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DATA ENVELOPMENT ANALYSIS: AN APPLICATION TO TURKISH BANKING INDUSTRY

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Abstract-DEA is a well-known decision making tool and efficiency measurement technique which was proposed by Charnes et al. in 1978 in EJOR. Since then, the technique had proved itself in many application in many areas like banking, agriculture, transportation and so on and many papers had written in many journals by authors around the worldwide.

DEA is a non-parametric technique and scope of DEA is to determine the efficiency of decision-making units (DMUs). It divides the DMUs into two group named as efficient and inefficient, then derives a piecewise linear frontier with pareto-efficient DMUs and gives an efficiency score of 1(one) to each of them. However, the DMUs below the frontier gets an efficiency score less than 1 (one). Their efficiency score is determined by the distance between the frontier and the coordinates of each of the inefficient DMUs. Furthermore, DEA also determines the source and the amount of inefficiency so that DEA becomes indispensable
for decision-makers, named as boss, manager director, owner or others. In this paper, despite of the classical usage, a new area for DEA would be introduced and a new definition would be made for decision-maker. In our work decision-maker is customer.

Key words- Data envelopment analysis (DEA), decision maker, customer, efficiency, banking

1. INTRODUCTION

A well-known realty is that the source of the world is limited but the needs of the human have no limits. Hence, using the source optimally becomes an important problem indeed. There are many disciplines that are trying to solve this matter with using their own solution methods, which reflect their look and aim. However, the objectives are all the same, allocating the source and using them efficiently and optimally.

As a result of cooperation, beneath the allocation of insufficient sources and using them as inputs, for getting outputs is a real problem indeed. Thus, monitoring the results and detecting that if the process works as desired or not is also important for indicating the success of operation researchers. For this purpose many techniques are in use, one of them is data envelopment analysis (DEA) which was proposed by Charnes et al. [1]. DEA is a non-parametric technique, which is used to measure the relative efficiency of DMUs. The essential feature of DEA is that despite of Farrell efficiency measure [2], it could be used in situations of multiple inputs and multiple outputs and does not require any functional form like regression equation or production function and any assumption on the distribution of error terms and/or on variables about the correlation between them. Thus, DEA is accepted by the authors because of above features and after the initial model (named as CCR ratio form) of Charnes et al. [1, 3], many new models and extensions had been developed for different purposes from 1978 to recent and used in many areas like banks, hospitals, schools, air and road transportation etc.

As in the examples given above DEA reflect the situation of interorganizations or intraorganizations and classically it was given to the manager, boss or the owner as a decision-maker. In this paper a new area of usage would be proposed for DEA. The definition of decision-maker would be changed and the customer that utilises the service of an organisation would be set as a decision-maker. In this manner a customer could use DEA results to select a company or shortly an organisation which he/she wants to get a service. In our work DEA would be applied to set of Turkish banks and the customers of banks would be decided as decision-makers.

Thereafter, in the remaining of this paper, main features of DEA and initial models named CCR ratio form and CCR linear programming form that are the parents of DEA would be given and BCC model which was proposed by Banker et al. [4] and used in our real world example introduced. Further, the two model namely CCR and BCC would be compared and illustrated with a hypothetical example. Then, the banking data, input and output variables introduced and the DEA results obtained from the banking data set would be given. Lastly, conclusions would be made.

2. DEA

2. 1. Beginning Of Dea

DEA had begun with the Ph.D. dissertation research of Eduardo Rhodes in Carnegie Melon University in USA. Under the supervision of W. W. Cooper and with the support of federal government Eduardo Rhodes had evaluated a program follow trough for disadvantaged students in US. Public schools. The collected data were in the form of inputs and outputs and also about the ability of students. As a result, that study had become a challenge of estimating the relative technical efficiency of schools with the information taken from the multiple inputs and multiple outputs and ended with the evaluation of CCR ratio form. It could be given below.

DEA ratio form :

$$\max_{u,v} \frac{\sum_r m_r y_{r0}}{\sum_i v_i x_{i0}}$$

st.

$$\begin{aligned} &\frac{\sum_r m_r y_{rj}}{\sum_i v_i x_{ij}} \le 1, \quad j = 1,\dots,n \\ &\frac{m_r}{\sum_i v_i x_{i0}} \ge \varepsilon \quad r = 1,\dots,p \\ &\frac{v_i}{\sum_i v_i x_{i0}} \ge \varepsilon \quad i = 1,\dots,k \end{aligned} \tag{1}$$

Ratio form of DEA depends on maximising the ratio of weighted sum of outputs and weighted sum of inputs or minimising the inverse and generalises the Farrell's single input single output efficiency measure. The numerator of the ratio could be thought as "virtual single output" because the weighted sum reduces the p outputs to a scalar number, analogously, it is the same for denominator and relates the DEA measure with

Farrell's efficiency measure. Because of some difficulties of ratio model the denominator was set to 1 and taken as a constraint and ratio model was transformed to equivalent LP form seen below by again Charnes et al. [5,1]

DEA CCR model :

$$\max \quad w_0 = \sum_r m_r y_{r0}$$

st.

$$\sum_i v_i x_{i0} = 1$$

$$\sum_r m_r y_{rj} - \sum_i v_i x_{ij} \le 0 \qquad (2)$$

$$\mu_r \ge \varepsilon$$

$$v_i \ge \varepsilon$$

m_r and v_i are the weights of outputs and inputs, y_{rj} (r = 1, ... , p), x_{ij} (i = 1, ... , k) represent outputs and inputs respectively (j = 1, ...n) and ε is a non-archimedian constant. The index 0 represents the DMU in the objective function whose efficiency would be computed. As we said before DEA is used to measure the relative efficiency of DMUs and any of above model have to be run n times, one for each DMU. After this process every DMUs got an efficiency score between 0 and 1 according to their inputs and outputs under the constraint of all the other DMUs. The efficiency score 1 meant being %100 efficient. The efficient DMUs evaluate an efficient frontier that envelops the other DMUs. Rather than the efficient units, inefficient DMUs are all below the frontier and their efficiency score is determined by the distance between the frontier and their coordinates. In fact their efficiency score is below 1(one).

2. 2. BCC Model

Since 1978, from the paper of Charnes et al., DEA takes very long way and used in the areas of non-profit organisations like schools [6], police stations [7], sports [8, 9] and also the areas of profit organisations like banking and finance [5, 10], transportation [11] , agriculture [12] and markets [13] and many other areas. As a result of great many applications and ability to answer different type of questions, many new models and extensions of DEA developed. In this paper, BCC model proposed by Banker et al. [4] would be used to determine the efficient banks that gives the best service to their customer. It could be thought that DEA would be a useful tool for customers who want to determine the bank, which he/she wants to work with. The primal and dual form of BCC model could be given as follows.

BBC-Primal :

$$\min_{\theta,\lambda,s^+,s^-} \quad z_0 = \theta - \varepsilon \sum_{r=1}^{p} s_r^+ - \varepsilon \sum_{i=1}^{m} s_i^-$$

st.;

BCC-Dual :

$$\max \quad w_0 = \sum_r m_r y_{r0} + u_0$$

st.;

$$\sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0} \quad r=1,\dots,p$$
$$\sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \theta\, x_{i0} \quad i=1,\dots,k$$
$$\sum_{j=1}^{n}\lambda_j = 1 \qquad (3)$$
$$\lambda_j, s_r^+, s_i^- > 0$$

$$\sum_i v_i x_{i0} = 1$$
$$\sum_r m_r y_{rj} \; - \; \sum_i v_i x_{ij} + u_0 \le 0$$
$$\mu_r \ge \varepsilon$$
$$v_i \ge \varepsilon$$
$$u_0 = free$$
$$r=1,\dots,p\,,\, i=1,\dots,k,\;\; j=1,\dots,n \qquad (4)$$

The notations in BCC model are the same as in CCR model and the main difference is the unconstrained variable u_0 in dual model. Its equivalence in primal model is the convexity constraint, ($\sum\lambda = 1$). Moreover, the variable θ in primal model gives efficiency score directly. We might explain the effect of new variables and the difference of two models by a hypothetical example.

2. 3. Hypothetical Example

The example has also been designed to demonstrate the Farrell's single input, single output efficiency measurement. There are 8 DMUs depicted as in Figure 1. , and using different amount of single input to produce different amount of single output. The straight line passing from origin trough C is the frontier evaluated by CCR model. DMU C that has the highest output / input ratio (4/5 = 0.8) gets the efficiency score of 1(one) as could be seen in Table 1. , all the other DMUs are not efficient. The amount of inefficiency for an inefficient DMU is the distance between the coordinates of DMU and efficient frontier. An inefficient DMU could be efficient if and only if one of the following conditions would be satisfied.

- By reducing its inputs to the level of efficient frontier while protecting its outputs
- By increasing its outputs to the level of efficient frontier while using the same amount of input.

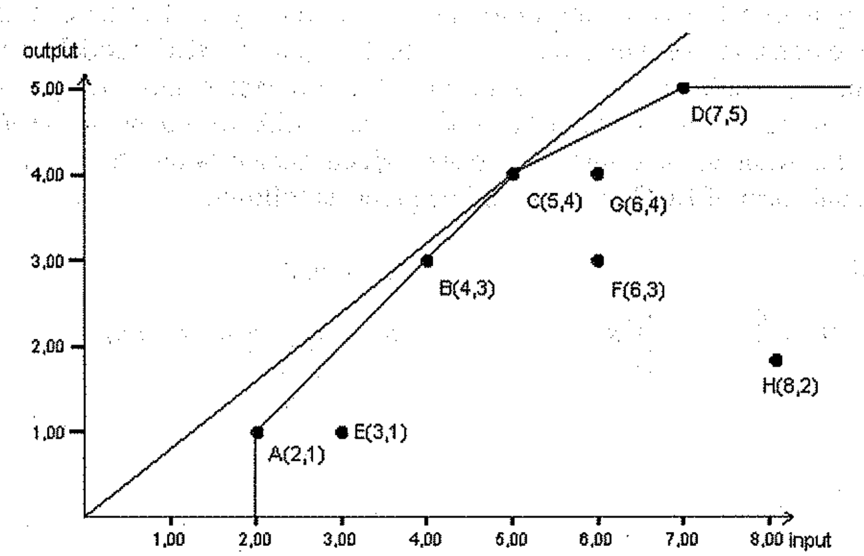

Figure 1. Graphical representation of CCR and BCC models

Table 1. The results of hypothetical example

DMU	CCR			BCC			
	w_j^*	m	v	w_j^*	m	v	u_0
A	0,6250	0,6250	0,5	1	0	0,5000	1
B	0,9375	0,3125	0,25	1	0,2500	0,2500	0,2500
C	1	0,2500	0,2	1	0,2500	0,2000	0
D	0,8929	0,1786	0,1429	1	0,2857	0,1429	0,4286
E	0,4167	0,4167	0,3333	0,6667	0	0,3333	0,6667
F	0,6250	0,2083	0,1667	0,6667	0,1667	0,1667	0,1667
G	0,8333	0,2083	0,1667	0,8333	0,2083	0,1667	0
H	0,3125	0,1563	0,1250	0,3750	0,1250	0,1250	0,1250

It is the same for BCC model as represented in model (3) and (4), but because of the convexity constraint ($\sum\lambda = 1$) (3) efficiency conditions are more realistic. Three more DMUs namely DMU A, DMU B, DMU D are also on the frontier with DMU C. The piecewise linear form passing trough A, B, C and D represents the frontier evaluated by BCC model. DMU C had highest output / input ratio as we said. DMU A is using the lowest input to produce output and KVB D is the highest output producer, lastly KVB B has output / input ratio better than DMU A and D. The other DMUs are still inefficient and they have to satisfy the efficiency conditions for being efficient. If the number of inputs and outputs are more than two, it can be difficult to show graphically as depicted in Figure 1. , but DEA measures the efficiency of DMUs with the same sense as it had been shown.

3. CUSTOMER BASED DATA ENVELOPMENT ANALYSIS

In this paper, DEA had been applied to a sample of Turkish banks for guidance to customer. The data* contains 21 banks (DMUs) and totally 6 variables, 3 of them is input and remaining 3 is outputs. In the sense that the interest of deposit accounts are the incomes of customers , the three interest rates namely Turkish Lira (TL), United States Dollar (USD) and German Mark (DM) are taken as output. Besides that, with the same sense that the interest of credits given by banks to customer is the expense of a customer. The three interest rates , consumer credits (TKT), automobile credits (TST) and dwelling credits (KON) are taken as output. The data is given in Table 2.

Table 2. The Banking Data

DMU NO	BANKS	INPUTS(%)			OUTPUTS(%)		
		TKT	TST	KON	TL	USD	DM
1	AKBANK	5,25	4,95	5,25	25,89	12,00	12,00
2	ANADOLUBANK	8,50	7,50	7,50	20,04	10,00	10,00
3	DEMİRBANK	6,50	5,75	5,75	20,88	9,50	7,00
4	DENİZBANK	6,50	5,50	7,00	29,33	6,75	4,50
5	DIŞBANK	9,50	7,50	7,50	39,25	8,00	7,50
6	EMLAKBANK	6,00	5,50	6,00	26,00	9,00	7,00
7	ESBANK	6,25	6,00	6,00	34,24	7,50	6,50
8	FİNANSBANK	6,50	5,00	5,00	28,39	10,00	7,50

* Our data is obtained from Turkish daily newspaper , Hürriyet in dated 02.18.2001

9	GARANTİ BANKASI	5,75	5,25	5,25	35,91	9,00	8,00
10	İŞ BANKASI	5,75	5,25	5,25	34,24	7,00	6,00
11	KENTBANK	7,75	7,25	8,00	26,72	8,50	7,50
12	KOÇBANK	6,95	5,65	5,65	36,74	8,25	6,75
13	OSMANLI BANKASI	8,50	5,95	5,95	33,40	7,50	5,25
14	OYAKBANK	5,95	5,45	5,45	37,58	11,00	9,25
15	PAMUKBANK	6,50	5,75	5,75	29,23	10,00	9,00
16	SİTEBANK	9,50	8,00	8,00	29,23	12,00	12,00
17	ŞEKERBANK	5,90	5,75	6,00	30,06	8,50	8,50
18	TEB	7,50	6,50	6,75	32,57	8,50	7,50
19	TOPRAKBANK	5,50	5,00	5,00	32,57	9,00	9,00
20	TÜRK TİCARET BANKASI	5,75	5,25	5,25	28,39	7,50	6,50
21	YAPI KREDİ BANKASI	5,25	5,00	5,25	26,72	9,50	7,50

Any of the DEA models could be applied to data given in Table 2. In this paper, BCC primal model (3) of DEA has been used to measure the customer efficiency of sample data set and the results are presented in Table 3. It is evident from the Table 3. that merely 8 DMUs had been efficient (1, 5, 8, 9, 14, 16, 19, 21) and therefore, remaining 13 DMUs had been inefficient.

Table 3. The Results of BCC Model

KVB	DEA Score (θ)	S^+_{TL}	S^+_{USD}	S^+_{DM}	S^-_{TKT}	S^-_{TST}	S^-_{KON}
1	1	0	0	0	0	0	0
2	0,677778	10,3033	0	0	0,3444	0,1	0
3	0,869564	9,6	0	1,25	0	0	0
4	0,904671	0	3,7051	5,955	0,5016	0	1,2115
5	1	0	0	0	0	0	0
6	0,900142	0	2,9506	4,9506	0,1468	0	0,155
7	0,899996	0	1,5	2	0	0,275	0,275
8	1	0	0	0	0	0	0
9	1	0	0	0	0	0	0
10	0,978256	0	2	2,5	0	0,0109	0,0109
11	0,683608	0	3,1272	4,1272	0,1696	0	0,2499
12	0,946793	0	1,744	1,8712	0,7308	0	0
13	0,850772	0	1,5	3,5015	0,6695	0	0
14	1	0	0	0	0	0	0
15	0,879999	0,5664	0	0,3	0	0,072	0
16	1	0	0	0	0	0	0
17	0,914205	0	0,7885	0	0	0,2582	0,3791
18	0,769229	0	0,5	1,5	0,2692	0	0,1923
19	1	0	0	0	0	0	0
20	0,954541	3,8763	1,6364	2,6364	0	0,0136	0
21	1	0	0	0	0	0	0

An important feature of DEA could here be recognised. The slack values of primal models in DEA are used to obtain the source and the amount of inefficiency. Because, BCC is an orienting model, inefficiency of a DMU could not be computed by the slack values directly. Inefficiency could be obtained by the following equations;

$$\hat{y}_{r0} - y_{r0} = s_{r0}^{+}$$
$$x_{i0} - \hat{x}_{i0} = (1-\theta)x_{i0} - s_{i0}^{-}$$

$\hat{y}_{r0}$ and $\hat{x}_{i0}$ are the efficient projection points and the difference between the corresponding input and output of inefficient DMU gives us the amount of inefficiency.

If you notice in Table 3. that all the slack values of efficient DMUs are all zero meanwhile the slack values of inefficient DMUs might not be zero. If we discuss a few example, DMU 1 namely Akbank got the efficiency score of 1 and its slacks are all zero, despite of it, DMU 6 namely Emlakbank had the efficiency score of % 90 and so that it is inefficient. As we said the source and the amount of inefficiency is determined by the help of slacks. Its slack values are $S_{TL}^{+} = 0$, $S_{USD}^{+} = 2.9506$, $S_{DM}^{+} = 4.9506$, $S_{TKT}^{-} = 0.1468$, $S_{TST}^{-} = 0$ and $S_{KON}^{-} = 0.155$. From these results we could say that Emlakbank gives interest to deposits of Turkish Liras as high as efficient banks but lower interests rates to deposits of Unites States Dollar and German Mark the amount is annually % 2.9506 and % 4.9506 respectively. Furthermore, this bank demands higher interest rates for automobile, consumer and dwelling credits from the customers, about the amount of % 0.4523 , % 0.5492 and % 0.4441 per mount respectively rather than efficient banks.

If an inefficient bank decided to be efficient for its customers, it would decrease the interest rates of credits and increase the interest rates of deposits about the amount of inefficiency, otherwise customers preference would be efficient banks.

4. CONCLUSIONS

In this paper, a new look and area of using had been represented to DEA. Consequently, this paper is original and customer based DEA could be used by customer as a decision maker for firm selection among the firms that are giving the same service.

In this study customer based DEA was applied to a sample of Turkish banks. The results has been tabulated, efficient and inefficient banks was determined and presented to the use of customer. If anybody who wants to work with a bank but are not a customer at present would choose the bank according to these results. If someone is still a customer of a bank, he/she might check the efficiency of his/her bank according to these results. if the results are satisfactory then there could not be any problem but if the bank is inefficient it might be a problem. The customers request from the bank is to be efficient. If his/her demand could not been accepted, the customer may change the bank which he/she work with and find an efficient bank. This is also important for banks because inefficient banks are under the risk of loosing their customer.

As a result customer based DEA could be applied successfully to similar problems like efficient bank selection.

REFERENCES

1. Charnes, A., Cooper, W.W. Rhodes, E., Measuring the Efficiency of Decision Making Units, European Journal of Operations Research, 1978, **2**(6), 429-444.
2. Farrell, M., J., The Measurement of Productive Efficiency , Journal of the Royal Statistical Society, Series A, 1957, **120**, 253-290.
3. Charnes, A., Cooper, W.,W., Lewin, A., Y., Seiford, L.,M., Data Envelopment Analysis: Theory, Methodology and Application, Kluwer Academic Publishers, 1994
4. Banker, R., D., Charnes, A., Cooper, W., W., Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis, Management Science, 1984, **30**, 1078-1092.
5. Camanho, A., S., Dyson, R., G., Efficiency, Size, Benchmarks and Targets for Bank Branches: An Application of Data Envelopment Analysis, Journal of Operational Research Society, 1999, **50**, 903-915.
6. Grosskopf, S., Moutray, C., Evaluating Performance in Chicago Public High Schools in the Wake of Decentralisation , Economics of Education Review, 2001,**20**(1),1-14.
7. Drake, L., Simper, R., Productivity Estimation and the Size-Efficiency Relationship in English and Welsh Police Forces: An Application of Data Envelopment Analysis and Multiple Discriminant Analysis, Int. Review of Law and Economics, 2000 , **20** , 53-73.
8. Alp, İ., Gölcükcü, A., Basketbolda Oyuncu Başarı Sıralaması ve Takıma Etkisi, Gazi Beden Eğitimi ve Spor Bilimleri Dergisi, 1999, **4**, 3-11.
9. Alp, İ., Gölcükcü, A., Etkinlik Analizi: Negatif ve Pozitif Etkili Değişkenlerle Performans Hesaplanması, Gazi Beden Eğitimi ve Spor Bilimleri Dergisi, 2000, **5**, 21-34.
10. Oral, M., Yolalan, R., An Empirical Study on Measuring Operating Efficiency and Profitability of Bank Branches, European Journal of Operations Research,1990,**46**,282-284.
11. Odeck, J., Assesing the Relative Efficiency and Productivity Growth of vehicle inspection services: An Application of DEA and Malmquist Indices, European Journal of Operational Research, 2000, **26**, 501-514.
12. Shafiq, M., Rehman, T., The Extent of Resource Inefficiencies in Pakistan's Punjab: An Application of Data Envelopment Analysis, Agricultural Echonomics,2000, **22** , 321, 330.
13. Alp, İ., Gölcükcü, A., Market Şubelerinin Etkinlik Analizi, Gazi Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 2001, **2**, 383-398.