*Mathematical & Computational Applications*, Vol. 7, No. 1, pp. 23-32, 2002



# ELASTIC - PLASTIC STRESS ANALYSIS AND RESIDUAL STRESSES IN METAL MATRIX LAMINATED PLATES UNDER IN-PLANE LOADING

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**Abstract-** Metal matrix composites reinforced by fibers give high strengths, specific stiffness, ductility, yield point and a good temperature performance. They provide a relatively new way of strengthening metals. In this study, continuous steel fiber reinforced aluminum matrix composite manufactured by die casting process has been used as a simply supported symmetric and antisymmetric laminated plate loaded by in-plane forces. An elastic- plastic numerical solution has been carried out by finite element technique for several load numbers. Residual stresses have been given in tables.

## 1. INTRODUCTION

Metal-matrix composites offer a relatively new way of strengthening metals. They consist of a ductile metal matrix and brittle or strong fibers. They provide high strengths, and stiffness, ductility, yield point and a good temperature performance. In recent years, several low cost manufacturing techniques have been developed for the metal matrix composites such as powder metallurgy processing, compo casting, squeeze casting and die casting techniques.

Chou et al. [1] reviewed the work on fiber-reinforced metal-matrix composites explaining fabrication methods, mechanical properties, secondary working techniques and interfaces. Kang and Kang [2] reviewed the on the short fiber-reinforced aluminum composite material, fabricated by stirring method and extruded at high temperatures at various extrusion ratios. Nan and Clarke [3] investigated the influence of particle size on particle fracture and the elastic-plastic deformation of metal-matrix composites.

Residual stresses in the composites are particularly important because they lead to premature failure or strengthening of the composites. Unger and Hansen [4] proposed a method to predict the effect of thermal residual stresses on the free-edge delamination behavior of fiber reinforced composite laminates. Jeronimidis and Parkyn [5] examined thermal residual stresses in carbon-fiber, thermoplastic matrix laminates. Bahei-El-Din and Dvorak [6] investigated the elastic-plastic behavior of symmetric metal-matrix composite laminates for the case of in-plane mechanical loading. Owen and Figuerias [7] studied elasto-plastic finite element analysis of anisotropic plates and shells by using the Huber-Mises yield criterion. Meijer et al.[8] investigated the influence of inclusion geometry and thermal residual stresses and strains in particle reinforced metal matrix composites.

Rolfes et al.[9] presented a post processing procedure for the evaluation of the transverse thermal stresses in laminated plates based on first-order shear deformation theory.

Sayman [10] carried out an elastic-plastic stress analysis in stainless steel fiber reinforced aluminun metal-matrix laminated plates under a transverse force. Karakuzu et al.[11,12] investigated residual stresses in aluminum metal-matrix composite laminates under mechanical loading by using the finite element method. Sayman et al.

studied elastic-plastic stresses in aluminum metal-matrix beams and laminated plates under mechanical loads by using the Tsai-Hill criterion[13-17].

In this study, an elastic-plastic stress analysis is carried out on steel reinforced aluminum metal-matrix laminated plates under in-plane loading. The Tsai-Hill theory is used as a yield criterion

## 2. MATHEMATICAL FORMULATION

The steel fiber reinforced aluminum metal-matrix laminated plates of constant thickness are composed of orthotropic laminates symmetrically and antisymmetrically stacking about the middle surface. The plate in Cartesian coordinates (x, y, z) is illustrated in Figure 1. Where the middle surface of the plate coincides with the x-y plane.

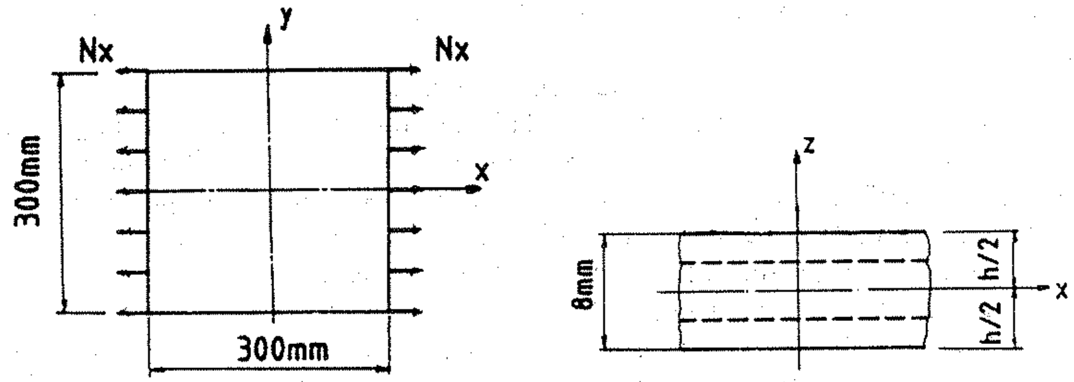


**Figure 1.** Laminated plate.

The solutions of the laminated plates are based on the theory of plates with transverse shear deformations included. The stress-strain relations for an orthotropic layer can be written as,

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & Q_{16}\\ Q_{12} & Q_{22} & Q_{26}\\ Q_{16} & Q_{26} & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix},\quad \begin{bmatrix}\tau_{yz}\\ \tau_{xz}\end{bmatrix}=\begin{bmatrix}Q_{44} & Q_{45}\\ Q_{45} & Q_{55}\end{bmatrix}\begin{bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{bmatrix} \tag{1}$$

where $Q_{ij}$ are the transformed reduced stiffness, which can be expressed in terms of the engineering constants with the coordinate transformations. In this solution, the first order shear deformation theory is used, thus the displacement field is given as,

$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\,\psi_x(x,y)\\ v(x,y,z) &= v_o(x,y) + z\,\psi_y(x,y)\\ w(x,y,z) &= w(x,y) \end{aligned} \tag{2}$$

where $u_o$, $v_o$, and $w$ represent the displacements of a point on the middle surface in the direction of x, y and z axes, respectively. The bending strains vary linearly through the plate thickness and are given by the curvatures of the plate using,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_o}{\partial x} \\ \frac{\partial v_o}{\partial y} \\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ -\frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial y} - \frac{\partial \psi_y}{\partial x} \end{Bmatrix} \tag{3}$$

whereas the transverse shear strains are assumed to be constant through the thickness.

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial y} - \psi_y \\ \frac{\partial w}{\partial x} + \psi_x \end{Bmatrix} \tag{4}$$

The total potential $\Pi$ is considered with in-plane forces $\overline{N_n^b}, \overline{N_s^b}$, in order to satisfy the element equilibrium equations in finite element solution.

$$\Pi = \frac{1}{2}\int_{-h/2}^{h/2}\left[\int_A (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})dA\right]dz + \frac{1}{2}\int_{-h/2}^{h/2}\left[\int_A (\tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz})dA\right]dz - \int_{\partial R} (\overline{N_n^b}u_n^o + \overline{N_s^b}u_s^o)ds \tag{5}$$

where dA = dx dy and in-plane forces are applied on the boundary $\partial R$. The resultant forces, moments and shearing forces per unit length of the cross section are calculated by integration of the stresses through the thickness as,

$$\begin{Bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (1, z) dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{6}$$

Equilibrium requires that $\Pi$ is stationary, i.e., $\delta\Pi = 0$, which can be regarded as the principle of virtual displacement for the plate element.

## 3. FINITE ELEMENT MODEL

In order to obtain the yield points of laminates, and the residual stresses in the laminated plates, a nine node finite element was used. Stacking four layers forms the symmetric or antisymmetric laminated plates. The laminated plates are divided into eight imaginary parts for obtaining the results more accurately. The plate is meshed into 64 elements and 289 nodes.

Bending and shear stiffness matrices of the plate are found by using the virtual energy principle as[19],

$$\left|K_b\right| = \int_A \left|B_b\right|^T \left|D_b\right| \left|B_b\right| dA$$

$$\left|K_s\right| = \int_A \left|B_s\right|^T \left|D_s\right| \left|B_s\right| dA \qquad (7)$$

where

$$\left|D_b\right| = \begin{vmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{vmatrix} \quad , \quad \left|D_s\right| = \begin{vmatrix} k_1^2 A_{44} & 0 \\ 0 & k_2^2 A_{55} \end{vmatrix}$$

$$\left(A_{ij},\ B_{ij},\ D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, z, z^2\right) dz \qquad (i, j = 1, 2, 6) \qquad (8)$$

$$\left(A_{44}, A_{55}\right) = \int_{-h/2}^{h/2} \left(Q_{44}, Q_{55}\right) dz$$

$D_b$ and $D_s$ are the bending and shear parts of the material, respectively. $k_1^2$ and $k_2^2$ denote the shear correction factors, for rectangular cross sections are given as $k_1^2 = k_2^2 = 5/6$, [18].

The Tsai-Hill theory is used as a yield criterion due to the same yield points of the layers in tension and compression. In the numerical solution, in-plane forces are increased incrementally. In such a solution, the calculated stresses and the true stresses must coincide. In order to satisfy this condition, the applied and the equivalent nodal forces have to be found and compared.

The unbalanced nodal forces are used to obtain increments and satisfy the convergence

$$\{R\}_{equivalent} = \int_{Vol} \left|B\right|^T \left|\sigma\right| dA = \int_{Vol} \left|B_b\right|^T \left|\sigma_b\right| dA + \int_{Vol} \left|B_s\right|^T \left|\sigma_s\right| dA \qquad (9)$$

tolerance in a nonlinear solution throughout the complete history per each step of the external load application.

$$\{R\}_{unbalanced} = \{R\}_{applied} - \{R\}_{equivalent} \qquad (10)$$

When the external forces are applied in opposite direction in the elastic solution of the problem, the resultant external forces become zero. Then the differences between elastic-plastic and elastic solution produce the residual stresses in the laminated plates.

## 4. PRODUCTION OF LAMINATED PLATE

A layer of stainless steel fiber-reinforced aluminum metal-matrix laminate was manufactured by moulds. The thickness of each mould was 1mm, thus the thickness of each layer was 2mm. They were heated by electrical resistance and insulated by glass-fibers. A pressure of 30 MPa was applied to the upper mould by a hydraulic pressure and heated up 600°C, as shown in Figure 2. Under these conditions the steel fiber and the aluminum matrix were bonded. Forces were applied to the layer in the principal material directions, and then the yield points of the layer were obtained, as shown in Table1. Mechanical properties of a layer were found by using the strain gages. The laminated plate was obtained by using four layers under a pressure of 30 MPa and 600°C.

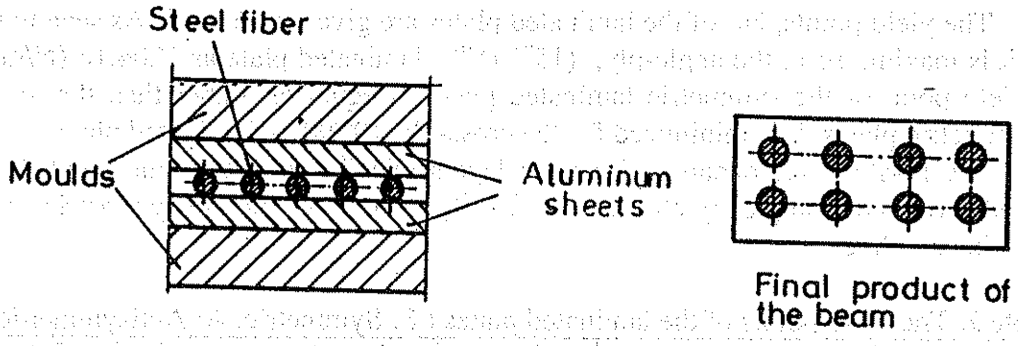


**Figure 2.** Production of the laminated plate

**Table 1.** Mechanical properties and yield points of a layer

| Mechanical properties | | Yield strengths and parameters | | |
|---|---|---|---|---|
| $E_1$ | 86 Gpa | Axial Strength | X | 228.3 MPa |
| $E_2$ | 74 Gpa | Transverse Strength | Y | 24.2 MPa |
| $G_{12}$ | 32 Gpa | Shear Strength | S | 47.6 MPa |
| $\nu_{12}$ | 0.30 | Hardening Parameter | k | 1254 MPa |
| | | Strain Hardening Parameter | n | 0.7 |

The stress-strain relation in plastic region is given by Ludwik equation as,

$$\sigma = \sigma_o + K \ \varepsilon_p^n \qquad (11)$$

In the plastic solution Tsai-Hill criterion was used as a yield criterion.

## 5. NUMERICAL RESULTS AND DISCUSSION

Laminated plates are assumed to be under uniform in-plane loading along the rectangular edges of the plates. The laminated plates are composed of four layers bonded symmetrically and antisymmetrically. Loading is incrementally increased up to the plastic zone allowing not being large. For small deformations the overall stiffness matrix of the plate can be taken as the same per iteration. The laminated plate is divided into eight imaginary parts for the more accurate numerical solution. 169 nodes and 36 elements are used in the numerical solution. They are supported in simply form. The solution is carried out for 200,400 and 600 load steps. The load is increased 0.01 per step.

The yield points, $N_x$, of the laminated plates are given in Table 2. As seen in this table it is maximized in the angle-ply, $(15°/-15°)_2$, laminated plate as 1204.16 (N/mm). The yield point of the symmetric laminated plates is equal or higher than that of the antisymmetric plates. It is minimized for the cross-ply, $(0°/90°)_2$, laminated plate.

The plastic zone expands along the layers of 90° orientation angle but the 0° oriented layers are completely elastic in, $(0°/90°)_2$, symmetric and antisymmetric cross-ply laminated plates.

**Table 2.** The yield points of the laminated plates ( S: Symmetric, A: Antisymmetric).

| | $(0°/90°)_2$ | $(30°/-30°)_2$ | $(15°/-15°)_2$ | $(15°/-30°)_2$ | $(15°/45°)_2$ | $(15°/75°)_2$ |
|---|---|---|---|---|---|---|
| S $N_x$(N/mm) | 207.60 | 616.48 | 1204.16 | 591.20 | 361.04 | 217.76 |
| A $N_x$(N/mm) | 199.12 | 616.48 | 1204.16 | 578.08 | 347.44 | 209.28 |

| | $(30°/45°)_2$ | $(30°/60°)_2$ | $(0°/30°)_2$ | $(0°/45°)_2$ | $(0°/60°)_2$ |
|---|---|---|---|---|---|
| S $N_x$(N/mm) | 351.76 | 254.96 | 609.60 | 369.84 | 265.84 |
| A $N_x$(N/mm) | 344.24 | 247.44 | 590.96 | 353.20 | 253.52 |

Elastic-plastic and residual stresses for the symmetric cross-ply, $(0°/90°)_2$, laminated plate of 600 load steps are given in Table 3. The residual stress components of $\sigma_x$ are 5.565 (MPa) and -5.565 (MPa) in the first (0°) and third (90°) layers, respectively. The absolute value of $\sigma_y$ is 1.523 (MPa) in all the layers.

**Table 3.** Elastic-plastic, elastic and residual stress components in $(0^o/90^o)_2$ cross-ply laminated plate for 600 load steps.

| | | $\sigma_x$ (MPa) | $\sigma_y$ (MPa) | $\tau_{xy}$ (MPa) | $\tau_{yz}$ (MPa) | $\tau_{xz}$ (MPa) |
|---|---|---|---|---|---|---|
| Elastic -Plastic Stresses | 1 | 34.347 | 0.669 | 0.000 | 0.000 | 0.000 |
| | 3 | 39.921 | 2.197 | -0.001 | 0.000 | 0.000 |
| Elastic Stresses | 1 | 34.347 | 0.669 | 0.000 | 0.000 | 0.000 |
| | 3 | 29.553 | -0.669 | 0.000 | 0.000 | 0.000 |
| Residual Stresses | 1 | 5.565 | 1.523 | -0.002 | 0.000 | 0.000 |
| | 3 | -5.565 | -1.523 | 0.002 | 0.000 | 0.000 |

Residual stress components of the cross-ply, $(0^o/90^o)_2$, laminated plate are given in Table 4 for 200, 400 and 600 load steps. As seen in this table, the residual stress components of the symmetric laminated plate are higher than that of for the antisymmetric laminated plate.

The residual stress components of the, $(15^o/-15^o)_2$, $(15^o/30^o)_2$, $(15^o/45^o)_2$, $(15^o/75^o)_2$, $(30^o/-30^o)_2$, $(30^o/45^o)_2$, $(30^o/60^o)_2$, angle-ply laminated plates for 200, 400 and 600 load steps are given in Table 5. As seen in this table, the residual stress components of symmetric laminated plates are equal or higher than that of antisymmetric laminated plates. When the difference between the absolute values of the orientations angles is increased the magnitude of the residual stress components becomes higher. Residual stress component of $\sigma_x$ for the symmetric angle-ply, $(15^o/30^o)_2$, $(15^o/45^o)_2$, $(15^o/60^o)_2$, laminated plate at 600 load steps are 0.785 (MPa), 1.740 (MPa) and 4.983 (MPa), respectively.

The residual stress component of $\sigma_x$ for the angle-ply, $(30^o/-30^o)_2$, $(30^o/-45^o)_2$ and $(30^o/60^o)_2$, laminated plates at 600 load steps is 0.000 (MPa), 1.771 (MPa) and 3.441 (MPa), respectively.

**Table 4.** Residual stresses in the cross-ply, $(0^o/90^o)_2$, laminated plate for 200, 400, 600 load steps ( S: Symmetric, A: Antisymmetric).

| | | $\sigma_x$ (MPa) | $\sigma_y$ (MPa) | $\tau_{xy}$ (MPa) | $\tau_{xz}$ (Mpa) | $\tau_{yz}$ (MPa) |
|---|---|---|---|---|---|---|
| 200 | S | 1.854 | 0.508 | -0.001 | 0.000 | 0.000 |
| | A | 1.205 | 0.022 | 0.133 | -0.031 | 0.001 |
| 400 | S | 3.714 | 1.016 | -0.002 | 0.000 | 0.000 |
| | A | 2.772 | 0.149 | -0.245 | -0.049 | 0.002 |
| 600 | S | 5.565 | 1.523 | -0.002 | 0.000 | 0.000 |
| | A | 5.102 | 0.275 | 0.650 | -0.110 | 0.004 |

**Table 5.** Residual stresses in the angle-ply, (15°/-15°)$_2$, (15°/30°)$_2$, (15°/45°)$_2$, (15°/75°)$_2$, (30°/-30°)$_2$, (30°/45°)$_2$ and (30°/60°)$_2$, laminated plates for 200, 400 and 600 load steps (S: Symmetric, A: Antisymmetric).

| | | | $\sigma_x$ (MPa) | $\sigma_y$ (MPa) | $\tau_{xy}$ (MPa) | $\tau_{xz}$ (MPa) | $\tau_{yz}$ (MPa) |
|---|---|---|---|---|---|---|---|
| (15°/-15°)$_2$ | S | 200 | 0.000 | 0.000 | 0.302 | 0.000 | 0.000 |
| | | 400 | 0.000 | 0.000 | 0.617 | 0.000 | 0.000 |
| | | 600 | 0.000 | 0.000 | 0.912 | 0.000 | 0.000 |
| | A | 200 | 0.000 | 0.000 | 0.599 | 0.000 | 0.000 |
| | | 400 | -0.006 | -0.053 | 0.632 | 0.001 | -0.001 |
| | | 600 | 0.000 | 0.000 | 0.912 | 0.000 | 0.000 |
| (15°/30°)$_2$ | S | 200 | 0.262 | 0.191 | -0.149 | 0.000 | 0.000 |
| | | 400 | 0.522 | 0.377 | -0.298 | 0.000 | 0.000 |
| | | 600 | 0.785 | 0.558 | -0.447 | 0.000 | 0.000 |
| | A | 200 | 0.034 | -0.004 | -0.047 | -0.002 | 0.000 |
| | | 400 | 0.263 | 0.033 | -0.167 | 0.000 | 0.000 |
| | | 600 | 0.613 | 0.428 | 0.396 | 0.011 | 0.016 |
| (15°/45°)$_2$ | S | 200 | 0.585 | 0.476 | -0.280 | 0.000 | 0.000 |
| | | 400 | 1.165 | 0.945 | -0.557 | 0.000 | 0.000 |
| | | 600 | 1.740 | 1.403 | -0.829 | 0.000 | 0.000 |
| | A | 200 | 0.160 | -0.051 | -0.028 | -0.008 | -0.003 |
| | | 400 | 0.696 | 0.095 | -0.153 | -0.008 | -0.005 |
| | | 600 | 1.225 | 0.230 | -0.272 | -0.008 | -0.008 |
| (15°/75°)$_2$ | S | 200 | 1.668 | 0.562 | -0.283 | 0.000 | 0.000 |
| | | 400 | 3.327 | 1.121 | -0.566 | 0.000 | 0.000 |
| | | 600 | 4.983 | 1.679 | -0.848 | 0.000 | 0.000 |
| | A | 200 | 0.831 | 0.037 | -0.153 | -0.012 | -0.002 |
| | | 400 | 2.580 | 0.208 | -0.355 | -0.023 | 0.000 |
| | | 600 | 4.586 | 0.171 | 1.574 | -0.201 | -0.119 |
| (30°/-30°)$_2$ | S | 200 | 0.000 | 0.000 | 0.331 | 0.000 | 0.000 |
| | | 400 | 0.000 | 0.000 | 0.660 | 0.000 | 0.000 |
| | | 600 | 0.000 | 0.000 | 0.988 | 0.000 | 0.000 |
| | A | 200 | 0.001 | 0.001 | 0.332 | 0.000 | 0.000 |
| | | 400 | 0.000 | 0.000 | 0.662 | 0.000 | 0.000 |
| | | 600 | 0.000 | 0.000 | 0.990 | 0.000 | 0.000 |
| (30°/45°)$_2$ | S | 200 | 0.597 | 0.497 | -0.288 | 0.000 | 0.000 |
| | | 400 | 1.185 | 0.984 | -0.573 | 0.000 | 0.000 |
| | | 600 | 1.771 | 1.463 | -0.855 | 0.000 | 0.000 |
| | A | 200 | 0.333 | 0.037 | -0.076 | -0.004 | -0.003 |
| | | 400 | 0.720 | 0.229 | -0.642 | 0.000 | -0.001 |
| | | 600 | 1.458 | 0.483 | 0.452 | 0.000 | 0.000 |
| (30°/60°)$_2$ | S | 200 | 1.153 | 0.623 | -0.401 | 0.000 | 0.000 |
| | | 400 | 2.307 | 1.243 | -0.799 | 0.000 | 0.000 |
| | | 600 | 3.441 | 1.853 | -1.192 | 0.000 | 0.000 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | 200 | 0.745 | 0.043 | -0.135 | -0.014 | -0.006 |
| | | 400 | 1.616 | 0.478 | -0.721 | -0.015 | -0.002 |
| | | 600 | 2.759 | 0.414 | -0.932 | -0.018 | -0.001 |

The residual stresses for the angle-ply, $(0^o/30^o)_2$, $(0^o/45^o)_2$, $(0^o/60^o)_2$ and $(0^o/90^o)_2$, laminated plates at 400 load steps are given in Table 6. As seen in this table, when the difference between orientation angles is increased, the residual stress component of $\sigma_x$ becomes higher. It is 0.522 (MPa), 1.137 (MPa), 2.204 (MPa) and 3.712 (MPa) for the symmetric, $(0^o/30^o)_2$, $(0^o/45^o)_2$, $(0^o/60^o)_2$ and $(0^o/90^o)_2$, laminated plates.

**Table 6.** Residual stresses in, $(0^o/30^o)_2$, $(0^o/45^o)_2$, $(0^o/60^o)_2$ and $(0^o/90^o)_2$, laminated plates for 400 load steps. ( S: Symmetric A: Antisymmetric).

| | | $\sigma_x$ | $\sigma_y$ | $\tau_{xy}$ | $\tau_{xz}$ | $\tau_{yz}$ |
|---|---|---|---|---|---|---|
| $(0^o/30^o)_2$ | S | 0.522 | 0.346 | -0.293 | 0.000 | 0.000 |
| | A | 0.225 | -0.003 | -0.063 | -0.003 | -0.001 |
| $(0^o/45^o)_2$ | S | 1.137 | 0.900 | -0.541 | 0.000 | 0.000 |
| | A | 0.602 | 0.049 | -0.125 | -0.010 | -0.005 |
| $(0^o/60^o)_2$ | S | 2.204 | 1.153 | -0.767 | 0.000 | 0.000 |
| | A | 1.574 | 0.127 | -0.267 | -0.028 | -0.010 |
| $(0^o/90^o)_2$ | S | 3.712 | 1.015 | -0.002 | 0.000 | 0.000 |
| | A | 2.772 | 0.149 | -0.245 | -0.049 | 0.002 |

## 6. CONCLUSIONS

Elastic-plastic stress analysis has been carried out by using the first order shear deformation theory in aluminum-steel fiber laminated plates under in-plane loading. The following results of elastic-plastic stress analysis are obtained.

The orientation angle affects the yield points of laminated plates.

In-plane loading, plastic yielding occurs in the $90^o$ oriented layers and the residual stress component of $\sigma_x$ is compressive and tensile in layers of $90^o$ and $0^o$ orientation angles, respectively.

The yield point of symmetric laminated plates is higher than that of antisymmetric laminated plates.

When the difference between the absolute values of the orientation angles is increased, the residual stress component of $\sigma_x$ becomes higher.

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