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PRESSURE DISTRIBUTION IN A SLIDER BEARING LUBRICATED WITH SECOND AND THIRD GRADE FLUIDS

M. Yürüsoy
Department of Mechanical Education
Technical Education Faculty, Afyon Kocatepe University
TR-03200, Afyon, Turkey. yurusoy@aku.edu.tr

Abstract- A Slider bearing with second and third grade fluids as lubricant is analysed in the present study. The analysis is based on perturbation technique. Choosing second and third grade effects to be smaller than the viscous effects, a perturbation solution is constructed. Under the thin film assumption, inertia terms are negligible compared to the viscous, second and third grade terms. The pressure distributions in the bearing are calculated approximately.

Key words- slider bearing, second and third grade fluids, perturbation analysis

1. INTRODUCTION

Lubrication of bearings is an important technological problem. The velocity and pressure distribution in the bearing should be known for a proper functioning. Much work has been done on the Newtonian type of lubrication. However, additives are frequently used in lubricating fluids, which makes the flow non-Newtonian.

In this work, second and third grade fluids are considered as a lubricant in a slider bearing. The stress tensor equation for second and third grade fluids is given by Rajagopal and Fosdick [1] as follows

$$\mathbf{T}=-p\mathbf{I}+\mu\mathbf{A}_1+\alpha_1\mathbf{A}_2+\alpha_2\mathbf{A}_1^2+\beta(\mathrm{tr}\mathbf{A}_1^2)\mathbf{A}_1 \quad (1)$$

where $\mathbf{T}$ is stress tensor, p is the pressure, μ is the viscosity and α_1, α_2 and β are material constants. The thermodynamic restrictions and the assumption that the free energy be minimum when the fluid is at rest leads to the relations [1]

$$\mu \geq 0,\ \alpha_1 \geq 0,\ \beta \geq 0,\ |\alpha_1+\alpha_2| \leq (24\mu\beta)^{1/2} \quad (2)$$

$\mathbf{A}_1$ and $\mathbf{A}_2$ are the first Rivlin-Ericksen tensors defined by

$$\mathbf{A}_1=\mathbf{L}+\mathbf{L}^T$$
$$\mathbf{A}_2=\dot{\mathbf{A}}_1+\mathbf{A}_1\mathbf{L}+\mathbf{L}^T\mathbf{A}_1 \quad (3)$$

where L= gradv and v is the velocity vector.

First the equations of motion for second and third grade fluids, in a slider bearing will be derived. Under the thin film assumption, viscous, second and third grade effects remain significant whereas inertial term can be neglected in a slider bearing flow. Then assuming that the second and third effects are small compared to the viscous effects a perturbation type of solution is constructed. The first term in the solution is due to Newtonian behaviour and non-Newtonian terms are added to the Newtonian solution as corrections. The pressure distributions are calculated approximately and the effect of non-Newtonian behaviour is shown in figures.

Some of the relevant studies on non-Newtonian lubrication in bearings are as follows: Ng and Saibel [2] used a special third grade fluid (second grade terms neglected) and studied the flow occuring in a slider bearing. Harnoy and Hanin [3] and Harnoy and Philippoff [4] studied the flow of a second grade fluid in a journal bearing. Bourgin and Gay [5] used a similar model with that of Ng and Saibel [2] to investigate the behaviour of flow in a journal bearing. Buckholz [6] used a power-law model as a non-Newtonian lubricant in a slider bearing. More recently Kacou *et al.* [7] studied the flow of a third grade fluid in a journal bearing and constructed a perturbative solution. The work is extended by the same authors (Kacou *et al.*[8]) by including thermal effects. Yürüsoy and Pakdemirli [9] studied the flow of a special third grade fluid in a slider bearing.

2. EQUATION OF MOTION

The slider bearing is shown in Figure 1. The continuity and linear momentum equations are

$$\mathrm{div}\,\mathbf{v}=0 \tag{4}$$

$$\mathrm{div}\mathbf{T}=\rho\frac{d\mathbf{v}}{dt} \tag{5}$$

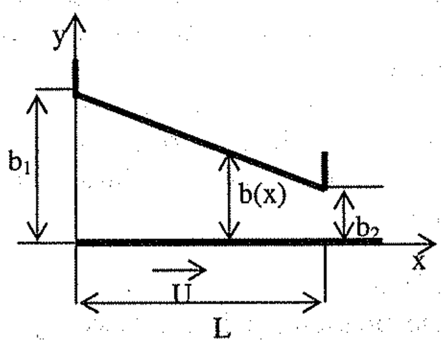

Fig. 1. Slider bearing

Let us introduce the following non-dimensional parameters:

$$x=\frac{x^*}{L},\ y=\frac{y^*}{b_1},\ u=\frac{u^*}{U},\ v=\frac{Lv^*}{b_1U},\ b=\frac{b^*}{b_1},\ p=\frac{p^*}{\rho U^2}\frac{b_1}{L} \tag{6}$$

Substituting equations (1)-(3) and (6) into (4) and (5), one has

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{1}{\delta}\frac{\partial p}{\partial x}+\frac{1}{Re}\frac{1}{\delta^2}\frac{\partial^2 u}{\partial y^2}+\frac{\gamma_1}{\delta^2}\left(v\frac{\partial^3 u}{\partial y^3}+u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)+2\frac{\gamma_1+\gamma_2}{\delta^2}\frac{\partial^2 u}{\partial x\partial y}\frac{\partial u}{\partial y}+6\frac{\gamma_3}{\delta^4}\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2 \tag{8}$$

$$\delta\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{1}{\delta^2}\frac{\partial p}{\partial y}+2\frac{2\gamma_1+\gamma_2}{\delta^3}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+2\frac{\gamma_3}{\delta^3}\left[\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial x\partial y}+4\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}\right] \tag{9}$$

In above equations, the non-dimensional parameters are

$$\mathrm{Re}=\frac{\rho UL}{\mu},\ \gamma_1=\frac{\alpha_1}{\rho L^2},\ \ \gamma_2=\frac{\alpha_2}{\rho L^2},\ \ \gamma_3=\frac{\beta U}{\rho L^3},\ \ \frac{1}{\delta}=\frac{L}{b_1} \tag{10}$$

In equations (7)-(9), only the largest terms in each group are retrieved. We may now assume that 1/Re is of order δ, γ_1 and γ_2 of order δ and finally γ_3 of order δ^3. Under these assumptions, the largest terms in equations (6)-(8) are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{11}$$

$$\frac{dp^*}{dx}=\frac{\partial^2 u}{\partial y^2}+k_1\left(v\frac{\partial^3 u}{\partial y^3}+u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)+6k_3\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

where

$$p^*=p-(2k_1+k_2)\left(\frac{\partial u}{\partial y}\right)^2 \tag{13}$$

Using (12) in (8), we obtain

$$\frac{\partial p^*}{\partial y}=0 \tag{14}$$

The boundary conditions for the problem are

$$u(0)=1,\quad u(b)=0,\quad v(0)=0,\quad v(b)=0 \tag{15}$$

3. VELOCITY PROFILE

In this section, velocity profile will be calculated approximately. Assuming that second and third grade terms are small compared to the viscous term, one may write

$$k_i=\in\bar{k}_i \qquad i=1,2,3 \tag{16}$$

where $\in$ is a small parameter. The approximate velocity profile in the x and y direction and the approximate pressure profile can then be written as

$$u=u_0+\in u_1 \qquad v=v_0+\in v_1 \qquad p^*=p_0^*+\in p_1^* \tag{17}$$

Substituting equations (16) and (17) into (11) and (12), one has

Order 1:

$$\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0 \tag{18a}$$

$$\frac{\partial^2 u_0}{\partial y^2}=\frac{dp_0^*}{dx} \tag{18b}$$

$$u_0(0)=1,\quad u_0(b)=0,\quad v_0(0)=0,\quad v_0(b)=0 \tag{18c}$$

Order $\in$:

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0 \tag{19a}$$

$$\frac{\partial^2 u_1}{\partial y^2}=\frac{dp_1^*}{dx}-\bar{k}_1\left(v_0\frac{\partial^3 u_0}{\partial y^3}+u_0\frac{\partial^3 u_0}{\partial x\partial y^2}+\frac{\partial u_0}{\partial x}\frac{\partial^2 u_0}{\partial y^2}-\frac{\partial u_0}{\partial y}\frac{\partial^2 u_0}{\partial x\partial y}\right)-6\bar{k}_3\frac{\partial^2 u_0}{\partial y^2}\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{19b}$$

$u_1(0)=0,\quad u_1(b)=0,\quad v_1(0)=0,\quad v_1(b)=0$ (19c)

Equations (18a-c) represent the Newtonian problem and the well-known solution is

$$u_0=\frac{1}{2}\frac{dp_0^*}{dx}\left(y^2-yb\right)+\left(1-\frac{y}{b}\right) \tag{20}$$

$$v_0=-\frac{1}{2}\frac{d}{dx}\left(\frac{dp_0^*}{dx}\left(\frac{y^3}{3}-b\frac{y^2}{2}\right)\right)-\frac{y^2}{2b^2}\frac{db}{dx} \tag{21}$$

$$\frac{d}{dx}\left(\frac{dp_0^*}{dx}b^3\right)=6\frac{db}{dx},\qquad p_0^*(0)=p_0^*(1)=0 \tag{22}$$

Equation (22) determines the Newtonian pressure.

Substituting equation (20) into equation (18b) and using the boundary conditions for one gets the correction term to the velocity profile

$$u_1=\frac{dp_1^*}{dx}\left(\frac{y^2}{2}-\frac{by}{2}\right)-\bar{k}_1\left(\frac{d}{dx}\left(\frac{dp_0^*}{dx}\right)\left(\frac{1}{2}-\frac{dp_0^*}{dx}\frac{b^2}{4}\right)+\frac{db}{dx}\left(\frac{1}{b^3}-\left(\frac{dp_0^*}{dx}\right)^2\frac{b}{4}\right)\right)$$
$$-\bar{k}_3\left(\left(\frac{dp_0^*}{dx}\right)^3\left(\frac{y^4}{2}-y^3b+\frac{3y^2b^2}{4}-\frac{yb^3}{4}\right)+\left(\frac{dp_0^*}{dx}\right)^2\left(-\frac{2y^3}{b}+3y^2-yb\right)+3\frac{dp_0^*}{dx}\left(\frac{y^2}{b^2}-\frac{y}{b}\right)\right) \tag{23}$$

Hence, the solution can be written as

$$u=\frac{dp^*}{dx}\left(\frac{y^2}{2}-\frac{by}{2}\right)+\left(1-\frac{y}{b}\right)-k_1\left(\frac{d}{dx}\left(\frac{dp_0^*}{dx}\right)\left(\frac{1}{2}-\frac{dp_0^*}{dx}\frac{b^2}{4}\right)+\frac{db}{dx}\left(\frac{1}{b^3}-\left(\frac{dp_0^*}{dx}\right)^2\frac{b}{4}\right)\right)$$
$$-k_3\left(\left(\frac{dp_0^*}{dx}\right)^3\left(\frac{y^4}{2}-y^3b+\frac{3y^2b^2}{4}-\frac{yb^3}{4}\right)+\left(\frac{dp_0^*}{dx}\right)^2\left(-\frac{2y^3}{b}+3y^2-yb\right)+3\frac{dp_0^*}{dx}\left(\frac{y^2}{b^2}-\frac{y}{b}\right)\right) \tag{24}$$

Using equations (19a) ,(19c) and(17), we have

$$v=-\frac{d}{dx}\left(\frac{dp^*}{dx}\left(\frac{y^2}{2}-\frac{by}{2}\right)+\left(1-\frac{y}{b}\right)\right)+k_1\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{dp_0^*}{dx}\right)\left(\frac{1}{2}-\frac{dp_0^*}{dx}\frac{b^2}{4}\right)+\frac{db}{dx}\left(\frac{1}{b^3}-\left(\frac{dp_0^*}{dx}\right)^2\frac{b}{4}\right)\right)$$
$$+k_3\frac{d}{dx}\left(\left(\frac{dp_0^*}{dx}\right)^3\left(\frac{y^5}{10}-\frac{y^4b}{4}+\frac{3y^3b^2}{12}-\frac{y^2b^3}{8}\right)+\left(\frac{dp_0^*}{dx}\right)^2\left(-\frac{y^4}{2b}+y^3-\frac{y^2b}{2}\right)+3\frac{dp_0^*}{dx}\left(\frac{y^3}{3b^2}-\frac{y^2}{2b}\right)\right) \tag{25}$$

$$-\left(\frac{dp_1^*}{dx}\frac{b^3}{12}\right) = -\bar{k}_1\left(-\frac{dp_0^*}{dx}\frac{d}{dx}\left(\frac{dp_0^*}{dx}\right)\frac{b^2}{4} - \left(\frac{dp_0^*}{dx}\right)^2\frac{db}{dx}\frac{b}{4} + \frac{1}{2}\frac{d}{dx}\left(\frac{dp_0^*}{dx}\right.\right.$$

$$\left.\left. + \frac{db}{dx}\frac{1}{b^3}\right) + \bar{k}_3\left(-\left(\frac{dp_0^*}{dx}\right)^3\frac{b^5}{40} - \frac{dp_0^*}{dx}\frac{b}{2}\right) + C_1 \tag{26}$$

$$p_1^*(0) = p_1^*(1) = 0$$

Equation (26) is non-Newtonian pressure equation. The pressure distribution $p_0^*(x)$ and $p_1^*(x)$ remain unknown in equations (22) and (26) respectively. The goal would then be to determine the pressure distribution approximately.

4. PRESSURE DISTRIBUTION

The solution to equation (22), which is the Newtonian solution, subject to the given boundary conditions, is

$$p_0^* = \frac{6x(b-r)}{b^2(1+r)} \tag{27}$$

where b and r are defined to be

$$b=(1-(1-r)x), \quad r=b_2/b_1 \tag{28}$$

Inserting this Newtonian pressure distribution into equation (26) and applying the associated boundary conditions one finally obtains

$$p_1^* = \bar{k}_1\left(-\frac{2(1-2r-x+xr^2)^2}{(1+r)^2b^4} + \frac{8(1-2r+2r^2-r^3)}{(1+r)^2(1-r)b^2} - \frac{6}{(1+r)^2}\right)$$
$$+\bar{k}_3\left(\frac{3888r^2}{25(1-r)(1+r)^2b^5} + \frac{168}{5(1-r)b^3} - \frac{432r^3}{5(1-r)(1+r)^3b^6} - \frac{576r}{5(1-r)(1+r)b^4}\right. \tag{29}$$
$$\left. + \frac{24(1-r)}{25r(1+r)^3b^2}[13(1+r^2)-r] - \frac{24(13+13r^2+8r)}{25(1-r)r(1+r)^3}\right)$$

The final pressure distribution would then be

$$p^* = \frac{6x(b-r)}{b^2(1+r)} + k_1\left(-\frac{2(1-2r-x+xr^2)^2}{(1+r)^2b^4} + \frac{8(1-2r+2r^2-r^3)}{(1+r)^2(1-r)b^2} - \frac{6}{(1+r)^2}\right)$$
$$+k_3\left(\frac{3888r^2}{25(1-r)(1+r)^2b^5} + \frac{168}{5(1-r)b^3} - \frac{432r^3}{5(1-r)(1+r)^3b^6} - \frac{576r}{5(1-r)(1+r)b^4}\right. \tag{30}$$
$$\left. + \frac{24(1-r)}{25r(1+r)^3b^2}[13(1+r^2)-r] - \frac{24(13+13r^2+8r)}{25(1-r)r(1+r)^3}\right)$$

In the next section, numerical plots of pressure distribution and velocity profiles will be given.

5. NUMERICAL RESULTS

In this section, the pressure distribution in the bearing is determined for various values of the parameters k_1, and k_3. Figure 2 indicates the variation of the pressure with respect to x when $k_3=0$ and k_1 is varied. It is seen that the pressure increases with increasing k_1. Figure 3 illustrates the manner in which pressure varies with k_1, when k_3 is held fixed at some nonzero value. As before, increasing k_1 increases the pressure. In Figure 4 for different k_3, k_1 is held fixed. As k_3 increases the pressure inside the bearing increases which means higher loading capacity for the bearing. In Figure 5 for $k_1=k_3=0.1$ the dimensionless length versus dimensionless pressure is plotted for different clearance ratios. Similar to Newtonian behaviour, in the non-Newtonian case also, pressure builds up in the bearing for lower clearance ratios.

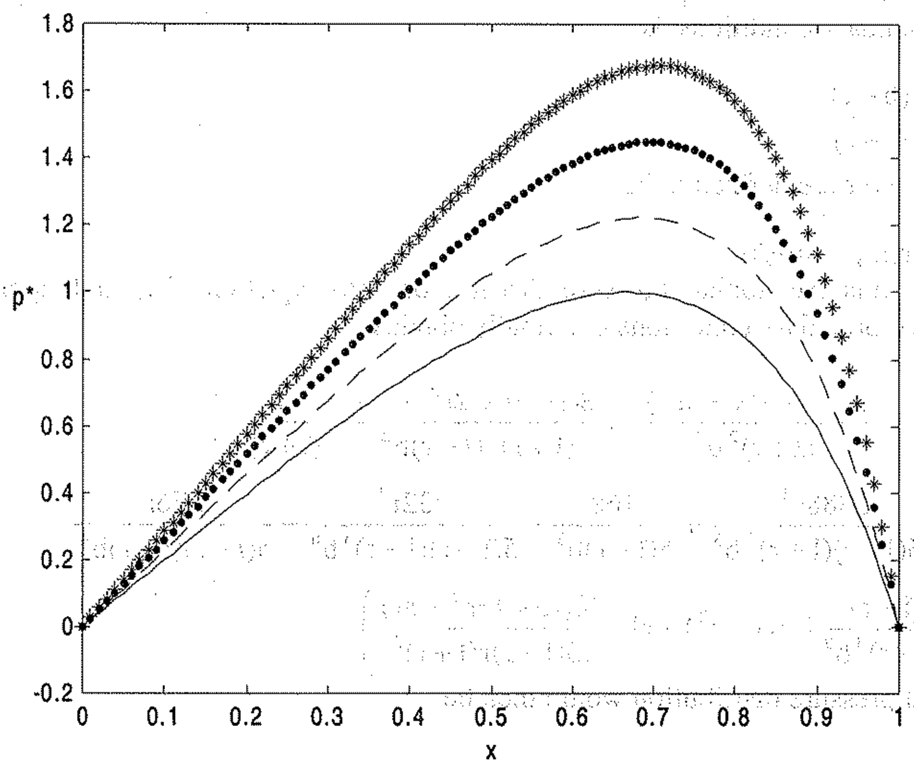

Fig.2 Pressure distribution in the bearing corresponding to various second grade effects for r=0.5 (— $k_1=k_3=0$ (Newtonian); --- $k_3=0$, $k_1=0.1$ ...... $k_3=0$, $k_1=0.2$; *** $k_3=0$,$k_1=0.3$)

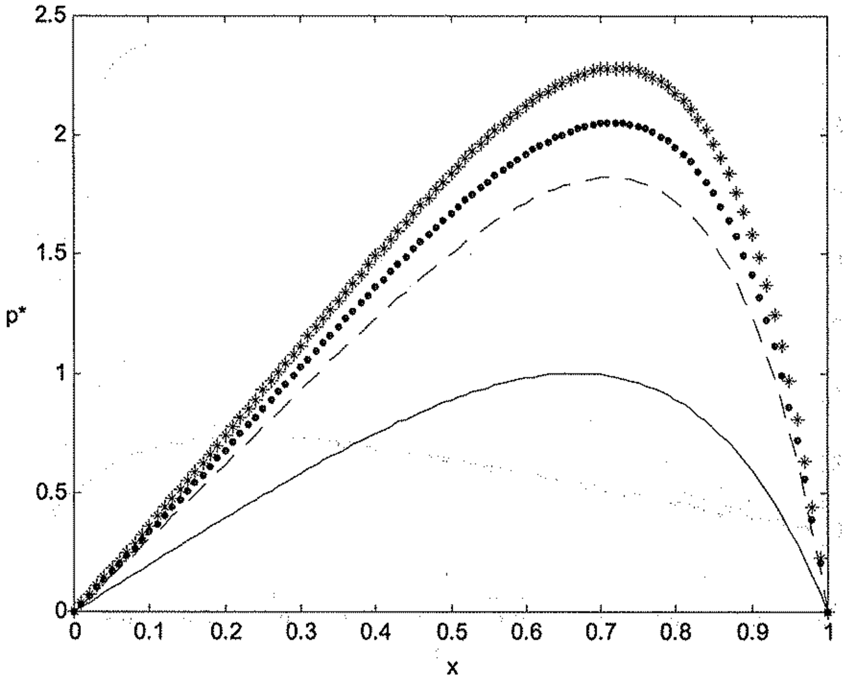

Fig.3 Pressure distribution in the bearing corresponding to various second grade effects with a constant third grade effects for r=0.5 (— $k_1=k_3=0$, (Newtonian); --- $k_3=0.04$, $k_1=0.1$ ...... $k_3=0.04$, $k_1=0.2$; *** $k_3=0.04$, ,$k_1=0.3$)

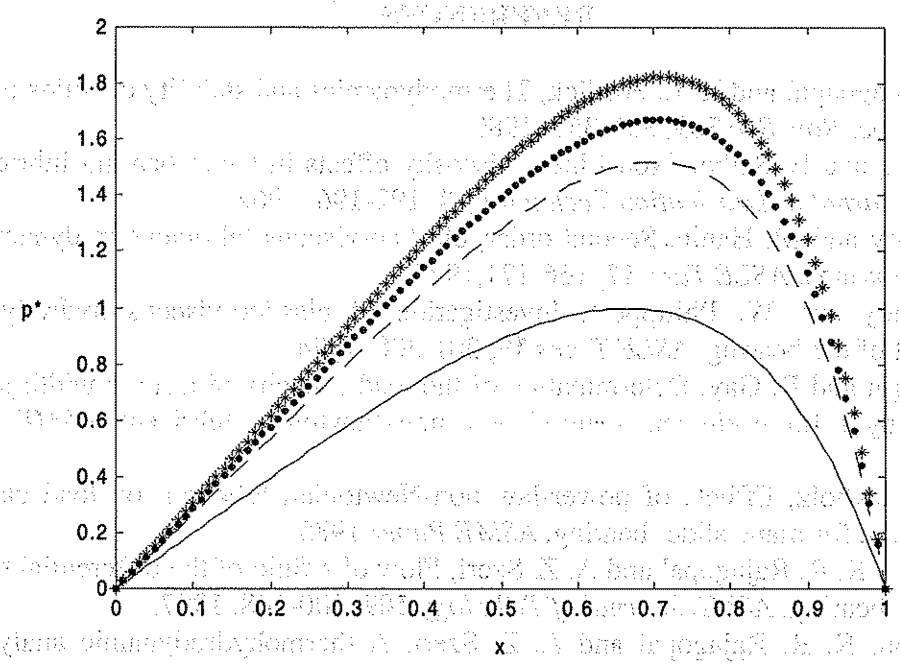

Fig.4 Pressure distribution in the bearing corresponding to various third grade effects with a constant second grade effects for r=0.5 (— $k_1=k_3=0$, (Newtonian); --- $k_3=0.02$, $k_1=0.1$ ...... $k_3=0.03$, $k_1=0.1$; *** $k_3=0.04$, ,$k_1=0.1$)

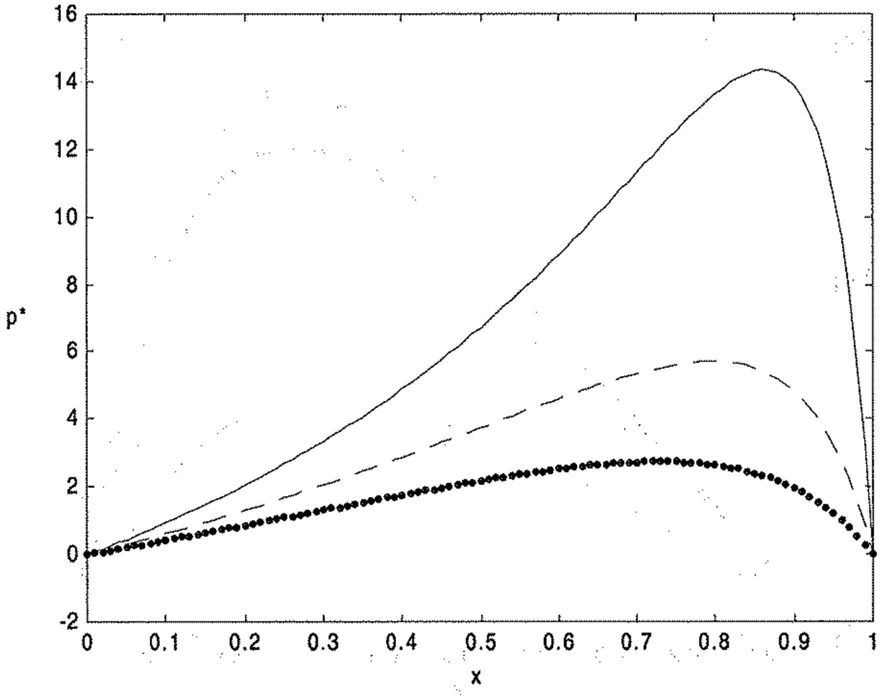

Fig.5 Pressure distribution in the bearing corresponding to different clearance radios for $k_1=k_3=0.1$ (— r =0.3; ---- r =0.4; ...... r =0.5)

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