	171	229	18	0	1	0	489
S ₃	0	350	519	64	0	0	1	1311

Source: SFPA 2013 annual report

Step (4) : select the entering basic variables

The entering basic variable is non-basic variable in the objective function row that has most negative coefficient in the simplex tableau Table (8) this is x₃ whose objective coefficient of -15 is more negative than the objective function coefficient of -8 for x₁ and x₂. The column for the entering basic variable is called the pivot column.

Table (8) Second Simplex Tableau:-

Non. Basic Variables									
Basic Variable	Z	x ₁	x ₂	x ₃ Entering Variable	s ₁	s ₂	s ₃	RHS	Ratio
Z	1	-8	-8	-15	0	0	0	0	0
s ₁	0	617	840	82	1	0	0	1800	22
s ₂	0	171	229	18	0	1	0	489	27
s ₃	0	350	519	64	0	0	1	1311	21
Leaving Variable				Pivot Element					
@	@	@	@	@	@	@	@	@	@

Source: own calculation based on SFPA 2013 annual report

@@@@ **Pivot Row**

Step(5) : select the leaving basic variable

The minimum ratio is used to determine the leaving basic variable by dividing (The right hand side \ the column for the entering basic variable) $((1800/82)$, $(489/18)$, $(1311/64)$) the solution is $(22, 27, 21)$.Then the leaving basic variable is s_3 .

Step (6): update the second simplex tableau

Table (6) is a step for putting the tableau in proper form.

Step (6.1): in the column entitled basic variable, replace the leaving basic variable listed for the pivot row by the entering basic variable.

In Table (7) the basic variable listed for equation is s_3 , but in Table (8) is x_3 .

Find the new values of basic variable x_3 by divided (the values in pivot row \ pivot element). Pivot element is the element where the pivot row and the pivot column intersect.

The new values of x_3 are:-

$$\begin{aligned} & [(0/64), (350/64), (519/64), (64/64), (0/64), (0/64), (1/64), (1311/64)] \\ & = \\ & [0, 6, 8, 1, 0, 0, (1/64), 21] \end{aligned}$$

Step (6.2): find the new values of z by multiply value of z in pivot column (-15) in new values of x_3 .

$$\begin{aligned} & -15 * [0 \quad 6 \quad 8 \quad 1 \quad 0 \quad 0 \quad (1/64) \quad 21] \\ & = \\ & [0 \quad -90 \quad -120 \quad -15 \quad 0 \quad 0 \quad -.23 \quad -315] \end{aligned}$$

The new values of z are:-

$$\begin{aligned} & [1 \quad -8 \quad -8 \quad -15 \quad 0 \quad 0 \quad 0 \quad 0] \\ & - \\ & [0 \quad 90 \quad -120 \quad -15 \quad 0 \quad 0 \quad .2 \quad -315] \end{aligned}$$

$$Z = [1 \quad 82 \quad 112 \quad 0 \quad 0 \quad 0 \quad .2 \quad 315]$$

Step (6.3): find the new coefficient of s_1

$s_1 = (\text{coefficient of } s_1 \text{ in pivot row} * \text{new coefficient of entering variable } (x_3))$

$$82 * [0 \quad 6 \quad 8 \quad 1 \quad 0 \quad 0 \quad (1/64) \quad 21]$$

$$= [0 \quad 492 \quad 656 \quad 82 \quad 0 \quad 0 \quad 1 \quad 1722]$$

New coefficient of $s_1 =$

$$[0 \quad 617 \quad 840 \quad 82 \quad 1 \quad 0 \quad 0 \quad 1800]$$

$$-$$

$$[0 \quad 492 \quad 656 \quad 82 \quad 0 \quad 0 \quad 1 \quad 1722]$$

$$s_1 = [0 \quad 125 \quad 184 \quad 0 \quad 1 \quad 0 \quad -1 \quad 78]$$

Step (6.4): Find the new coefficient of s_2

= (Coefficient of s_2 in pivot row * new coefficient of entering variable x_3)

$$18 * [0 \quad 6 \quad 8 \quad 1 \quad 0 \quad 0 \quad (1/64) \quad 21]$$

$$=$$

$$[0 \quad 108 \quad 144 \quad 18 \quad 0 \quad 0 \quad .3 \quad 378]$$

Then the new coefficients of s_2 are

$$[0 \quad 171 \quad 229 \quad 18 \quad 0 \quad 1 \quad 0 \quad 489]$$

$$-$$

$$[0 \quad 108 \quad 144 \quad 18 \quad 0 \quad 0 \quad .3 \quad 378]$$

$$s_2 = [0 \quad 63 \quad 85 \quad 0 \quad 0 \quad 1 \quad -.3 \quad 111]$$

Table (9) Last Simplex Tableau

	Z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
Z	1	82	112	0	0	0	.2	315
s_1	0	125	184	0	1	0	-1	78
s_2	0	63	85	0	0	1	-.3	111
x_3	0	6	8	1	0	0	0	21

Source: own calculation based on SFPA 2013 annual report

Table (9) shows the greatest value of the objective function (315). Then the maximum return achieved by mobile clinics was SDG (315), when distributed (21) Injections.

7.4 Maximization of the Return of RH Commodities Distributed by Integrated Health Center:

Integrated health center distributes five methods of RH commodities Mono hormonal pills, Bi hormonal pills, Injection, Implant and IUDs. Table below shows the RH commodities distributed by integrated health center.

Table (10) RH Commodities and Sale Prices by SFPA and Integrated Health Center.

	RH Commodities				
	Mono Hormonal Pills	Bi Hormonal Pills	Injection	Implant	IUD
Denote	x_1	x_2	x_3	x_4	x_5
The Sale Prices by SPFA	2	2	10	125	20
The Sale Prices by Integrated Health Center	10	10	30	175	75
The Profit	8	8	20	50	55

Source: SFPA 2013 annual report

Table (10) shows the RH commodities distributed by integrated health center and the sale prices of RH commodities sold by SFPA and integrated health center.

Table (11) Constraints, RH Commodities Distributed by Integrated health center and Commodities Targeted Annually

Constraints	RH Commodities					Target
	Mono Hormonal Pills	Bi Hormonal Pills	Injection	Implant	IUD	
Quantities of RH Commodities.	1414	1658	144	19	26	4230
Number of Women Who Use RH Commodities for the First Time.	388	437	61	19	3	1177
Number of Women Who Ever Use RH Commodities.	961	1147	83	0	23	3053

Source: SFPA 2013 annual report

Table (11) shows the activities (constraints), quantities of RH commodities, number of women who use RH commodities for the first time and number of women who ever use RH commodities. Also the table shows the number of RH commodities distributed by means of constraints and the RH commodities targeted annually.

For presenting the problem Tables (10) and (11) can be used.

$$\text{Maximize } Z = 8x_1 + 8x_2 + 20x_3 + 50x_4 + 55x_5$$

Subject: to

$$1414x_1 + 1658x_2 + 144x_3 + 19x_4 + 26x_5 \leq 4230$$

$$388x_1 + 437x_2 + 61x_3 + 19x_4 + 3x_5 \leq 1177$$

$$961x_1 + 1147x_2 + 83x_3 + 23x_5 \leq 3053$$

Further; x_1, x_2, x_3, x_4 and $x_5 \geq 0$.

Table (12) Optimum Solution of RH Commodities Distributed by Integrated Health Center

Decision Variables	Solution Value	Unit Profit	Total Contribution
X ₁	0	8	0
X ₂	0	8	0
X ₃	0	20	0
X ₄	40.9	50	2049.3
X ₅	132	55	7300.7
Objective	Function	Max =	9350.0

Source: QSB Result Sheet Based on Tables (4-8) and (4-9).

Table (12) shows that integrated health center achieved maximum return SDG (9350) by distributed 41 Implants and 132 IUDs.

7.5 Maximization of the Return of RH Commodities Distributed by CMoH:

CMoH distribute five methods of RH commodities: Mono hormonal pills, Bi hormonal pills, Injection, Implant and IUD. Table below shows the RH commodities distributed by CMoH.

Table (13) RH Commodities and Sale Prices by SFPA and CMoH

	RH Commodities				
	Mono Hormonal Pills	Bi Hormonal Pills	Injection	Implant	IUD
Denote	x1	x2	x3	x4	x5
The Sale Prices by SFPA	2	2	10	125	20
The Sale Prices by CMoH	10	10	30	200	100
The Profit	8	8	20	75	80

Source: SFPA 2013 annual report

Table (13) shows the RH commodities distributed by CMoH and the sale prices of RH commodities sold by SFPA and CMoH.

Table (14) Constraints, RH Commodities Distributed by CMoH and RH Commodities Targeted Annually

Constraints	RH Commodities					Target
	Mono Hormonal Pills	Bi Hormonal Pills	Injection	Implant	IUD	
Quantities of RH Commodities.	1318	1833	58	14	55	5630
Number of Women Who Use RH Commodities for The First Time.	259	480	29	14	24	1384
Number of Women Who Ever Use RH Commodities.	848	1003	29	0	31	4246

Source: SFPA 2013 annual report

Table (14) shows the activities, quantities of RH commodities, number of women who use RH commodities for the first time and number of women who ever use RH commodities. Also the table shows the number of RH commodities distributed by means of constraints and the RH commodities targeted annually.

For presenting the problem Tables (13) and (14) can be used.

$$\text{Maximize } Z = 8x_1 + 8x_2 + 20x_3 + 75x_4 + 80x_5$$

Subject: to

$$318x_1 + 1833x_2 + 58x_3 + 14x_4 + 55x_5 \leq 5630$$

$$259x_1 + 480x_2 + 29x_3 + 14x_4 + 24x_5 \leq 1384$$

$$848x_1 + 1003x_2 + 29x_3 + 31x_5 \leq 4246$$

$$\text{Further; } x_1, x_2, x_3, x_4 \text{ and } x_5 \geq 0$$

Table (15) Optimum Solution of RH Commodities Distributed by CMoH

Decision Variables	Solution Value	Unit Profit	Total Contribution
X ₁	0	8	0
X ₂	0	8	0
X ₃	0	20	0
X ₄	98.8	75	7414
X ₅	0	80	0
Objective	Function	Max =	7414

Source: QSB Result Sheet Based on Tables(4-11)and (4-12).

Table (15) presents the optimum solution of RH commodities distributed by CMoH. The results show that CMoH achieved maximum return SDG (7414), by distributed 98.8 Implants.

7.6. Application of LP Using Data Envelopment Analysis:

Data Envelopment Analysis (DEA) measure the relative efficiency of distribution sources (midwives, moving clinics, integrated health center, CMoH) for SFPA and measure their performance using input and output constraints. Tables (16) and (17) below show the output and input of SFPA (constraints of the model).

Table (16) Output of SFPA

Sources Distribution RH Commodities	Quantities of RH Commodities Distributed by Sources	No. of Women Who Use RH Commodities for the First Time	No. of Women Who Ever Use RH Commodities
Midwives	3292	757	2217
Moving Clinics	1539	418	933
Integrated Health Center	3261	414	2214
CMoH	3278	806	1911

Source: SFPA 2013 annual report

Table (17) Input of SFPA

Sources Distribution RH commodities	Quantities of RH Commodities provided by SFPA
Midwives	4000
Moving clinics	1800
Complete health center	4230
CMOH	5630

Source: SFPA 2013 annual report

Table (16) and (17) use to present the problem of the distribution sources of RH commodities:

First: Presenting the Problem of Midwives

$$\text{Maximize } \theta = 3292u_1 + 754u_2 + 2217u_3$$

Subject: to

$$4000v_1 = 1$$

$$3292u_1 + 754u_2 + 2217u_3 \leq 4000v_1$$

$$1539u_1 + 418u_2 + 933u_3 \leq 1800v_2$$

$$3261u_1 + 414u_2 + 2214u_3 \leq 4230v_3$$

$$3278u_1 + 806u_2 + 1911u_3 \leq 5630v_4$$

$$u_1, u_2, u_3, v_1 \geq 0$$

Second : presenting the problem of mobile clinics

Maximize $\theta = 1539u_1 + 418u_2 + 933u_3$

Subject: to

$$1800v_2 = 1$$

$$3292u_1 + 754u_2 + 2217u_3 \leq 4000 v_1$$

$$1539u_1 + 418u_2 + 933u_3 \leq 1800v_2$$

$$3261u_1 + 414u_2 + 2214u_3 \leq 4230v_3$$

$$3278u_1 + 806u_2 + 1911u_3 \leq 5630v_4$$

$$u_1, u_2, u_3, v_1 \geq 0$$

Third: presenting the problem of integrated health center

Maximize $\theta 3261u_1 + 414u_2 + 2214u_3$

Subject: to

$$4230v_3 = 1$$

$$3292u_1 + 754u_2 + 2217u_3 \leq 4000 v_1$$

$$1539u_1 + 418u_2 + 933u_3 \leq 1800v_2$$

$$3261u_1 + 414u_2 + 2214u_3 \leq 4230v_3$$

$$3278u_1 + 806u_2 + 1911u_3 \leq 5630v_4$$

$$u_1, u_2, u_3, v_1 \geq 0$$

Fourth: Presenting the Problem of CMoH

Maximize $\theta 3278u_1 + 806u_2 + 1911u_3$

Subject: to

$$5630v_4 = 1$$

$$3292u_1 + 754u_2 + 2217u_3 \leq 4000 v_4$$

$$1539u_1 + 418u_2 + 933u_3 \leq 1800v_4$$

$$3261u_1 + 414u_2 + 2214u_3 \leq 4230v_4$$

$$3278u_1 + 806u_2 + 1911u_3 \leq 5630v_4$$

$$u_1, u_2, u_3, v_1 \geq 0$$

Table (18) the Relative Efficiency of the Sources Distribute RH Commodities

Sources distribute RH Commodities	The Relative Efficiency
Midwives	106
Moving clinics	100
Complete health center	94
CMOH	68

Source: QSB Result Sheet Based on Tables (4-14) and (4-15).

Table (18) shows the relative efficiency of the sources distribute RH commodities. The results present that the relative efficiency of midwives distribute RH commodities was (106%) that means midwives have more relative efficiency in the distribution RH commodities. The relative efficiency of mobile clinics distribute RH commodities was (100%) and that means it is relatively efficient. The complete health center and CMOH were not relatively efficient in the distribution RH commodities.

7.7 Sudan Family Planning Commodity Poverty Index (SFPCPI). The

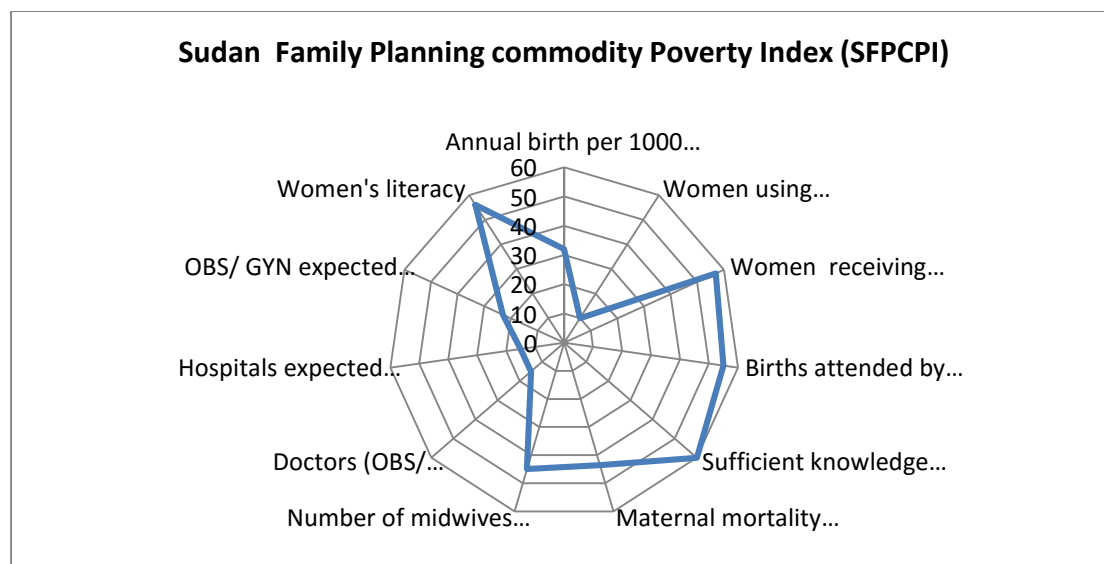
Population Action International (PAI), 2008, ranked 133 countries of the world's population on Risk Reduction Index (RRI) composed of 10 indicators of RH.

Countries were classified according to their family planning commodity poverty in the following table.

Table (19) Classified Countries According to their Family Planning Commodity Poverty

19	26	28	35	45
Very poor	Poor	Moderate poverty	Rich	Very Rich

Source: PAI report card 2008



Source: PAI report card 2008

Figure (2): Sudan Family Planning commodity Poverty Index.

Sudan was ranked in the very poor category indicating low standard RH. In fact SFPCPI was identified in terms of RH risks and poverty status as reflected in the nested shape Fig (2). SFPCPI is based on seven indicators which are Maternal Mortality Rate (MMR), annual births per 1000 women 15-19 years, women using contraception % , women receiving prenatal care % , birth attended by skilled personnel, Total Fertility Rate and sufficient knowledge of HIV/AIDS .The correlation between indicators as calculated by the PSC students shown in Table (20) The table single out birth attended by skilled personnel as highly negatively correlated with maternal mortality while fertility in general shows medium positive correlation with maternal mortality.

Table (20) Indicators of SFPCPI

Indicators	The Values
MMR	1.0
Annual Births per 1000 Women 15-19 years	0.35
Women Using Contraception %	0.24
Women Receiving Prenatal Care %	0.17
Birth Attended by Skilled Personnel	0.66
Total Fertility Rate	0.40
Sufficient knowledge of HIV/AIDS	0.23

Source: PSC 2013

From table (18) the

$$\begin{aligned} \text{FPCPI} &= \sqrt[7]{1 \times 0.35 \times 0.24 \times 0.17 \times 0.66 \times 0.40 \times 0.23} \\ &= \sqrt[7]{0.000867} = 3.68 \end{aligned}$$

According to Classified Countries by their Family Planning Commodity Poverty index : Gezira state was extremely very poor in terms of reproductive health commodities availability and accessibility.

8. Conclusion and Recommendations

8.1 Conclusion:

The study was carried out to estimate the optimum quantities of RH commodities distributed by the sources of SFPA, investigate the women characteristics who used RH commodities and to measure their level attitude towards used RH commodities.

The study found that the optimum quantities distributed by the sources of SFPA and yield the greatest return were; (2.47) tape of mono hormonal pills distributed by midwives and yielded the greatest return of SDG (19.77) , (21) injections distributed by mobile clinics and yielded the greatest return of SDG (315) , (40) implants and (132) IUDs distributed by integrated health center and a yielded the greatest return of SDG (9350) and (98) implants distributed by CMoH and yielded the greatest return of SDG (7414).

8.2 Recommendations:

- The linear programming can be utilized as a useful tool for decision making to optimize the decision and satisfy the simultaneous constraints.
- Encourage institutions to use (LP) to allocate scarce resources in an optimal way.
- Investing in RH service and making them available especially in remote rural areas.
- The government and private health institutions should provide more written information to their clients on usage of RH commodities their risks and benefits.
- To conduct more research in the future in unmet need for Family Planning methods.

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