LOGIC IN THE 14^{th} CENTURY AFTER OCKHAM

Catarina Dutilh Novaes

This chapter is meant to complement the previous chapter on Ockham's and Buridan's respective semantic systems, and the chapters on modalities, on self-referential paradoxes and on supposition in this volume. Here, I intend to cover for as much as possible the important material from the 14^{th} century that is not covered by these other chapters.

The 14^{th} century was a period of intense intellectual activity in Christian Europe, in spite of the image of decline and disaster often associated with this period. By that time, the Universities of Paris and Oxford, whose birth had taken place in the previous centuries, had acquired maturity as institutions, and the different forms for intellectual investigation had been laid down. Even the Black Death in the mid-14th century did not provoke a total decline in the degree of sophistication of the knowledge being produced at the time, in spite of having taken the lives of some of its brightest masters (e.g. Bradwardine, cf. [Read, 2006b]).

Logic occupied a privileged position in the medieval curriculum; it was part of the *trivium* along with rhetoric and grammar, the three subjects a medieval student worked on at the very beginning of his career. In many senses, logic was thought to be the general method with which any student had to have a high degree of familiarity before proceeding to any other topic (cf. [Zupko, 2003, ch. 2]). So, on the one hand, at least some of the logic of that period was really meant for very young students just beginning their intellectual career; on the other hand, while it was indeed the most common for masters to move on to more 'serious' topics (especially theology) at later stages of their careers (but this was not always the case; Buridan is the most prominent but not the only example of a master having stayed at the faculty of Arts throughout his career — cf. [Courtenay, 2004]), many of them viewed logic not only as the matter to be covered by very young students. Indeed, the 14^{th} century corpus on logic presents logical analysis of the highest quality.

But first, of course, we must clarify what was meant by 'logic' in the 14^{th} century. That medieval logic is very different from what we call logic in 21^{st} century is almost a truism. However, a case can be made for the non-equivocal use of the term 'logic' applied to these two radically different traditions (cf. [Dutilh Novaes, forthcoming, 'Conclusion']), insofar as some of the most basic traits of what is/was thought to be logic in each of these periods seem to converge in significant aspects. But this is not the place for such a conceptual, intensional analysis; rather, for the present purposes, it is sufficient to present the **extension** of the term 'logic' in the 14th century — that is, the kinds of theories that were treated under the heading of logic in that period.

As the list of 14th century authors and texts (such as in [Spade, 1996, 329]) will show, the main logical topics treated in that period were: *insolubilia* (paradoxical propositions); modal propositions; supposition; the analysis ('proof') of propositions; obligationes; and consequence. One also encounters works bearing the title '*sophismata'* (William Heytesbury, Albert of Saxony, Richard Kilvington), but *sophismata* are not theories in themselves; rather, *sophismata* are logical and/or philosophical puzzles (cf. [Pironet, 2005]). The apparatus to be used to solve a given sophisma obviously depends on the source of the problem in question, and may come from any of the familiar theoretical frameworks most used then (theories of supposition, theories of fallacies etc.).

As for the first two topics of this list, *insolubilia* and modal propositions (two of the main topics in 14th century logic), they are treated in detail elsewhere this volume, so I will not approach them here. We are thus left with the other four prominent logical genres in the 14th century: supposition, the analysis ('proof') of propositions, obligationes, and consequence. Indeed, this chapter is composed of three main parts, each of them dedicated to one of these topics – under the common heading of 'semantics', I treat supposition and, briefly, the theory of proofs of propositions. But before I move to the study of these three topics, in a preliminary section I give an overview of names, dates and places, so as to provide the reader with some of the historical background before we proceed to the conceptual analysis. The reader may also choose to turn directly to the thematic sections, if the historical aspects are not her main interest.

1 NAMES, DATES AND PLACES

While it is true that there has been growing interest in 14th century philosophy over the last decades, and that the number of publications on this subject has grown exponentially, we are still nowhere near a complete account of the historical and bibliographical events involving the authors in question. It is significant, for example, that even the exact date of death of an author as influential as John Buridan remains unknown (cf. [Zupko, 2002a, section 1]). It may be a matter of further work on the extant manuscripts and records, which remain largely unstudied in libraries, but it may also be that some of these details will never be revealed for lack of extant records.

Be that as it may, and although there is definitely a significant amount of work still to be done on manuscripts and records, scholars have managed to compile an impressive, albeit incomplete, amount of information on the philosophers of that period. Here I attempt to present the main lines of our current state of knowledge on 14th century philosophers and logicians, especially in view of the conceptual analyses to follow. Naturally, several important texts remain without definitive authorial attribution and are listed as 'Anonymous'; whenever relevant, such texts will be mentioned in the thematic sections to follow this one.

1.1 Beyond Paris and Oxford

What is perhaps most interesting in the historical development of logic in the 14th century is the spread of logical and philosophical knowledge to places other than the two traditional centers, Paris and Oxford. Indeed, even though it would be incorrect to say that Paris and Oxford were the only centers of intellectual and academic development in Christian Europe in the centuries preceding the 14th century (the University of Bologna is generally considered to be the oldest university in Christian Europe, and there were important *studia*, i.e. schools of higher learning, in many European cities), it is undeniable that Oxford and especially Paris (which was the great center for theology from the 12th up to the 15th century) were the two great poles of development concerning the *Ars* in general (i.e. logic, rhetoric, grammar, geometry, astronomy, music and arithmetic, plus the three 'philosophies': moral philosophy, metaphysics and natural philosophy), and concerning logic in particular (cf. [De Libera, 1982], on the Paris and Oxford traditions in the 13th century). In the 14th century, however, especially in the second half of the century, this was no longer the case.

The regional element was always an important one in how academic learning was organized in the Faculty of Paris, which was divided in 'nations' in such a way that it was most common for a student to study under a master originally coming from his own home region (cf. [Courtenay, 2004]).[1] But with the creation of several new universities in different locations in Europe, by the end of the 14th century it was no longer necessary for a student to go to Paris or Oxford to obtain his degree; he could often stay within the boundaries of his own country. By the same token, the two traditional centers were no longer the only places where original and influential work in philosophy and logic was being done. It may still be useful, though, for explanatory purposes, to draw a distinction between the British and the Paris traditions in logic in the 14th century (even though there are of course multiple points of contact and mutual influence between the two traditions), and to track how each of these traditions was exported to and reworked in new centers of knowledge. Indeed, even at the time this distinction was recognized: Continental authors usually referred to the authors of the British tradition (in particular those currently referred to as the 'Mertonians') as '*Anglici*' or '*Britannici*' (cf. [Sylla, 1982, 541]).

Two interesting examples of the transmission of the British or Parisian traditions in logic are the influence of British logic in Italy and the influence of Parisian logic in Eastern Europe. Take Italy, for example: even though the country already had a great tradition of institutions of knowledge, an interesting phenomenon is

[1]The four Parisian nations were: Normandy, Picardy, France and the English/German nation. Other universities often followed Paris' example and were organized in (usually four) nations as well (but naturally, different nations than the original Parisian ones). The University of Prague, for example, was organized in the Bohemian, Bavarian, Saxon and Polish nations (see [Ashworth, 2006, 212]).

the spread of 'British logic' in Italy in the 14th century, especially in the second half of the century. In this period, more and more Italian students were sent to Oxford[2], and often brought back to Italy the knowledge they acquired there (cf. [Courtenay, 1982, p. 17]); British masters such as Ockham and Burley also visited and worked in Italy (cf. [Courtenay, 1982, p. 17]). It is very telling that he who is perhaps the most influential logician of the first half of the 15th century was an Italian having studied in Oxford in the last decade of the 14th century, namely Paul of Venice. After obtaining his degree, he taught in Italy for most of his career (cf. [Conti, 2005b]), and this exemplifies the aforementioned phenomenon of spread of knowledge beyond the traditional centers of Oxford and Paris that took place in the 14th century.[3]

A similar phenomenon occurred in Eastern Europe with the exportation of Parisian knowledge (logic in particular), which is made even more evident by the foundation of several influential universities in the region in the second half of the 14th century — the Universities of Prague[4] in 1348, Vienna in 1365 (but to be re-founded in 1384), Erfurt (papal bull in 1379, but inaugurated in 1392), Heidelberg in 1385 and Cologne in 1388/9 are the main examples. Some of the most influential masters of the second half of the 14th century were directly related to the foundation of these universities, most notably Albert of Saxony for the University of Vienna[5] (cf. [Biard, 2004, section 1]) and Marsilius of Inghen for the University of Heidelberg (cf. [Hoenen, 2001, section 1]). These two masters are particularly representative of the spread of 'Parisian' trends (especially the 'Buridanian' approach to logic[6]) into other regions.

[2]As a result of the papal schism (1378–1417, a period during which there were two and sometimes even three 'popes', each of them considering the other(s) to be an usurper), Italians could no longer go to Paris, since Italy and France supported opposing parties in the papal dispute.

[3]Even before Paul of Venice, Italy had already an important tradition of logicians. See for example the introduction to Blaise of Parme's logical *Questiones* [Blaise of Parme, 2001].

[4]In this respect, the University of Prague differs slightly from the other Eastern European universities at the time in that not only the teaching of Parisian masters was influential; Prague enjoyed equally close relations with Oxford. Thus, not only Buridan and Marsilius of Inghen were influential in Prague, but also Heytesbury and later Wyclif (see [Ashworth, 2006, 212]).

[5]However, Albert of Saxony stayed only for a year in Vienna (cf. [Shank, 1988, 13]). For several political and social reasons, the University of Vienna did not really come to existence before its re-foundation in 1384; but at that point it was again the importation of Parisian knowledge that marked its rebirth, as three of the most distinguished Germanic theologians trained in Paris (Henry of Langenstein, Henry of Oyta and Gerard of Kalkar) were recruited to be at the head of the reborn university (cf. [Shank, 1988, 17]). However, even before that, the Parisian master Thomas of Cleves was appointed chief schoolmaster at St. Stephen's Cathedral School in Vienna (the basis for the soon-to-be re-founded University of Vienna) (cf. [Read, 1991, 61]).

[6]Buridan was certainly one of the most influential logicians in the 14th century, probably more than Ockham himself (who, ironically, was more influential in Paris in the first half of the 14th century than in his own home country, England — cf. [Courtenay, 1984; 1987a]). See also [Markowski, 1984] for a detailed account of the reception of Buridan's texts in Eastern Europe.

1.2 A survey of the traditions

Establishing the relations of mutual influence between the different authors and trends in 14th century logic is definitely not a straightforward matter, and one often winds up with an oversimplification of the facts. But given this caveat, in the following sections I will still attempt to present a survey of 14th century logic with respect to names, dates and places, following the thread of these two main traditions, British and Parisian logic.

1.2.1 The British tradition

Let us start with the British tradition; it is somewhat easier to follow than the Parisian/continental tradition, as it developed in a relatively more compact way. (For a detailed overview of the British tradition, [Courtenay, 1987b; Ashworth and Spade 1992] are invaluable sources of information and references to other works on the topic).

The origin of the British tradition in logic is still a matter of debate among scholars. While it seems clear that one cannot speak of a British tradition in the 12th century — a time in which activity in logic was virtually entirely concentrated in Paris — there are important British authors from the 13th century, such as William of Sherwood, Roger Bacon, Simon of Faversham and Robert Kilwardby (who nevertheless all studied and/or worked in Paris). In fact, it has been argued that, while Paris was taken over by the 'modist' fashion at the end of the 13th century, Oxford remained faithful to the 'older' terminist tradition (cf. [Ebbesen, 1985]), which was in turn re-imported into the continent at the beginning of the 14th century, and which is the matrix for the developments in the 14th century to be discussed here. But this theory also encountered opposition, to an extent that we cannot as of now speak of an entirely clear picture of these developments.

The most important British philosopher of the very beginning of the 14th century is, beyond any doubt, Walter Burley. He was extremely influential in his own time as well as in the remaining of the century (his date of death is estimated at around 1344); he exemplifies the introduction of 'new' logical and semantic tools and techniques characteristic of the late medieval period, but in his case often used to defend rather conservative views – he is viewed as the main representative of late-medieval 'realism', as opposed to the nominalism of Ockham and Buridan (for the relations of criticism but also of mutual influence between Ockham and Burley, see [Conti, 2004]).

In the early stages of his career, Burley was linked to Merton College in Oxford, the college to which most of the important British masters of the first half of the 14th century were connected (more on Merton College shortly). It is also noteworthy that he obtained his doctorate in theology in Paris, which also shows that one cannot speak of entirely independent developments in the British and Parisian traditions. Throughout his life, Burley traveled extensively in Europe for several diplomatic missions, while at the same time never stopping his scholarly work, and thus can be seen as one of the actors in the dissemination of British logic in the continent, especially in Italy.[7]

Burley's work is also representative of the topics that were to become the logical topics par excellence throughout the 14^{th} century: he wrote a treatise on consequences early in his career [Walter Burley, 1980] (Green-Pedersen argues that this treatise was certainly written before 1302 — cf. [Green-Pedersen, 1981], as well as a treatise on supposition, and an influential treatise on obligations [Green, 1963]. But he is perhaps most famous for his *On the Purity of the Art of Logic* [Walter Burley, 2000], of which he wrote a shorter and a longer version.[8] For the present purposes, Burley's treatises on consequences and on obligations are particularly important, as well as the parts of the *Purity* concerning consequences; they will be the starting point for the conceptual analyses of each of these topics in this chapter.

Ockham, perhaps the most famous 14^{th} century philosopher now as well as then, was slightly younger than Burley. He led an agitated life, most notably marked by his quarrels with the Avignon popes.[9] He wrote on logic for only a very brief period of his life, before his departure to Avignon; but his work on logic, especially his *Summa Logicae* [William of Ockham, 1974] was to have a significant impact in subsequent developments. However, Ockham's writings will not be among the main objects of analysis in this chapter: his semantics is already thoroughly examined elsewhere in this volume, and his theory of obligations (described in his *Summa Logicae* III-3, chaps. 39-45) is not particularly important for the development of these theories in the 14^{th} century. Only his writings on consequence (*Summa Logicae* III-3) will be examined in the appropriate section below.

Of the same period, Adam Wodeham is now mostly known as the secretary and assistant of Ockham in the period in which the latter was writing the *Summa Logicae* (in the 1320s); although Wodeham has made important contributions as diffusor and also critic of Ockham (especially with respect to his epistemology), his contribution to logic does not seem to have been significant. He is, though, currently thought to be the creator of the doctrine of *complexe significabile*, which was later to be defended by the Parisian theologian Gregory of Rimini (see below in the next section), and was to become an influential theory concerning the meaning and truthmakers of propositions, with applications to epistemology.

Besides the very famous Ockham and Burley, several other extremely innovative and bright masters were active in the first half of the 14^{th} century in England (the Black Death in 1349 is usually considered as a convenient divisor between this period and the one to follow, each having quite distinct characteristics — cf. [Ashworth and Spade, 1992]). As already mentioned, most of these masters were connected to Merton College in Oxford, and are thus often referred to as the 'Mertonians' (often also as the 'Oxford Calculators' — see [Sylla, 1982]). The Merto-

[7]For brief but informative overviews of Burley's life and influence, see [Spade, 2000; Conti, 2004].

[8]On the rather awkward title of Burley's masterpiece, which might be better translated as *On the Essence of the Art of Logic*, see [Spade and Menn, 2003].

[9]For an account of Ockham's life and influence, see [Courtenay, 1999].

nians excelled not only in logic; their works on natural philosophy were probably even more influential; they are often viewed as precursors of the 'mathematical turn' in physics to take place a few centuries later (see [Sylla, 1982, 541]).

The main authors among the Mertonians of the first half of the 14^{th} century (for our present purposes) are: Thomas Bradwardine, William Heytesbury, Richard Billingham, Roger Swyneshed (not to be confounded with the famous Richard Swyneshed, also a Mertonian and in fact known as 'The Calculator'), and Richard Kilvington (for a more detailed presentation of each of these authors, see [Ashworth and Spade, 1992]).

Until not so long ago, it was though that Thomas Bradwardine's contribution to logic was, to say the least, meager, and that his main contributions were to be found in the field of natural philosophy, mathematics and theology. But a recent interest in Bradwardine's *insolubilia* (including a new edition of the text now in preparation by Stephen Read) has arisen, showing that his work on Liar-like paradoxes was extremely innovative and sophisticated (cf. [Read, 2006b]). Since I will not be dealing with *insolubilia* in this chapter (as they are the topic of a different chapter in this volume), Bradwardine will not figure prominently in the analyses to follow. But his historical as well as philosophical importance must not be overlooked.

William Heytesbury was seemingly more prolific than Bradwardine. The list of his still extant works is rather long, and includes treatises on consequence and obligations (which are only to be found in manuscripts — cf. [Longeway, 2003, section 2]), an influential treatise on divided and composite senses, some work on *insolubilia*, among others. But he is most famous for his *Regulae solvendi sophismata* (1335)[10], a work composed of six parts, where logical, semantic and physical sophisms are dealt with. In the first part he deals with the paradoxical propositions known as *insolubilia*, and his approach to them was later to be influential, especially in the continent (e.g. Peter of Ailly's treatment of *insolubilia*). The second part is dedicated to what we now call 'reference in opaque contexts'; the third and fourth part deal with semantic puzzles (related to the supposition of relative pronouns, in the third, and to the terms 'begins' and 'ceases' in the fourth); the last two concern physical puzzles. Thus, since the subject-matter of the first two parts is to be dealt with elsewhere in this volume, and since the last two parts do not concern logic and/or semantics directly, in the analysis to follow Heytesbury will not figure prominently. Nevertheless, it is important to notice that Heytesbury was to become very influential in Italy in the 15^{th} century (while almost entirely forgotten in England), again exemplifying the exportation of British logic to Italy in the 14^{th} and 15^{th} centuries (see [Braakhuis, 1982]).

Richard Billingham, another influential Mertonian (apparently a few years younger than Bradwardine and Heytesbury), was most known in his own time for one of his works, his *Speculum puerorum* [Maierú, 1970; de Rijk, 1975; 1982], even though he also wrote on all the traditional topics in 14^{th} century logic (obligations — also

[10] A transcription of the text by F. Pironet, en route for a critical edition, can be found at http://mapageweb.umontreal.ca/pironetf/Sophismata.html

influential, cf. (Ashworth 1985) — consequences, insolubilia, supposition etc. — for complete list of his still extant works, see the bibliography in [Richard Billingham, 2003]). His *Speculum puerorum* is dedicated to what seems to be a 14th century invention, the theory of the 'proofs of propositions'. To 'prove' a proposition is, in a general sense, to show it to be true, but not necessarily in a rigorous, formal way (as we now understand the notion of 'proof'); the basic idea is that there are some propositions that are basic, that is, whose truth does not depend on the truth of other propositions, but that the majority of propositions are not of this nature. The task is thus to 'unfold' these propositions that are not basic into basic propositions, so that it becomes clear what the truth of these non-basic propositions depends on. For this reason, the theory of the proof of propositions is essentially a semantic theory, that is, a theory intended to explain the meaning of some complex propositions in terms of more primitive ones, to which they can be reduced, and therefore will be (briefly) treated under the heading 'Semantics' below.

Billingham's treatise on the proof of propositions (*Speculum puerorum*) is not the first and probably not even the most remarkable among the treatises on the genre in the 14th century (see [De Rijk, 1975; Ashworth and Spade, 1992, 42]); it was, however, very influential in its time (see [De Rijk, 1976]), more than any of his other writings. Furthermore, his treatise on consequences has been given a modern edition recently [Richard Billingham, 2003], and will be mentioned in the section dedicated to consequences below.

Roger Swyneshed and Richard Kilvington are both minor figures if compared to the influential Bradwardine, Heytesbury and Billingham, but they both composed works that had considerable impact in later developments. Swyneshed is known for his treatise on *insolubilia* [Spade, 1979] — which, in spite of being heavily attacked by Heytesbury, eventually became quite popular in the 15th century — and for his treatise on obligations [Spade, 1977], which seemingly initiated a new trend in obligational disputations, the so-called *nova responsio* (as opposed to the *antiqua responsio*, exemplified by Burley's treatise). Indeed, Swyneshed's treatise on obligations will be one of the main objects of analysis in the section on obligations below. Kilvington will be mentioned in the same section, as parts of his quite popular *Sophismata* (cf. [Kilvington, 1990a] for the Latin text and [Kilvington, 1990b)]for the translation), which otherwise mostly deals with problems of motion and change from a logical perspective (see [Jung, 2001]), present important views on obligational disputations as well.

The next period in the history of British logic in the 14th century is, according to Ashworth and Spade [1992, 39] 'a period still of sophistication, even if no longer of great originality, during which earlier issues and doctrines were developed, consolidated and transmitted to the rest of Europe'. They also mention as characteristic features of this period a tendency to produce summary treatments of different logical topics, which seem to have been intended for teaching rather than to be original contributions; a strong interest in the theory of 'proofs of propositions', in the fashion of Billingham's *Speculum puerorum*, and in the problem of truth and signification of propositions. For our purposes, the main authors of this period are: Ralph Strode, Richard Lavenham, Richard Ferrybridge, John Wyclif, and Paul of Venice (the last two being beyond any doubt the best known to us and probably the most influential ones in their time too).

Richard Lavenham was neither particularly influential nor particularly original, but his (usually short) writings in many senses illustrate exceptionally well the general themes and theories of this period. It is perhaps for this reason that many of his writings have been given modern editions (see in particular [Spade, 1974], where his treatises on consequences and on supposition are edited, and [Spade, 1978], for his treatise on obligations; see also [Spade, 1980] for a general presentation of Lavenham); this obviously means that he is a key figure for anyone wishing to understand the logic of the second half of the 14$^{\text{th}}$ century, given the easy access to his writings.

Ralph Strode and Richard Ferrybridge were both more influential figures than Lavenham (they were to be particularly influential in Italy — see [Maieru, 1982a] and [Del Punta, 1982]), but unfortunately most of their works have not yet been given modern editions. Ralph Strode has written a *Logica* that exemplifies perfectly the main interests of logicians in the 14$^{\text{th}}$ century, composed of the following treatises: two introductory chapters on the principles of logic, one on consequence, one on supposition, one on obligationes, and finally one on *insolubilia* (on the order of the treatises within the *Logica*, see [Maieru, 1982a]). His treatise on consequence has been given a modern edition [Seaton, 1973], and there is an ongoing project to edit the rest of Strode's *Logica*, but which so far has not been completed. Strode will be mentioned in the sections below dedicated to obligationes and to consequences.

Ferrybridge wrote two known works in logic, a 'Logic, or treatise on the truth of propositions' and a treatise on consequences (cf. [Ashworth and Spade, 1992, 57]). To my knowledge, neither has been given a full modern edition, but the two first chapters of the *Logica* can be found in [Del Punta, 1982], and many passages of his treatise on consequences can be found in [Pozzi, 1978]; the latter will be commented upon in the section dedicated to consequences below.

By contrast, John Wyclif and Paul of Venice are much better known to us (see [Conti, 2005a; 2005b]); their works are often easily accessible to the modern reader, including translations. Wyclif is most known for his metaphysical positions (he is the main advocate of realism in the second half of the 14$^{\text{th}}$ century), but his importance in the history of logic must not be underestimated. His logical doctrines are indeed most often intimately related to some metaphysical problem; for example, the issue of universals led him to reflect on the notion of predication (cf. [Conti, 2005a, section 2.3]). He did write a *Logica* and a sequel to it (ed. Dziewicki, 1893-99), where again his realist metaphysics plays a prominent role; his discussion of the notion of supposition, heavily borrowed from Burley but with important modifications, will be briefly examined below. He also wrote on the issue of the truth of propositions, on insolubilia and on the 'proof of propositions' (cf. [Ashworth and Spade, 1992]).

At first sight it may seem strange to place Paul of Venice under the heading of ‘British logic’; he was after all an Italian who spent almost his entire life working in Italy. He did though spend a short period (at least three years, it would seem[11]) in Oxford in his formative years, and the logic he learned in Oxford remained his main source of influence in his subsequent writings. It makes thus good sense to place him among the ‘British’ logicians; moreover, in the 15$^{\text{th}}$ century it was mainly in Italy that ‘British’ logic flourished, as in Britain properly speaking a period of stagnation in logic occurred.

Paul’s work covers an impressive array of themes, as is attested for example by the length of his *Logica Magna* (of which several parts have been recently edited and translated into English — see bibliography). His *Logica Parva* (complied around 1395, at Oxford) was one of the most influential logic textbooks in the 15$^{\text{th}}$ century (Paul of Venice 1984). True enough, most of Paul’s career took place in the 15$^{\text{th}}$ century, so one might think that he should be treated elsewhere in this volume, and not in this chapter dedicated to the 14$^{\text{th}}$ century; but in many senses he epitomizes 14$^{\text{th}}$ century logic. Not only did he deal with virtually all of the important logical topics of this century (supposition, obligations, the truth of propositions etc.); he also usually summarized the logical knowledge produced in this century in his discussions, often quoting verbatim from his sources (for example, his use of Strode in his treatise on obligations — cf. [Ashworth and Spade, 1992, fn.99]), while also making original contributions to the discussions.

Another author who was of Italian origin (in fact he was Greek-born) and who adopted much of the Oxford logic framework after having studied there is Peter of Candia, later Pope Alexander V. Mostly a theologian, among his logical works are a treatise on obligationes and one on consequence (cf. [Green-Pedersen, 1985]).

Less influential figures still worth being mentioned are Henry Hopton (in particular his discussion of the truth of propositions), Robert Fland (in particular his works on consequence and on obligations, cf. [Spade, 1976; 1980c], Martinus Anglicus (in particular his works on consequence and on obligations), Johannes Venator (his *Logica* is in the spirit of Billingham’s theory of ‘proofs of propositions’ — cf. [de Rijk, 1982] and edited in [Johannes Venator, 1999]), Robert Alington (a follower of John Wyclif) and Richard Brinkley (in particular his theory of the signification of propositions and his obligationes — [Brinkley, 1987; 1995]). John of Holland is an interesting case of an author in some senses belonging to both traditions, British and Continental. While his writings (cf. [John of Holland, 1985]) show a familiarity with British logicians, which seems to indicate that he may have studied at Oxford, he is best known for his career at the University of Prague; therefore, he will be treated in more detail in the section dedicated to the continental tradition below.

As already said, the end of the 14$^{\text{th}}$ century coincides with a general decline in British logic. According to Ashworth and Spade [1982, 35], it is a period of ‘logic stagnation leading eventually in the sixteenth century to the rejection of the ‘thorns’ of scholastic logic’. In the 15$^{\text{th}}$ century, the most interesting and

[11]Cf. [Ashworth and Spade, 1992, 60].

innovative contributions within the tradition of 'British' logic were to take place elsewhere, in particular in Italy.

1.2.2 The Parisian/continental tradition

The continental tradition in logic in the 14th century begins with what could be described as a hiatus; according to our current state of knowledge about that period, it appears that, in the first three decades of the 14th century, no significant novelties were put forward by Parisian logicians. However, it must be said that the first half of the 14th century in Paris, and in the continent generally speaking, is as of now not as well studied as the same period in Oxford with respect to logic; but it is to be hoped that, with further research, our knowledge of this period in Paris will become more thorough in the coming years.

For as far as we can tell at present, this period in Paris was still very much market by the Modist theories, a late-13th century creation (which is treated elsewhere in this volume); indeed, what is perhaps the most important text of the Modist tradition, namely Thomas of Erfurt's *De modis significandi*, appears to have been written in the first decade of the 14th century (cf. [Zupko, 2002b]. Important Parisian Art Masters of this period (also within the general Modist trend) were Radulphus Brito and Siger of Courtrai, but one cannot speak of them as having made particularly original contributions in the domain of logic besides their influence in the development of the Modist doctrines. Another important figure of this period in Paris was Peter Auriol (see [Friedman, 2002]), who was predominantly a theologian, but whose doctrines had implications for the theory of cognition (cf. [Tachau, 1982]).

There is, however, one author of this period who is worth being mentioned in connection with the development of logic in Paris, namely Giraldus Odonis. His *Logica* [Giraldus Odonis, 1997] seems to have been written at some point in the first half of the 1320s (cf. [de Rijk, 1997, 24]); what is interesting is that it is nothing like the works in logic of the generation to follow, such as Buridan's, so apparently it was not particularly influential for subsequent developments. It was written roughly at the same time as Ockham's *Summa Logicae*, and according to de Rijk [1997, 24], 'neither in Girald's work nor in Ockham's *Summa Logicae* [...] is there any trace of acquaintance with each other's work'. The historical significance of Girald's logic is to show that there was seemingly activity within the terminist tradition in Paris at this time — contrary to the idea that the first three decades of the 14th in Paris were totally dominated by the Modist tradition, and that the terminist tradition was only practiced in Britain. Granted, Girald was familiar with Burley's work (recall that, at this period, Burley was a student of theology in Paris), so it is still possible that the terminist tradition was reintroduced in Paris as a British import, as claimed by Ebbesen [1985]. But what is very significant is how different Girald's theory of supposition is from Burley's, to the point that one wonders whether there wasn't indeed a genuine Parisian terminist tradition in this period, which however seemingly became surpassed by what can be loosely referred to as the 'British approach' in later works, such as those of Buridan.

By contrast, an Englishman[12] thought to have been working in Paris in the 1320s and 1330s, Thomas Maulevelt, attests of the early penetration of Ockhamist logical doctrines in Paris (the history of which deserves closer attention — cf. [Courtenay, 1984]). Very little is known about him (cf. [Lorenz, 1996]), and the fact that his name is spelled in several different ways (Maulvelt, Manlevelt, Maulfield, among others), does not make the job of tracing his steps any easier. But we do know, for example, that his works in the *parva logicalia* ('textbook'-style logic) were to be very influential in Germany and Eastern Europe. Modern editions of some of his texts are now in preparation, but so far none of his works is available in print. What is in any case clear is that Thomas, while working in Paris, was already following the footsteps of Ockham in logic, and sometimes taking the Ockhamist project of ontological reduction even further than Ockham himself (cf. [Andrews, 2005], on Maulevelt's denial of the category of substance).

Be that as it may, by the end of the third decade of the 14$^{\text{th}}$ century, 'a new academic generation with different concerns was emerging [...], but its directions and importance would not become visible for almost a decade' [Courtenay, 1999a, 5]. The most famous member of this new generation is John Buridan, but other inspired masters were Nicholas of Autrecourt, Nicholas of Oresme and Gregory of Rimini. Strictly speaking, the contributions in logic of the three latter authors are not particularly significant; however, they were important figures for the general development of the *Ars* in that period. Gregory of Rimini, for example, was an Italian Augustinian who came in contact with the works of Oxford scholars in Italy (after having studied theology in Paris in the 1320s), and upon his return to Paris in the 1340s is thought to have been particularly instrumental in the spread of Oxford philosophy in general and Oxford logic in particular in Paris (cf. [Schabel, 2001, section 2]) — in the 1330s, very little attention was paid in Paris to the revolutionary works of English logicians of that period, such as Bradwardine, Heytesbury etc. (cf. [Courtenay, 1984, p. 46]).[13] Predominantly a theologian, Gregory is also known for defending the doctrine of *complexe significabile*, the doctrine according to which the object of knowledge is neither propositions nor the things in the external world signified by its terms, but rather that which is signified by the proposition (*complexe significabile*), whose ontological status was seen by some (such as Autrecourt — cf. [Thijssen, 2001, section 7]) as problematic.

John Buridan is, as already mentioned, without a doubt one of the most influ-

[12]Although English, Thomas is treated in the Parisian/continental section here because he is known to have worked in Paris, and because his subsequent influence was particularly noticeable in Europe.

[13]Explicit or implicit signs of Oxford logic in Paris are always historically important, but less so the other way round. This is because one can virtually take for granted that the British logicians were always very much aware of what was going on in Paris, but the converse was not necessarily the case. For example, a manuscript on obligationes partially edited by P. V. Spade and attributed by him to a certain John of Wesel (a set of questions disputed in Paris — cf. [Spade, 1996b]) is significant insofar as it seems to show that as early as 1344 there may have been knowledge in Paris of Swyneshed's obligationes treatise and of his *nova responsio*.

ential philosophers of the 14^{th} century. His writings range over a wide variety of topics of the *Ars* curriculum (logic, natural philosophy (physics), psychology and moral philosophy, metaphysics — he commented on all major Aristotelian texts), but, as often noted, he never moved on to 'higher' levels of intellectual activity (such as law, medicine or theology) (cf. [Zupko, 2002a]). While it was not the most usual path for a master to remain in the Arts faculty throughout his career, Buridan was not the only one to have had such a trajectory (cf. [Courtenay, 2004]). In any case, this meant that Buridan spent his entire career focusing on the subjects of the Arts curriculum, producing a large corpus of extremely sophisticated philosophical texts.

For Buridan, logic was the basic methodology permeating not only all intellectual investigation, but also a key component for the political life of a good citizen (cf. the preface to his *Summulae* [Buridan, 2001, 3]); in other words, the importance of logic for Buridan can hardly be overestimated. Of course, it must be understood that what Buridan conceived logic to be goes beyond the narrower conception that the discipline currently has (this, in fact, holds of the whole medieval tradition): for him, logic encompassed investigations that we would now consider to belong to the fields of semantics, formal epistemology, philosophy of language, metaphysics, among others. Still, Buridan produced a sophisticated and coherent system of doctrines, which has been the object of growing interest over the last couple of decades. Like Ockham, Buridan was a nominalist, that is, a defender of ontological and theoretical parsimony, but while sharing a certain common base, his doctrines differed in content and in general approach from those of Ockham's in many significant aspects. Buridan, not Ockham, is usually thought to be the pioneer of a whole new approach to logic that was to be influential for at least another century, the so-called '*via moderna*' of 'via *Buridanii*'.

For our purposes, his most important texts are his long *Summulae de Dialectica* (a heavily modified commentary of Peter of Spain's *Summulae* — available in English in [Buridan, 2001], and in Latin in several volumes, as part of an ongoing project of critically editing the whole text of the *Summulae*) and his *Treatise on Consequences* [Buridan, 1976]. In fact, since his semantics will be treated elsewhere in this volume, and since he did not write on obligations, he will be discussed in the section dedicated to consequences and, more briefly, in the analysis of the concept of supposition; however, in the section on consequence, he will indeed feature as a most prominent figure, as his treatise on consequences and his remarks on the topic in the *Summulae* are in many respects the most interesting of such medieval texts.

Albert of Saxony was once thought to have been a pupil of Buridan's, but this is now considered as highly unlikely, since they belonged to two different nations (Buridan to the Picardy nation and Albert to the English-German nation); as noted above, the most customary was for a pupil to be trained under a master of his own nation (often of his own home region). However, the influence of Buridan's doctrines over Albert is evident; in fact, Albert's work is often seen as a synthesis of Ockham's and Buridan's ideas (cf. [Biard, 2004, section 1]). This is to some extent true, but the importance of Albert as an original thinker should not be underestimated. His most important logical work is his *Perutilis Logica* (Very useful logic — [Albert of Saxony, 1988; Kann, 1993] for the second treatise[14]), where he deals extensively with properties of terms, in particular supposition, with consequences, fallacies, *insolubilia* and obligations — in sum, the traditional logical topics in the 14^{th} century. His treatise on obligations in the *Perutilis Logica* is one of the only three genuinely 'continental' treatises on obligations of this period (the others being overwhelmingly British or written under British influence — cf. [Braakhuis, 1993]). He also wrote a *Sophismata* and several question commentaries in logic (cf. [Biard, 2004, section 1]) (one of such sets of questions has received a modern edition — [Albert of Saxony, 2002]). We shall be interested in particular in his treatment of supposition, and, to some extent, his treatment of consequence.

William Buser may have been a pupil of Albert of Saxony (they were members of the same English-German nation in Paris), and in turn Thomas of Cleves and Marsilius of Inghen (who will be discussed below) were later pupils of William (cf. [Read, 1991, 71]). Besides these interesting relations of 'intellectual hereditariness', the importance of William Buser for the present purposes is mainly that he is the author of one of the only three continental treatises on obligations. Other than his treatise on obligationes, no other logical text by him is known (in fact the only other written record by him still extant is his last will — cf. [Kneepkens, 1993, 343]).

Thomas of Cleves is another still obscure Parisian figure of the mid-14^{th} century (for his biography, see [Bos and Read, 2001, 15–18]). He was a pupil of William Buser, and appears to have become a full master of arts in 1365, in Paris. One interesting aspect of his biography is the fact that he became the schoolmaster of the St. Stephen's Cathedral School in Vienna, which (as already mentioned) was the foundation for the University of Vienna to be re-founded in 1384, exemplifying thus the spread of the 'Buridanian' tradition in logic in Eastern Europe. As far as his writings are concerned, we have now recent editions of his treatise on concepts [Bos and Read, 2001] and a reconstruction of his *Logica* (in [Bos, 2004]). For the present purposes, his position with respect the fourth mode of personal supposition, i.e. collective supposition, will be particularly important in the section on supposition theory below.

Marsilius of Inghen, who was a few years younger than Albert of Saxony (born around 1340) and also a pupil of William Buser, had a decisive role in the establishment of the *via Buridanii* as one of the two main approaches to logic in the late 14^{th} and 15^{th} century. In particular, as already mentioned, he was one of the founders and many times the rector of the University of Heidelberg, again exemplifying the spread of Parisian logic in Eastern Europe. At a later stage of his life he eventually obtained his degree in theology, but for most of his career he was writing predominantly on logic, natural philosophy and metaphysics (see [Hoenen, 2001, section 1]). Noteworthy are his treatises on the properties of terms: on

[14] Selections from Albert's masterpiece are available in English translation by T. Parsons *et al.* at `http://www.humnet.ucla.edu/humnet/phil/faculty/tparsons/download/AlbertSL.pdf`

supposition, ampliation, appellation, restriction; and his treatises on obligations, insolubles, and consequences. Unfortunately, so far only his treatises on the properties of terms have received a modern edition [Bos, 1983]. Marsilius will be a central figure in our discussion of supposition below.

John of Holland is another interesting case of an author somehow belonging to both the Continental and the British traditions. While there is no conclusive evidence to the effect that he did study at Oxford, his familiarity with the works of some Oxford logicians such as Heytesbury and Bradwardine is an indication that this might have been the case (cf. [Bos, 1985, *14*]). In fact, little is known about him, but we do know that he studied in Prague[15], and was later to become Dean of the faculty of arts in Prague in 1369. His treatises on supposition, fallacies, obligation and *insolubilia* have received modern editions [John of Holland, 1985].

Peter of Ailly was a Parisian master who wrote his best known work in 1372, his *Concepts and Insolubles* [Ailly, 1980]. This work deals with mental language and in particular the signification of mental and spoken terms, and, as the title says, with *insolubles*. His definition of signification was to be very influential in the 15^{th} century. Although influential for subsequent developments, Ailly will not be treated in any of the analyses to follow, since his main contribution to logic concerns *insolubilia* and semantic notions such as that of signification, which for reasons of space will not be dealt with here.

Besides France, Britain, Central Europe and Italy, there was also vivid intellectual activity in Spain; indeed, a handful of universities were founded in Spain in the 14^{th} century. But contrasting with the 15^{th} and 16^{th} centuries, when Spain was to become one of the main centers for original work within the Scholastic tradition, we know of no Spanish logicians in the 14^{th} century having had the same influence and importance as later authors such as Domingo de Soto. Worth noticing, however, is that the (otherwise) famous St. Vincent Ferrer composed an interesting treatise on supposition around 1372 (edited in [Trentman, 1977]). Particularly significant is the fact that St. Vincent Ferrer went through his whole student career in his native Spain; so while he seemed to be acquainted with most of the important logical texts of the 14^{th} century, he was in practice outside the circle of influence of the main centers, and claimed that his main source of inspiration was St. Thomas Aquinas. St. Vincent will be briefly mentioned in the section on supposition below.

By contrast, an author such as Blaise of Parme indicates that, in Italy, the 'usual' 14^{th} century authors were indeed very influential, such as Ockham and Buridan. The logic taught at Italian universities then was referred to (and dismissed) by humanists such as Petrarca as 'Ockhamist logic', and Blaise is perhaps the most prominent example thereof. His only surviving logical text is a set of questions on Peter of Spain's *Tractatus* (edited recently by J. Biard and G. Federici Vescovini — [Blaise of Parme, 2001]), which is in many ways idiosyncratic for a 14^{th} century logical work in that it does not treat of supposition and other

[15]But remember that, unlike other Eastern European universities, Oxford logic was quite influential in Prague, so John's knowledge of British logic may have been acquired in Prague.

typical 14th century topics. It does have a section on consequence, but it does not consider for example the crucial material vs. formal distinction with respect to consequence. Blaise of Parme is, in any case, an essential author for the understanding of the development of logic in Italy.

John Dorp, working at the very end of the 14th century, is most famous for his commentary on Buridan's logic (John Dorp 1499), which in 1393 became required reading material for a student to obtain the degree of Bachelor of Arts in Paris (cf. [Lorenz, 1996, 148]). Dorp's commentary consisted of the main text from Buridan's *Summulae* (the parts supposedly taken from Peter of Spain, but with significant modifications), and Buridan's own commentary was replaced by Dorp's more concise commentary (this was indeed the form of the early printed editions of Buridan's *Summulae*, that is, in fact with Dorp's commentary — cf. [Klima, 2001, xxxii]). Dorp deals with and even solves many of the tensions that could be felt in Buridan's logical doctrines, for example with respect to the effect of the negation over the personal supposition of terms (cf. [Karger, 1993]) — his ingenious solution to this problem will be discussed in the section on supposition below.

1.3 Conclusion

From the foregoing considerations, the picture that emerges of the 14th century is of an extremely active period of intellectual and academic activity, in particular with respect to logic. Many were the authors involved in these activities, and many were their contributions to the field. We will now see that the result of all these activities was exceptionally sophisticated logical analysis.

2 SEMANTICS

2.1 Supposition

While at the beginning of the 14th century the concept of supposition and the doctrines built upon it were already respectable and mature elements of the terminist logic tradition, one can surely speak of a further development of these doctrines in the 14th century. The concept of supposition was one of the most important conceptual tools used in fields as wide-ranging as natural philosophy and theology; another typical place to capture the development of the concept of supposition and its uses is in the *sophismata* literature. But here, I will focus on treatises on supposition properly speaking.

Moreover, for reasons of space, the discussion to follow is not intended to be comprehensive in a historical sense: it is impossible to mention the doctrines held by every single significant author, or to discuss all the important aspects concerning the concept of supposition in this period. Rather, the discussion here is thematic in that it focuses on a few interesting conceptual developments concerning the notion of supposition, and particular authors are mentioned only insofar as they are representative of a given position.

In the post-Ockham and post-Buridan period, the development of supposition theories can be summarized as their successors essentially dealing with the 'loose ends' left by these earlier authors in their theories. While it is undeniable that Ockham as well as Buridan had constructed impressive (and mutually different) semantic systems where the notion of supposition was central, a few aspects and conclusions that could be drawn from their systems had not been discussed by them as thoroughly as one might wish. So several of the masters in the generations to follow saw it fit to draw and discuss some of these conclusions and to offer solutions to the difficulties emerging from Ockham's and Buridan's systems, as we shall see. (For a comprehensive and systematic approach to supposition theory, including the definition of its main concepts, I refer to T. Parson's piece in this volume.)

2.1.1 Simple supposition: yes or no?

One of the recurring debates concerning the concept of supposition in the 14^{th} century regarded simple supposition, more explicitly whether it should be included among the main kinds of supposition. Traditionally, there are three main kinds of supposition: personal, simple and material (such as in Peter of Spain and William of Sherwood). Personal supposition occurs when a term in a proposition stands for thing(s) that fall(s) under it. For example, if the term 'man' in a given proposition would stand for actual men, then it would have personal supposition. Material supposition occurs when a term in a proposition stands for a word (in particular for itself). Finally, simple supposition occurs when a term in a proposition stands for the corresponding universal, that is, in the case of 'man', the abstract universal 'manhood'.

Ockham notoriously denied existence to those 'universals' for which terms in simple supposition were said to stand, and contended that there is no such thing as a common nature that Plato and Socrates shared insofar as they were both men. The only thing common to Socrates and Plato in this respect is that the (mental, written or spoken) term 'man' can be correctly predicated of each of them. So, for Ockham, if simple supposition were to be the supposition for universals, it would at best be a completely idle concept, since there are no such things in his ontology. At this point, two options were open to Ockham: (i) either to exclude simple supposition from his theory of supposition altogether, (ii) or else to keep it but with a significant reformulation. Ockham opted for (ii): for him, simple supposition becomes the supposition for a mental term (a concept) (in chapter 68 of Ockham's *Summa Logicae*). One may wonder why he chose to maintain and reformulate simple supposition instead of simply getting rid of it (as Buridan later did), and a good guess would be that he did so for the sake of conservativeness and respect for the tradition.

By contrast, Buridan, who shared Ockham's denial of universals, opted for (i). For Buridan, it makes no sense to distinguish the supposition for mental terms (simple supposition for Ockham) from the supposition for written and/or spoken terms (material supposition for Ockham); as far as Buridan is concerned, they are both supposition for terms, and thus both material supposition (cf. chapter 4.3.2 of Buridan's *Summulae*). He added that, being the conventionalist about language that he was, he did not really care whether some people prefer to call the supposition for mental terms simple supposition [Buridan, 2001, 253]. But from the point of view of the theoretical simplicity advocated by Ockham himself, it would seem more reasonable not to multiply concepts and terms unnecessarily; if the concept of simple supposition becomes theoretically superfluous, one may as well get rid of it. So it would seem that Buridan's position was overall more coherent.

Another related point is that, while Ockham ascribed different kinds of supposition to terms in mental language as well (yielding a few counterintuitive results related to equivocation in mental language[16]), Buridan only allowed for personal supposition in mental language [Buridan, 2001, 522], whereas in written and spoken language both personal and material supposition could occur. In this respect too Buridan's position was more coherent, as argued in the literature (cf. [Spade, 1980b]).

However, this was not the end of the story. Authors of the following generations kept on debating whether simple supposition was indeed required or in fact superfluous for a theory of supposition. Most nominalists, i.e. those who followed Ockham and Buridan in their denial of the existence of universals, ultimately opted for simplicity and followed Buridan in his exclusion of simple supposition. This was in particular the case of Marsilius of Inghen. Moreover, in the second half of the 14$^{\text{th}}$ century, with the revival of realism about universals with Wyclif and his followers, simple supposition became again an important theoretical tool within this trend.[17] But interestingly, there was an author who followed Ockham both in his ontology (denial of universals) and in his inclusion of simple supposition in the theory of supposition — a position that, as already argued, is not entirely straightforward. This author boldly holding Ockham's banner was Albert of Saxony (cf. [Berger, 1991]).

Moreover, Albert's position with respect to simple supposition and to the supposition of mental terms is also rather idiosyncratic: he accepted personal, simple and material supposition in spoken and written language, but only personal and

[16]Cf. [Spade, 1980b; Normore, 1997]. The problem is essentially the following: ambiguity in spoken/written language (equivocation/amphiboly) is accounted for by Ockham in terms of a one-many mapping between spoken/written expressions and mental expressions (an ambiguous expression is one corresponding to more than one mental expression). However, how can he account for ambiguity in mental language if there is no super-mental level to play the role that the mental level plays for the spoken/written level?

[17]The realist tradition in 14$^{\text{th}}$ century logic is still largely understudied. Its first exponent was, as already mentioned, Burley, who for example maintained the traditional definition of simple supposition as the supposition for a universal. In the second half of the century, realism underwent a revival, mainly with Wyclif and his followers (Robert Alyington, Johannes Sharpe — see the entries on these authors in the *Stanford Encyclopedia of Philosophy*, and a whole issue of *Vivarium* 43(1) (2005) dedicated to the realists).

material supposition in mental language.[18] This is rather surprising, since one could expect him either to side with Ockham in maintaining that all the different kinds of supposition that occur in spoken and written language also occur in mental language, or to side with Buridan in maintaining that there is only personal supposition in mental language. However, there is a striking coherence in his position, as I shall argue.

The first element to be taken into account is that his formulations of these three modes of supposition differ slightly but significantly from those of Ockham. Personal supposition is defined roughly in the same way as by Ockham, as the supposition for the things that the term signifies; material supposition, by contrast, is defined as the supposition for **terms** but not necessarily spoken or written terms, as it is by Ockham. According to Albert's definition, a term having material supposition can supposit for itself or for a similar term (be they written, spoken or mental). It is clear thus that, in mental language, if a given mental term supposits for itself or for another term (mental or otherwise), Albert's definition allows for it to be a case of material supposition, whereas for Ockham it had to be a case of simple supposition, since his definition of material supposition necessitated that the supposition be for a spoken or written term. Albert, however, with the definitions of personal and material suppositions thus stated, could already account for all the different kinds of supposition in mental language, making thus simple supposition in the mental realm superfluous.

He was still left with the cases of terms in spoken or written language suppositing for mental terms, which were presumably excluded from his definition of material supposition. Following Ockham and against Buridan, he prefers to keep such cases in a category of their own — perhaps in order to maintain a more fine-grained and discriminating taxonomy — a category in which the supposition of a term for itself could not occur. From this point of view, what characterizes and gives unity to the concept of material supposition is above all the possibility of supposition of a term **for itself** (a spoken term for a spoken term, a written term for a written term, and a mental term for a mental term), whereas simple supposition would deal with the 'left-over' cases where this could not occur, namely the cases where a spoken or written term was explicitly meant to supposit for a concept.

As for the risk of provoking equivocation in mental language by the ascription of different kinds of supposition to mental terms, such as in Ockham, Albert avoids this problem with another feature of his logical system: unlike most of his immediate predecessors such as Ockham and Buridan and his own contemporaries, Albert denies that some propositions must be distinguished, that is, that they are ambiguous.[19] Now, if there is no equivocation with respect to propositions in written and spoken language, where there are three kinds of supposition, then a

[18] "[...] a mental term cannot have simple supposition, but only material or personal supposition." (Translation by Parsons *et al*, p. 12, on the website mentioned above).

[19] See [Ashworth, 1991, 156]. Albert contends that the propositions that must be distinguished according to others in fact correspond to the conjunction or disjunction (depending on the case) of each of their possible readings.

fortiori there will not be this kind of equivocation in mental language either.

In sum, Albert's preservation of simple supposition seems to be better motivated than Ockham's, and that on purely logical/semantic grounds — that is, it is not motivated by ontological considerations (i.e. realism about universals) as it is in the case of Wyclif and Paul of Venice (see chap. 2 of Paul's treatise on supposition in his *Logica Magna* — [Paul of Venice, 1971]).[20] Moreover, thanks to the position he holds with respect to propositions in general, i.e. that they should not be distinguished, he escapes the risk of introducing equivocation in mental language even though he accepts different kinds of supposition in mental language.

However, the simplicity of Buridan's doctrines, namely the exclusion of simple supposition from spoken and written language and of all kinds of supposition except for personal supposition from mental language, remained very appealing; it is not by chance that the vast majority of nominalists sided with Buridan and not with Albert, and that virtually all other upholders of simple supposition were essentially motivated by ontological considerations.

2.1.2 A fourth mode of personal supposition?

As shown in T. Parson's contribution to this volume, in the 14$^{\text{th}}$ century the modes of personal supposition were virtually always associated to relations of ascent and descent between propositions and the corresponding singular propositions. The *descensii ad inferiora* are 'certain types of inferences in which the common terms of which the mode of supposition is being characterized is replaced by singular terms falling under it, appearing in either nominal or propositional conjunctions or disjunctions.' [Klima and Sandu, 1990, 177]. Singular terms are proper names or, as most frequent in the case of descents, demonstrative pronouns (usually accompanied by the appropriate common term). For example, in the case of 'Every man is an animal', if it is the supposition of the term 'man' that is at stake, then the singular propositions in question would be of the form 'This man is an animal', 'That man is an animal' etc. (pointing at each individual falling under the term 'man', i.e. each man), and the question is then how the descent to these singular propositions can be made, i.e. either nominally or propositionally, and either conjunctively or disjunctively.

Let the basic form of categorical propositions be represented as ΦA is ΦB[21], where A and B are terms and Φ stands for any syncategorematic expression (such as 'Every', 'Some', 'No' etc.) or the absence thereof. The different kinds of propositional descent can then be characterized as:

[20] Notice though that in his *Logica Parva* Paul of Venice only recognizes personal and material supposition.

[21] In fact, most often than not there was no quantifying expression preceding the predicate. However, in a few cases there was such an expression (such as in an example to be discussed below, 'Socrates differs from every man'), and therefore for the sake of generality I introduce a place-holder for a quantifying expression also in front of the predicate. See also [Karger, 1993] for more of such examples.

> Propositional conjunctive descent for A (from 'ΦA is ΦB' one can descent to 'This A is ΦB and that A is ΦB and ...'
>
> Propositional disjunctive descent for A (from 'ΦA is ΦB' one can descent to 'This A is ΦB or that A is ΦB or ...'
>
> Nominal disjunctive descent for A (from 'ΦA is ΦB' one can descent to 'This A or that A or ... is ΦB.'
>
> Nominal conjunctive descent for A (from 'ΦA is ΦB' one can descent to 'This A and that A and ... is ΦB.'

These definitions apply, *mutatis mutandi*, to the predicates of a proposition as well.

With respect to the main propositional forms (A, E, I, O), it is evident that at least three types of descent are required to account for the supposition of their terms:

(A) Every S is P (Propositional conjunctive descent is possible for the subject and nominal disjunctive descent is possible for the predicate: 'This S is P and that S is P and ...' and 'Every S is this P or that P or etc.'

(E) No S is P (Propositional conjunctive descent is possible for both subject and predicate: 'This S is not P and that S is not P and etc.' and 'No S is this P and no S is that P and ...'

(i) Some S is P (Propositional disjunctive descent is possible for both subject and predicate: 'This S is P or that S is P or etc.' and 'Some S is this P or some S is that P or ...'

(O) Some S is not P (Propositional disjunctive descent is possible for the subject and propositional conjunctive descent is possible for the predicate: 'This S is not P or that S is not P or etc.' and 'Some S is not this P and some S is not that P and ...'

Notice that, whenever (conjunctive or disjunctive) propositional descent is possible, so is the corresponding nominal descent (since propositional descent corresponds to wider scope and thus to a stronger reading of the proposition), and whenever conjunctive descent is possible, so is disjunctive descent (due to the logical properties of conjunctions and disjunctions).[22] We thus have:

Propositional conjunctive descent is possible	$\Rightarrow$	Propositional disjunctive descent is possible Nominal conjunctive descent is possible Nominal disjunctive descent is possible
Propositional disjunctive descent is possible	$\Rightarrow$	Nominal disjunctive descent is possible
Nominal conjunctive descent is possible	$\Rightarrow$	Nominal disjunctive descent is possible

[22]Naturally, these two claims ought to receive a formal proof, but I take them to be sufficiently intuitive so that these proofs are not necessary in the present context.

In the early 14^{th} century, three of these patterns of descent were associated to modes of personal supposition:

	Conjunctive	Disjunctive
Nominal	XXXXXXX	Merely confused
Propositional	Confused and distributive	Determinate

Naturally, since nominal disjunctive descent is always possible, merely confused supposition is not defined as the supposition of a term whenever nominal disjunctive descent is possible (otherwise all terms would have merely confused supposition); rather, it is (usually) defined as the supposition of a term when **only** nominal disjunctive supposition is possible. Similarly, determinate supposition is not defined as the supposition of a term whenever propositional disjunctive descent is possible; rather, it is defined as the supposition of a term when propositional disjunctive descent is possible but not propositional conjunctive descent (otherwise the categories of confused and distributive, and determinate supposition would overlap — cf. [Read, 1991b, 74]).

It is easy to see why at first no interest was paid to nominal conjunctive descent: it does not correspond to any of the terms in the four traditional categorical propositional forms. Accordingly, since it was tacitly assumed that all other propositions could be in some way or another reduced to one of these four forms, it was thought that only those three kinds of personal supposition were required to account for the supposition of terms in propositions.

But even at its early stages, the suppositional framework was also applied to cases other that these four traditional propositional forms. For example, in the case of exceptive propositions of the form 'Only S is P', it was recognized that S has merely confused supposition (cf. [Marsilius of Inghen, 1983, 59] — see below). In other cases, it was necessary to bend the concepts of the modes of personal to such an extent (for instance, by rephrasing the original sentence so that it would fit into one of the recognized propositional forms, but with rather implausible results) that one cannot help but wonder whether supposition theory with only these three modes of personal supposition was complete in the sense of being able to account for all, or at least most, propositions. But, since in most formulations of the modes of personal supposition merely confused supposition was in practice a 'catch-all' category (cf. [Read, 1991b, 75]) (since nominal disjunctive descent is always possible, as noted above), in the end no semantic phenomenon was excluded from the taxonomy with these three modes of personal supposition.

But at some point in the first half of the 14^{th} century, some authors were led to acknowledge at least the logical possibility of a fourth mode of descent, namely nominal conjunctive descent. According to [Read, 1991b, 74], the first mention to nominal conjunctive descent that we know of is to be found in Thomas Maulevelt's *De Suppositionibus*: there, Maulevelt says that when merely confused supposition occurs, nominal disjunctive **as well** as nominal conjunctive descent are possible.

Maulevelt's example of the supposition of a term which is best accounted for by nominal conjunctive descent instead of nominal disjunctive descent is 'Socrates differs from every man'. According to Maulevelt, the descent allowed for the term 'man' giving the intended meaning of the proposition is 'Socrates differs from this man and that man and…', and not 'Socrates differs from this man or that man or…' (although in principle, the second descent should also be allowed, since nominal conjunction should imply the corresponding nominal disjunction). In other words, according to Maulevelt, nominal conjunctive descent is not only a logical possibility; it is also the actual descent required by some real cases.

Albert of Saxony, writing after Maulevelt, explicitly rejects his analysis of 'Socrates differs from every man' as requiring nominal conjunctive descent; according to him, 'man' in this case should have determinate and not merely confused supposition (cf. [Read, 1991b, 80]). Moreover, Albert criticizes the inclusion of the clause for nominal conjunctive descent in the definition of merely confused supposition, thus implicitly defending the idea that nominal conjunctive descent is not a phenomenon that needs to be taken into account in the definition of modes of personal supposition (being at best only a logical possibility). Obviously, for those who reject nominal conjunctive descent as a relevant phenomenon, such as Albert of Saxony, three modes of personal supposition provide a complete picture of the (personal) supposition of all terms not only in the sense that all cases were taken into account (which happens anyway if merely confused supposition is defined as a catch-all clause), but also in the sense that these three categories are sufficiently fine-grained and discriminating so as to give a coherent grouping of these semantic phenomena, since they correspond to the three relevant kinds of descent.

Among those who recognize nominal conjunctive descent as an important phenomenon, two positions are possible; either to associate nominal conjunctive descent to merely confused supposition, together with nominal disjunctive descent (as did Maulevelt and later Paul of Venice — cf. [Read, 1991a, 53]), yielding thus a rather heterogeneous notion of merely confused supposition as a 'miscellaneous' category; or to associate nominal conjunctive descent to a fourth mode of personal supposition altogether. According to our current state of knowledge, while the notion of a *descensus copulatim* (nominal conjunctive supposition) seems to have been familiar in Paris and Oxford in the 1350s and 1360s, the first to associate a fourth mode of supposition to nominal conjunctive descent seems to have been Thomas of Cleves (cf. [Read, 1991a; 1991b]) in his *Suppositiones* written in the first half of the 1370s in Paris[23], before he moved on to be the rector of St. Stephen's cathedral school in Vienna This fourth mode of personal supposition became known as *collective supposition*.

Some of the examples usually associated with nominal conjunctive descent (from [Read, 1991a]) were: 'You are not every man', 'No animal is every man', and 'Some penny will be seen by every man' (in all three cases with respect to 'man'). But

[23]At this point, no surviving manuscript of Thomas of Cleves' *Suppositiones* has been located; but a reconstruction of it has been made recently (in [Bos, 2004]) on the basis of references made to this work in other texts.

in such cases, the opponents of *descensus copulatim* usually proceeded by showing that, if this kind of descent was possible at all, so were other kinds of descent, and therefore the supposition of such terms could be classified among the three usual modes of personal supposition (since, presumably, the category of collective supposition would be defined as the cases where **only** nominal conjunctive descent would be possible).[24] Their usual strategy consisted of an appeal to Ockham's Razor, to the effect that if the job of accounting for the different modes of personal supposition could be done with only three categories, then there was no need to posit a fourth one.[25]

But a fourth type of examples poses more serious difficulties. It is epitomized by the proposition 'All the apostles of God are twelve'. The nominal conjunctive descent under 'the apostles of God' is a very natural one indeed, either with demonstrative pronouns or even with proper names: 'Peter and James and John and Judas etc. are twelve'. But nominal disjunctive descent seems not to be allowed, since it is not of each of them that the predicate 'twelve' can be predicated, but rather of all of them taken collectively.[26] In other words, only nominal conjunctive descent seems to be allowed, and if this kind of descent is not accounted for in the definitions of the (three) modes of personal supposition, then no mode of personal supposition can be assigned to 'the apostles of God'.

There were different replies from those who rejected the notion of a fourth mode of personal supposition and nominal conjunctive descent, with various degrees of plausibility. Some proposed to treat 'all the apostles of God' as a singular term used to refer to the collection of apostles of God, having thus discrete supposition; others implausibly rephrased the proposition as 'All all of the apostles of God are twelve' and attributed confused and distributive supposition to the subject (cf. Read 1991a, 80).

It is worth noticing that, in his 1372 treatise on supposition (thus roughly at the same time as Thomas of Cleves was presumably writing his treatise on supposition), Vincent Ferrer advocates a rather idiosyncratic position with respect to the modes of personal supposition: he also recognizes three modes of common personal supposition, but not the three traditional ones. According to him, com-

[24]That poses a logical problem since, according to the usual laws for conjunction and disjunction, whenever nominal conjunctive descent is possible, so is nominal disjunctive descent, as indeed nominal disjunctive descent is always possible. So arguing that, in such cases what we have are cases of merely confused supposition because nominal disjunctive descent is possible is in some sense fallacious, since this holds of the other modes of personal supposition as well. In this sense, collective supposition should be defined as the cases where nominal conjunctive and nominal disjunctive descents are possible, but no propositional descent is possible. However, as we shall see shortly, there are cases where nominal conjunctive descent seems to be possible but **not** nominal disjunctive descent, which would violate the usual rules for conjunction and disjunction.

[25]An anonymous author of a commentary on Marsilius' Parva Logicalia says: "everything can be explained without positing collective supposition", and "one should not multiply entities without necessity" (cf. [Read, 1991a, 79]).

[26]Indeed, this seems to indicate that the logical behavior of nominal conjunctive and disjunctive descents, or in any case the semantics of collective nouns, is more complicated than what the mere truth-functional properties of conjunction and disjunction can account for.

mon personal supposition is subdivided into determinate, distributive confused, and collective confused (Trentman 1977, 134); in other words, he does recognize Thomas of Cleves's 'fourth' mode of personal supposition, but to him this is not a fourth but rather a third mode, because he does not recognize merely confused personal supposition. While this may seem an awkward position at first sight, it is not altogether implausible considering that, for Vincent Ferrer, and following very early (12th and 13th century) notions of supposition, it is only the **subject** of a categorical proposition that has supposition, not the predicate (cf. [Trentman, 1977, 89-92]). Given that, usually, merely confused personal supposition concerns the supposition of the predicate (in particular in universal affirmative propositions), if one does not attribute supposition to the predicate, then there may seem to be no need to recognize merely confused personal supposition.

But the fact that Ferrer recognized collective supposition poses a historiographic problem. While there is a clear line of continuity between the other authors mentioned so far (Maulfelt, Albert of Saxony, Thomas of Cleves, all roughly belonging to the same Parisian nominalist tradition), Vincent Ferrer was, as already mentioned, educated in Spain, and saw himself as belonging to a Thomist tradition, thus completely out of the circle of influence of the nominalist tradition. Read [1991a, 74] argues that Ferrer's treatise 'does not read like that of an author inventing an original theory'; that is, presumably collective supposition was already a recognized mode of supposition in the tradition within which he was schooled. We could thus have two independent 'inventions' of the concept of collective supposition. But our knowledge of the 14th century Spanish logical tradition is as of yet still insufficient in order to tell us whether the concept of collective supposition was indeed independently developed within this tradition (which seems unlikely, given the coincidence in terminology), or whether there were earlier points of contact between the two traditions in such a way that there might have been influence in one direction or in the other (or both).

In sum, the debate concerning nominal conjunctive descent and a fourth mode of supposition was a heated one in the second half of the 14th century. While it is fair to say that the majority of the authors preferred to maintain the traditional scheme with three modes of common personal supposition, for the sake of theoretical parsimony and probably also out of respect for the tradition, many authors nevertheless recognized the fourth kind of descent. But all in all, considered from a systematic point of view, it seems that supposition theory containing four modes of personal supposition is indeed a more sophisticated version of the traditional theory: it respects the logical symmetry of nominal and propositional descent, and it allows for a very intuitive account of the semantics of some terms (even if some of them can also be accounted for with only the three traditional kinds of descent).

2.1.3 Different modes of supposition also for material supposition

In the traditional formulations of theories of supposition, in particular with the 13th century summulists (William of Sherwood, Peter of Spain), but also with authors of the first half of the 14th century such as Ockham and Buridan, there is a remarkable asymmetry between personal supposition on the one hand and simple/material supposition on the other hand. While personal supposition is further divided into the different modes of personal supposition, as just discussed, the same does not occur with simple and material supposition.

The different modes of personal supposition concern the fact that, in a given proposition, we do not always wish to talk about every single object that falls under a given term; for example, when using the term 'man', we sometimes wish to talk about all men, but sometimes only about some of them. In anachronistic terms, the modes of personal supposition are the medieval counterpart of theories of quantification, and, roughly speaking, allow us to determine how many individuals are being 'talked about' with a given proposition.

At first sight, it may seem natural that personal supposition should have the privilege of being further subdivided, while simple and material supposition should not: given that simple and material supposition are not the significative kinds of supposition, why should we quantify over things that do not fall under a given term? What is the need to quantify over the word that a term stands for if it has material supposition, or, even more awkwardly, over the universal that it stands for if it has simple supposition? For example, if the term 'man' has simple supposition, then there is only one thing it can stand for, namely the unique universal 'manhood', so there is no point in discussing how many universals the term 'man' stands for if it has simple supposition.

But further reflection quickly shows that there may be indeed a point in further distinguishing the kinds of supposition other than personal supposition. In the 14th century, material supposition is usually not defined exclusively by the fact that the term in question supposits for itself; it may also supposit for other terms, either its equiform occurrences or even non-equiform terms (for example, a term in a given case — nominative, accusative etc. — may supposit for the same term but in a different case, thus generally not being equiform). So it is clear that, just as much as with personal supposition, there is a rather wide range of objects (different occurrences of different words) that a given term can stand for if it has material supposition (and that has led some to conclude that material supposition is such a wide-ranging concept that it winds up becoming ineffective — cf. [Read, 1999]).

As for simple supposition, for as long as it was exclusively the supposition for the unique universal corresponding to a given term, there was indeed not much point in further distinguishing different kinds of simple supposition with respect to quantification — notice that Burley, for example, distinguished simple supposition into absolute and compared simple supposition [Burley, 2000, 92], but this subdivision concerned different ways in which a term had simple supposition, and not the number of things being talked about. However, with Ockham's reformulation of the notion of simple supposition as the (non-significative) supposition for concepts, this issue acquired an entirely different shape. For Ockham (in his mature theory) concepts are simply the very acts of conceiving (things) by a given intellect; that is, they are (temporary) attributes of the intellect. In other words, concepts, just as much as spoken words, have different temporal occurrences: the concept *man* does not correspond to one single concept that perdures in the intellect, but rather to each and every occurrence of this concept in the intellect every time it conceives of men, just as much as the spoken word 'man' corresponds to each of its temporal occurrences.

Hence, if both simple supposition and material supposition concern a wide range of objects for which a term can supposit (different occurrences of a given concept or of a given word), just as much as personal supposition, then it seems counterintuitive that only personal supposition should receive further distinctions. In fact, Ockham's and Buridan's nominalism seems almost to require these distinctions; for both of them, all that exists are the different actual occurrences of written, spoken and mental terms – that is, what we now call 'tokens', but not the corresponding types. For this reason, also in simple and material supposition it seems necessary to consider the number of entities (concepts, words) being talked about with a given proposition, just as much as with personal supposition.

Buridan seems to be well aware of the fact that one can quantify over the different occurrences of a term. Take this passage, from the first treatise of the *Summulae*:

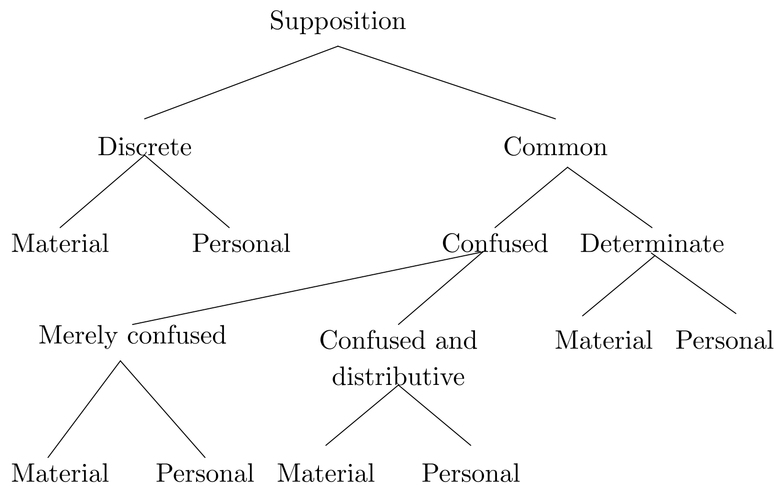
> Next, we should say that if the subjects in the aforementioned propositions supposit materially, then the proposition "[A] man is a species" is [...] indefinite, for the term 'man' is not to be understood as suppositing only for itself, but indifferently for other similar terms as well [...]. And in this way "Every man is a species" would be universal, and "Some man is a species", i.e. "Some term man is a species" would be particular, and "This man is a species", i.e. "This term man is a species", would be singular. [Buridan, 2001, 92][27]

But Buridan does not go as far as actually applying the different modes of personal supposition to material supposition, even though from saying that a proposition is universal to acknowledging that its subject has confused and distributive supposition is but a small step (as it was widely recognized that the subject of an affirmative universal proposition has confused and distributive supposition, and similarly that the subject of a particular or indefinite supposition has determinate supposition and the subject of a singular proposition has discrete supposition).

To my knowledge, the first author to have made the small but significant step of introducing the distinctions originally pertaining only to personal supposition also to other kinds of supposition is Buridan's follower, Marsilius of Inghen. In his treatise on supposition [Marsilius of Inghen, 1983, pp. 52-97], Marsilius presents a compelling and elegant way of structuring the different kinds of supposition and their subdivisions. What is most remarkable about his doctrine is that, unlike his predecessors, who began by dividing proper supposition into personal, simple and material, Marsilius actually begins by the division of what were traditionally the

[27] I am indebted to Gyula Klima for having drawn my attention to this passage.

different modes of personal supposition. This is a sensible move, considering that the subdivisions of the different modes of supposition are considerably more complicated than the division between personal and material supposition (recall that Marsilius, following Buridan, does not recognize simple supposition as a class of its own and views the supposition for mental terms as a kind of material supposition). Here is a tree representing his divisions:



We thus have four kinds of material supposition, just as much as four kinds of personal supposition. Thereby, Marsilius is able to present a more fine-grained account of the phenomenon of terms standing for other terms. Indeed, a term may stand for:

- one specific non-ultimate significate[28] only – discrete material supposition, for example if I say 'This 'man' is written in red' pointing at a specific occurrence of the word 'man' (cf. [Marsilius of Inghen, 1983, 55]);
- any non-ultimate sigificate in a disjunctive way — determinate material supposition, for example if I say 'Man is written on this page' meaning that there is at least one occurrence of the word 'man' in a given page (cf. [Marsilius of Inghen, 1983, 57]);
- any non-ultimate significate with disjunction of the term — merely confused material supposition, for example with 'Only man is a monosyllabic word', from which follows 'Only this [occurrence of] man or that [occurrence of] man etc. is a monosyllabic word' (cf. [Marsilius of Inghen, 1983, 59]);

[28]For Marsilius, following Buridan, the ultimate significates of terms are the things that fall under them, such as men for 'man', concepts for 'concept' etc., and the non-ultimate sigificates of terms are the things that they also signify — the corresponding mental, spoken and written terms — but not ultimately.

- every non-ultimate significate in a conjunctive way — confused distributive material supposition, for example with 'Every man is a monosyllabic word', meaning that every single occurrence of 'man' is monosyllabic (cf. [Marsilius of Inghen, 1983, 59]).

Indeed, in these divisions we see the recognition of individual concepts, inscriptions and utterances as legitimate members of the ontology (something that was already crucial for Buridan), and the possibility of talking about them in a much more refined way. With these divisions, we can attribute a certain property to one specific occurrence of a term, to some or even to all its occurrences; in other words, here we certainly have a conceptual predecessor of the important token-type distinction, which was to be fully developed only in the 20$^{\text{th}}$ century.

Marsilius's reformulation (or, perhaps better put, improvement) of supposition theory so as to attribute the so-called modes of personal supposition to material supposition as well seems to have become the standard practice in the 15$^{\text{th}}$ century. It is symptomatic that Paul of Venice, in his very influential *Logica Parva* [Paul of Venice, 1984, 147], also adopted these distinctions of modes of supposition for material supposition.

2.1.4 Problems and solutions for the negation

Both in Buridan's treatise on supposition and in the part of Ockham's *Summa Logicae* dedicated to supposition (the last chapters of Part I), one of the main topics are the syntactic rules determining which mode of (personal) supposition a term has on the basis of the syncategorematic terms present in a proposition (quantifying terms such as 'some', 'every', negating terms etc.) and word order. We learn for example that an affirmative sign of universality ('every') causes the term immediately following it to have confused and distributive supposition, and that it causes a term mediately (i.e. not immediately) following it to have merely confused supposition (so in 'Every man is an animal', the subject, which follows 'every' immediately, has confused and distributive supposition, while the predicate, which follows 'every' mediately, has merely confused supposition). We also learn that a negative sign of universality ('no') causes all terms following it (immediately and mediately) to have confused and distributive supposition, and that when the negation is placed relative to the copula (as in 'A man is not a stone'), it causes the predicate to have confused and distributive supposition. As for determinate supposition, it occurs when a term immediately follows a sign of particularity ('some') or when it does not follow any syncategorematic term (as in 'a man is an animal').

But matters become significantly more complicated once one departs from the four traditional propositional forms ('Every A is B', 'Some A is B', 'No A is B' and 'Some A is not B'), especially when iteration of syncategorematic terms occur. As argued by Parsons in [Parsons, 1997] and in his piece in this volume, in the 14$^{\text{th}}$ century it became current to classify the modes of personal supposition globally rather than locally, as had been done in the 13$^{\text{th}}$ century — that is, taking into account the whole propositional context, and not only the syncategorematic term immediately preceding a given term. For this reason, the occurrence of several syncategorematic terms all having the same categorematic terms under their scope posed the problem of the effect of those embedded syncategorematic terms over each other with respect to the (personal[29]) supposition of the categorematic terms in question.

In particular, the treatment of the negation requires a great deal of ingenuity. The basic problem is: what is the effect of a negating term over the supposition of a term which, if the negating term was removed, would have such-and-such personal supposition in a given proposition? In other words, rules determining the kind of supposition that a term would have if a negation is added to the proposition where it stands are required for all three cases, namely if the term in the original proposition had determinate, confused and distributive or merely confused supposition. And this is where the issue arises.

Buridan, for example, offers an explicit rule concerning the effect of the negation over a term that, without the negation, would have determinate or merely confused supposition: *A negating negation distributes every common term following it that without it would not be distributive and does not distribute anything that precedes it.* [Buridan, 2001, 269]

That means that, if in a proposition $\mathbf{P}$, a term A has determinate supposition, and if a negation is added to $\mathbf{P}$ (yielding $\mathbf{P}^*$) in such a way that A follows the negation (immediate or mediately), then A will have confused and distributive supposition. For the purposes of clarity in the exposition, let me introduce a few notations in order to express this rule more precisely:

$\mathrm{Det}(A)_P$	$\Leftrightarrow$	The term A has determinate supposition in proposition P.
$\mathrm{Dist}(A)_P$	$\Leftrightarrow$	The term A has confused and determinate supposition in proposition P.
$\mathrm{Conf}(A)_P$	$\Leftrightarrow$	The term A has merely confused supposition in proposition P.
$\langle \sim, A\rangle_P$	$\Leftrightarrow$	The negation is followed by term A in proposition P.

Buridan's rule can then be formulated as follows:

Rule 1 $\quad \mathrm{Det}(A)_P \& \langle \sim, A\rangle_{P^*} \rightarrow \mathrm{Dist}(A)_{P^*}$

Rule 2 $\quad \mathrm{Conf}(A)_P \& \langle \sim, A\rangle_{P^*} \rightarrow \mathrm{Dist}(A)_{P^*}$

There is, however, a serious problem concerning the effect of a negating sign upon a term that, without the negation, would have confused and distributive supposition. Given the structure of the theory, it seems at first sight impossible to provide a general rule for the negation and for confused and distributive supposition, for the following reason. Consider the four traditional kinds of categorical propositions:

[29]Or material supposition, if one follows Marsilius of Inghen in applying the traditional modes of personal supposition also to material supposition.

Some A is B (1)	$\xrightarrow{\text{is the contradictory of}}$	No A is B (2)	(contradiction 1)
Every A is B (3)	$\xrightarrow{\text{is the contradictory of}}$	Some A is not B (4)	(contradiction 2)

'Some A is B' (1) should be equivalent to 'Not: No A is B' ($2'$) (the contradictory of (2)) and 'Every A is B' (3) should be equivalent to 'Not: Some A is not B' ($4'$) (the contradictory of (4)). If these equivalences hold, then the supposition of the terms in (1) and ($2'$) should be the same: A and B have determinate supposition in (1), so they should have the same kind of supposition in ($2'$).

For this to happen, the effect of the negation in ($2'$) should be to turn the confused and distributive supposition of A and B in 'No A is B' into determinate supposition. This is indeed the rule proposed by Ockham: *Nevertheless, it should be noted that the aforementioned rules hold only in the case where the term in question would not stand confusedly and distributively if the negation sign or the relevant verb or name were taken away. For if the term were to stand confusedly and distributively when one of these expressions [negation sign] were taken away, then with the addition of such an expression it would stand determinately.* [Ockham, 1998, 214]

Ockham's rule can be formulated as follows:

Rule 3o $\quad \mathrm{Dist}(A)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Det}(A)_{P^*}$

But what about the equivalence between (3) and ($4'$)? In (3) A has confused and distributive supposition and B has merely confused supposition. So the same should occur in ($4'$). However, in 'Some A is not B', A has determinate supposition and B has confused and distributive supposition. According to rule 1, the negation would make A have confused and distributive supposition in ($4'$), that is, the same supposition of A in (3) (so far, so good). But what about B? According to the rule proposed by Ockham, since it has confused and distributive supposition in 'Some A is not B', it would have **determinate** supposition in ($4'$), under the effect of the negation. But in fact it ought to have **merely confused** supposition, because of the equivalence between (3) and ($4'$). So the rule stated by Ockham does not safeguard this equivalence.

Buridan, on the other hand, presents a rule that does safeguard the equivalence between (3) and ($4'$): *A common term is confused nondistributively by two distributive [parts of speech] preceding it, either of which would distribute it without the other.* [Buridan, 2001, 275]

Buridan's rule can be formulated as follows:

Rule 3b $\quad \mathrm{Dist}(A)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Conf}(A)_{P^*}$

That is, under the effect of two negations, B in ($4'$) would have merely confused supposition, which is the desired result. But then the equivalence between (1) and

$(2')$ would no longer be preserved: B would have merely confused supposition in $(2')$, whereas it ought to have determinate supposition, as in (1).

Thus, neither of these rules is able to preserve both equivalences: thus formulated, a rule concerning the effect of the negation upon a term originally with confused and distributive supposition either preserves the equivalence between (1) and $(2')$ (Ockham's rule) or it preserves the equivalence between (3) and $(4')$ (Buridan's rule). This is due to the following asymmetry: in the case of contradiction 1, the opposition in the supposition of B in each proposition is between determinate supposition and confused and distributive supposition, whereas in contradiction 2 the same opposition is between merely confused supposition and confused and distributive supposition. Therefore, it would seem impossible to provide a homogeneous account of the effect of the negation (or other distributive term) upon terms with confused and distributive supposition (for a systematic approach to this problem, see part 8 of Parson's contribution to this volume).

However (and fortunately for the general robustness of supposition theory as a semantic framework), later masters were well aware of this difficulty and proposed ways to deal with it. Already at the end of the 14^{th} century, in his popular commentary to Buridan's *Summulae*, John Dorp [1499] proposed a method to determine the supposition of terms following a negation based on the idea that the proposition should be rephrased in such a way that the negation would be all the way at the end of the proposition, in which case the usual (i.e. positive) rules for the determination of the personal supposition of a term could be applied. On this approach, '[t]he problem of assigning the mode of supposition to a term following a negation becomes that of determining a procedure for bringing negative sentences into a non-ordinary form [one where the negation only precedes the verb] such that the mode of supposition of each term remains unchanged.' [Karger, 1993, 419]

On the basis of Dorp's examples, Karger proposes a reconstruction of what this procedure would be like, essentially based in the idea of bringing the negation towards the end of the proposition, immediately before the verb, and introducing universally quantifying signs (to recover the distributive effect of the negation) where there was none, and deleting such universally quantifying terms previously present. Here is an example (cf. [Karger, 1993, 419], and [Read, 1991a, fn. 8] for Dorp's text):

> *Nullum animal omnis homo est* (No animal is every man) is rephrased as
> *Omne animal homo non est* (Every animal a man is not).

Nullum is replaced by *Omne*, *omnis* preceding *homo* in the original proposition is deleted, and the negation comes to be followed only by the verb *est*. Now, it is clear that *animal* has confused and distributive supposition (which was clear already in the original proposition), but moreover it becomes apparent that *homo* has merely confused supposition (as it follows mediately the universal sign), whereas in the original proposition it would have been unclear (from 3o and 3b alone) whether it had determinate or merely confused supposition.

A different approach to the same issue can be found in the writings of the early 17^{th} century philosopher and theologian John of St. Thomas (admittedly a few centuries off our period); there, one finds a precise account of the effect of distributive terms (the negation in particular) upon terms already having confused and distributive supposition. For this purpose, one has to consider the whole propositional context, i.e. the supposition of the other term in the proposition. Here is how John formulates it: *If two universal signs simultaneously affect the same term, then you must see how it remains after the first negation or universal sign is removed; and if it remains distributive with reference to a term having determinate supposition, then it originally had confused supposition; if however the term remains distributive with reference to a term having confused supposition, it originally was determinate.* [John of St. Thomas, 1955, 69]

Here are his own examples: 'For example, if I said, *No man is not an animal*, then when the first negative, i.e. the *no*, is taken away, *animal* becomes distributive with reference to *man*, which is determinate. Thus originally *animal* had confused supposition. However, if I said, *Not every man is an animal*, then when I take the *not* away, *man* becomes distributive with reference to *animal* which is confused. And thus *man* originally had determinate supposition.' [John of St. Thomas, 1955, 69]

Making use of the symbolism introduced here, these rules can be formulated as:

Rule 3o$'$ $\mathrm{Dist}(A)_P \& \ \mathrm{Conf}(B)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Det}(A)_{P^*}$

Rule 3b$'$ $\mathrm{Dist}(A)_P \& \ \mathrm{Det}(B)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Conf}(A)_{P^*}$

If a uniform account of the effect of distributive terms upon terms already having distributive and confused supposition could not be provided, this would have been a serious drawback for theories of supposition as a whole. Seemingly, at the time of Ockham and Buridan a solution for this issue had not yet been found; however, later authors such as John Dorp and John of St. Thomas were clearly aware of the problem, and succeeded in finding appropriate rules to deal with it. Clearly, many other cases may seem problematic and appear to be, at first sight, unaccountable for within supposition theory; but the reformulation of the rules for confused and distributive supposition above shows that the supposition framework is more resourceful than one might expect at first sight, allowing for constant refinement.[30]

[30]See (Klima and Sandu 1991) for the use of supposition theory to account for complex quantificational cases.

2.2 Other important developments

2.2.1 The doctrine of proof of terms/propositions

Besides the 'older' semantic tradition based on the concept of supposition, another semantic tradition became very influential in the 14^{th} century.[31] This tradition was known as the doctrine of the proof of terms (*probationes terminorum*) or propositions, and its most influential text was Billingham's *Speculum Puerorum*. Billingham's text was not the first in this tradition (for example, de Rijk dates Martin of Alnwick's text also edited in [De Rijk, 1982] as earlier than Billigham's), but it seems to have been the main source for the popularity of this genre in the second part of the 14^{th} century.

'Proof' here is not to be understood in its mathematical/logical sense, as demonstration; in this sense, to prove a proposition is to show what its truth depends on, in particular which simpler propositions must be true in order for a given proposition to be true. It is essentially an analytic procedure, in which the meaning of a 'difficult' proposition is decomposed in terms of simpler propositions, known as immediate propositions, on the basis of the analysis of the term(s) causing the proposition to be a 'difficult' one. Immediate propositions are those, according to Billingham, which cannot be proved (verified) except by a direct appeal to the senses or the understanding. These would be primarily propositions with directly referential pronouns or adverbs such as 'I', 'this', 'that', 'here', 'now' etc (cf. Billigham, *Probationes Terminorum*, in [De Rijk, 1982, 49]).

There are three basic techniques to 'prove' a proposition, according to whether the proposition is an exponible, a resoluble or an officiable proposition. An exponible proposition is one that corresponds to several propositions taken in conjunction, such as 'Socrates begins to be white', which corresponds to 'Socrates was not white and Socrates is now white'. Propositions containing comparative and superlative terms, or the verbs 'begins', 'ceases' and 'differs from' are analyzed in this fashion. A resoluble proposition is one that involves the descent from a general term to discrete terms, such as 'A man is running', which is proved by an appeal to sense experience codified by the propositions 'This is a man' and 'This is running'. Finally, an officiable proposition is one containing the nominalization of a proposition with the accusative-plus-infinitive construction, and corresponds roughly to what is now known as 'opaque contexts' (modalities and verbs related to propositional attitudes such as 'think', 'believe' etc.).

Although we dispose of several texts presenting the doctrine of the proof of propositions/terms, it is still in fact quite understudied. Indeed, it is fair to say that we still do not really understand the purpose and the mechanisms defining it (see [Spade, 2000, part IV]). For this reason, it is to be hoped that scholars will at some point take up the challenge of analyzing this doctrine systematically, as

[31] This approach was sometimes used instead of supposition theories (cf. Johannes Venator's *Logica* which uses the doctrine of the proof of propositions exclusively), but at other times both theories co-existed together, for example as distinct chapters of the same work (such as Paul of Venice's *Logica Parva*).

it is still by and large murky terrain.

2.2.2 Other developments worth mentioning

For reasons of space, it is impossible here to treat the totality of influential 14^{th} century semantic theories; but before we move on to the next section, a few more of these should be at least mentioned, so that the interested reader may further pursue his/her investigations. One of them is the doctrine of the supposition of relative pronouns, a vivid topic already in Ockham's *Summa Logicae* (Part I, chap. 76), which is the medieval counterpart of modern theories of anaphora (cf. [King, 2005]).

Another important aspect which, for reasons of space, could not be addressed here are the fascinating discussions on the semantics of propositions. In the same way that the medieval authors were interested in what single terms stood (supposited) for, many 14^{th} century authors raised similar questions concerning phrases and propositions (see for example Chapter 6 of [Spade, 1996]). One of such questions was what, in the extra-mental physical realm, the accusative-plus-infinitive constructions then known as *dicta* (which are in fact nominalizations of indicative propositions) corresponded to. Another important question concerned what in the physical world, if anything, makes a true proposition true. Some of the authors who addressed such issues are Burley (cf. [Cesalli, 2001]), Wyclif (cf. [Conti, 2005a, 2.1; Cesalli, 2005]), Henry Hopton (cf. [Ashworth and Spade, 1992, 51]), [Richard Brinkley, 1987]; Paul of Venice (part II, fascicule 6 of his *Logica Magna*), among others, often in treatises bearing the conspicuous titles of *De significato propositionis* and *De veritate et falsitate propositionis*, or similar ones.

In sum, while 14^{th} century semantics has been a popular topic of research in the last decades, this tradition itself is so rich that much of it still remains to be further studied and better understood. Therefore, the foregoing analyses should not be seen as an exhaustive account of this tradition but rather as a starting point for further research.

3 CONSEQUENCES

Theories of consequences are considered to be genuine medieval inventions. Of course, investigations on the nature of logical and inferential relations between propositions have existed ever since logic has existed; but medieval theories of consequence present a characteristic approach to the issue and a level of systematization that is arguably not to be found in previous investigations. Some (see [Moody, 1953]) see in them the forerunners of the 'propositional turn' in logic that took place to its full extent only in the 19^{th} century with Frege, after millennia of predominance of term logic — that is, of logical systems whose basic units were terms, such as in traditional Aristotelian logic, and not propositions.[32]

[32] With the notable exception of Stoic logic, which is usually recognized as the first propositional system of logic in the history of logic (see [Mates, 1973]).

This picture, however, is not entirely accurate, as the logic of terms also occupies a prominent place in 14^{th} century theories of consequence, as we shall see. But 14^{th} century logicians were probably the first to attempt a systematization of the propositional rules of inference that we now take as fundamental, such as contraposition, *ex falso*, the behavior of conjunctions, disjunctions etc.

Medieval logicians sought not only to establish the validity of such basic rules; they also made inquiries on the very nature of logical consequence and inference. In this sense, their investigations overlap not only with modern 'proof theory', but also with modern philosophy of logic (as exemplified by modern discussions on the nature of logical consequence such as [Etchemendy, 1990]).

In this section, I begin with a brief historical overview of this logical genre, and then move on to the three main subjects that the medievals discussed with respect to consequence: a general definition/criterion of what is to count as consequence; the distinctions of different kinds of consequence; and the most widely accepted rules of consequence. Finally, I show that at that time too there was no absolute consensus as to what rules of inference should be accepted, and that a few authors questioned some of the rules that were otherwise widely accepted.

3.1 Historical development

3.1.1 Origins

The precise historical origin of 14^{th} century theories of consequence is still controversial among specialists. It is still something of a mystery why and how, all of a sudden, at the beginning of the 14^{th} century, treatises bearing the title *De consequentiis* or the like began to appear. Why then, and not before? Naturally, the subject itself, that is, the logical and inferential relations between propositions, was very often discussed by earlier authors; the very term '*consequentia*' was in constant usage (in the same sense) since at least the 12^{th} century, and dates as far back as Boethius in the 5-6^{th} century (cf. [Boh, 1982, 302]). But no treatises or chapters were specifically dedicated to the topic or bore such titles before the 14^{th} century.

According to an influential hypothesis concerning the origin of theories of consequences, they stemmed essentially from the tradition on the Topics (cf. [Bird, 1961; Stump, 1982]). The *Topics* was the fifth book of Aristotle's *Organon*, a book that can be described as a rather loose collection of rules for the conduction of non-demonstrative reasoning and argumentation. At first sight, this hypothesis makes good sense: in the tradition of Aristotelian logic, the role of the Topics was often that of accounting for the patterns of (correct) inference and reasoning that did not fit into the syllogistic system presented in the *Prior Analytics*. While it is a wonder of systematicity and formality, syllogistic is not a very wide-ranging theory in that it accounts for only a small portion of the patterns of reasoning that we are prepared to accept as valid. The Topics, even though not as rock-solid as syllogistic patterns, provided an account of many more of such patterns of reasoning. So, conceptually, it would seem quite natural that the tradition on the Topics would be at the origin of theories of consequences, as these are essentially theories about the relations between propositions that go beyond the patterns recognized by syllogistic. Moreover, some earlier investigations on the notion of consequence were made explicitly within the context of an analysis of the Topics; Abelard, in the 12^{th} century, developed a sophisticated theory of the logical relations between propositions precisely in the part of his *Dialectica* [Abelard, 1956] dedicated to the topics.

However, this hypothesis did not receive the historical confirmation that one could have expected. It has been argued [Green-Pedersen, 1984, 270] that the late 13^{th} century literature on the Topics, that is, the period immediately preceding the emergence of treatises on consequences, gives absolutely no clue of what was to come; that is, there is no significant similarity between the contents of these 13^{th} century treatises on the topics and 14^{th} century treatises on consequences. Therefore, it has been concluded that the Topics could not have been the main source for 14^{th} century theories of consequences.[33]

Although our current state of knowledge on the matter still does not allow for a conclusive account of these developments, the picture that at this point seems more plausible is that different strands of traditional Aristotelian logic converged in order to give rise to the 14^{th} century theories of consequences. It seems that at least three other traditions contributed to the development of theories of consequence: treatises on *syncategoremata*, especially in connection with the *syncategorema* '*si*' (corresponding to the 'if ... then' structure in English); the analysis of hypothetical syllogisms, a concept absent from Aristotle's logic and introduced by Boethius in the 6^{th} century AD (his treatise *De hypotheticis syllogismis* is referred to 6 times in Burley's *De puritate artis logicae* — cf. [Green-Pedersen, 1984]); and commentaries on the *Prior Analytics* — indeed, it is in the *Prior Analytics* that Aristotle explicitly states a formulation of the notion of 'following' that is arguably the (remote) source for the most fundamental definition of consequence in the 14^{th} century.[34]

Be that as it may, the importance of the Topics for the development of 14^{th} century theories of consequences should not be altogether dismissed. It is worth noticing that two of the first authors having written explicitly on consequence, Ockham and Burley, are both in some way or another influenced by the Topics. Burley explicitly says that all valid consequences are based on dialectical Topics [Burley, 2000, 158]. By contrast, the relation of Ockham's theory of consequence to the Topics is more convoluted; Green-Pedersen argued convincingly that Bird's reconstruction of Ockham's theory within the framework of the Topics [Bird, 1961] is not satisfactory [Green-Pedersen, 1984, 268], but he also confirms that Ockham's 'intrinsic' and 'extrinsic' middles, crucial concepts for his theory of consequence, are concepts essentially taken (albeit heavily modified) from the topical framework.

[33] Chapter E of [Green-Pedersen, 1984], on topics and the theory of consequence, is the most comprehensive survey of these developments that I am aware of.

[34] "A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so." *Prior Analytics* 24^b19-20.

In sum, while we are not yet able to reconstruct a complete history of the development of these theories, at this point it seems that the most plausible hypothesis is that at least these four traditions — topics, *syncategoremata*, hypothetical syllogisms and *Prior Analytics* — must be taken into account to explain the rise of theories of consequences in the 14th century. Different aspects of each of these traditions contributed to the development of different aspects of the theories of consequence.[35] Green-Pedersen [1984, 295] argues for example that the late 13th century treatises that most resemble early 14th century treatises on consequences are 'the treatises on syncategorematic words and a number of sophism-collections arranged after syncategoremes.' Some of these connections will be commented upon in what follows, when specific aspects of theories of consequence are discussed.

3.1.2 Development in the 14th century

The 14th century treatises on consequences can be divided in roughly four groups:[36]

1. The treatises on consequences from the very beginning of the 14th century (Burley's *De consequentiis* and two anonymous treatises of roughly the same time — cf. [Green-Pedersen, 1981]) are in fact rather unsystematic collections of rules of consequence/inference. It seems as though their purpose was solely to provide 'rules of thumb' to deal with *sophismata* related to some syncategorematic terms; no conceptual or systematic discussion of the nature of consequences is presented therein.

2. The second stage of their development is represented by Burley's *De Puritate* [Burley, 2000], the chapters on consequence in Ockham's *Summa* (III-3), a few Pseudo-Ockham treatises and the *Liber consequentiarium* edited in [Schupp, 1988]. I group these texts together because in them the concept of (intrinsic and extrinsic) middles plays a crucial role and thus the presence of the (reworked) topical framework is more clearly perceived. They display a much deeper interest in the very nature of consequences than the previous group, presenting general definitions and criteria of what is to count as a consequence as well as divisions of kinds of consequence.

3. The third stage is represented by Buridan's treatise (Buridan 1976) and the treatises inspired by it, most notably Albert of Saxony's (a section of his *Perutilis logica*), the commentary on the *Prior Analytics* formerly attributed to Scotus (Pseudo-Scotus 2001)[37], and Marsilius of Inghen's (which is still only to be found in manuscripts). In these treatises the doctrine of intrinsic and

[35] For several of these tentative connections, see chapter E of [Green-Pedersen, 1984].

[36] My division is inspired by the division presented in [King, 2001, fn. 1], but with a few modifications. Here I refer mostly to treatises that can be found in print. For a list of these printed treatises, see [King, 2001]; for a list of British treatises only available in manuscripts, see [Green-Pedersen, 1985].

[37] On the identity of the author of this text, see [Read, 1993, fn. 10].

extrinsic middles has disappeared completely, and they present sophisticated analyses of the nature of consequences. What characterizes them as a group is the definition of formal consequence based on the substitutivity criterion (more on it below). This tradition can be referred to as the continental tradition on consequences.

4. The fourth group of treatises is predominantly British, and significantly more represented in number of treatises than group (3). It is represented by the treatises by Robert Fland, John of Holland, Richard Billingham, Richard Lavenham, Ralph Strode, among others. What characterizes this group as such is the definition of formal consequence in terms of containment of the consequent in the antecedent; it is a distinctly epistemic definition of formal consequence, when compared to that of group (3).[38]

In sum, the development of theories of consequence in the 14^{th} century is characterized by an early and rather 'primitive' stage, then by a stage of further development but with emphasis on the idea that consequences need something extrinsic to validate them (most generally quasi-Topics), and then by two further traditions that run more or less in parallel, the British tradition and the continental tradition.[39]

3.2 General definition of consequence

Before we inspect the different general definitions of consequence, we must first address the question of the relation between consequences and conditionals. For explanatory purposes, it may be convenient to consider three related but distinct concepts:

- A conditional sentence, a sentence that relates logically two embedded phrases, which are themselves not assertions properly speaking.
- A consequence, which is a logical relation between assertions or propositions.
- An inference, which is the action performed by somebody of inferring a conclusion from a (set of) (asserted) premise(s).

The medieval authors were well aware of these distinctions, or at least of the distinction between conditional sentences and consequences/inferences, but they often treated these different notions simultaneously, causing some confusion among modern interpreters.[40] Conditional sentences were generally treated by the medievals under the general heading of 'hypothetical propositions', along with conjunctions, disjunctions and others (cf. for example Chapter 1.7.3 of Buridan's

[38] For a compelling account of how this epistemic notion of formal consequence may have been the background for the emergence of the Cartesian notion of inference, see [Normore, 1993].

[39] This is in any case the general picture, but of course I do not claim that there are no exceptions to it — that is, there may very well be treatises that do not fit this description.

[40] For example: if the medieval notion of *consequentia* is not to be assimilated to that of a conditional sentence or implication, as I will argue, then the discussion on whether Ockham knew of material implication based on his theory of consequence seems rather misguided.

Summulae — [Buridan, 2001]); however, many of them recognized that a true conditional corresponds to a valid consequence/inference[41] (cf. Ockham, *Summa Logicae*, Part II, chapter 31, where he says that he will not discuss conditionals extensively since their logical properties correspond to the logical properties of consequences). Moreover, if the historical hypothesis of the influence of theories on the syncategoremata and on hypothetical syllogisms for the development of theories of consequences is correct, it becomes then patent that the two notions of conditionals and consequences were intimately related for our authors.

Nevertheless, as argued in [King, 2001], there are various reasons to conclude that the medievals were not only aware of these distinctions, but also that they viewed their *consequentiae* as primarily corresponding to arguments and inferences, i.e. to relations between statements, and not to conditional statements. Perhaps the most conspicuous of these reasons, taken from the structure of Ockham's *Summa*, is that consequences are treated there in Part III, dedicated to arguments, and not in part II, dedicated to propositions. Buridan explicitly addresses this distinction between conditionals and arguments in 7.4.5 of his *Summulae de Dialectica*, and while he says that *consequentia* is a twofold concept (covering conditionals and arguments), in most cases the logical properties of conditionals are treated under the heading 'conditionals', and what is dealt with in sections on consequence is predominantly logical properties of arguments. King [2001, 123] also discusses how Burley presents rules of consequences involving conditional propositions, explicitly contrasting 'the conditional sentences that enter into such reasoning with the consequences made out of them'. Hence, for the purposes of the present analysis, medieval consequences are not to be understood as conditional sentences but rather as inferences/arguments.

Now, as for the general definition of consequence, most authors of the 14$^{\text{th}}$ century accept at least as a necessary condition for a (valid) consequence that the antecedent cannot be true while the consequent is false; many accept this as a sufficient condition as well. This is, of course, the very familiar modal definition of consequence, present in Aristotle and also widely accepted in current (philosophy of) logic (at least as a necessary condition). Another formulation of the same idea is that a consequence is valid if from the contradictory of the consequent the contradictory of the antecedent follows [Ockham, 1974, 728][42], or similarly that the contradictory of the consequent is incompatible with (the truth of) the antecedent [Burley, 2000, 149]. In fact both Ockham and Burley give several equivalent formulations of this core idea, but the key point is obvious: for our authors, the most fundamental characteristic of a consequence is that the truth of the antecedent is incompatible with the falsity of the consequent.

Buridan's formulation of the fundamental criterion of what is to count as a consequence follows the same idea, but it is more convoluted because he has to take

[41] We shall see in due course that absolute consequences correspond to necessarily true conditionals, while as-of-now consequences correspond to contingently true propositions.

[42] Naturally, this should not be seen as a definition, as it is obviously circular, but rather as a rule of thumb for the recognition of putative consequences.

into account a few extreme counterexamples related to his commitment to actually formed propositions as the bearers of truth value (for the details of Buridan's discussion, see [Dutilh Novaes, 2005c]). His final formulation is 'a proposition is antecedent to another when it is related to it [the other] in such a way that it is impossible for things to be in whatever way the first signifies them to be without their being in whatever way the other signifies them to be, when these propositions are put forward together.' [Buridan, 1976, 22]. Pseudo-Scotus [2001] also considers this formulation, but he notices that it is not immune to a form of what is now known as Curry paradox[43], so he adds a clause to the effect that the definition does not hold in such extreme cases.

In the British tradition, variations of the same basic definition of consequence can be found, as for example in Billingham [Billingham, 2003, 80], Strode (in [Pozzi, 1978, 237]) and Paul of Venice [Paul of Venice, 1983, 167]; in fact, these authors present this definition without much discussion or analysis, as opposed to what one can find in Buridan's treatise for example. That seems to indicate that at later stages the definition was seen as unproblematic. It must also be noted that, while earlier authors such as Ockham and Buridan consider to be consequences only those pairs of propositions that satisfy the criterion, later authors such as Billingham and Paul of Venice (cf. same as above) recognized invalid consequences — that is, those that do not satisfy this criterion — as consequences nevertheless, making thus the distinction between valid and invalid consequences (while for Buridan and others, an invalid consequence was simply not a consequence).

3.3 Types of consequence

Medieval logicians recognized that the class of pairs (or triplets etc.) satisfying the general modal definition of consequence is rather heterogeneous. Indeed, one of the main focuses of treatises on consequences is the distinction of different kinds of consequences. There are three main kinds of distinctions: natural vs. accidental consequences, absolute vs. as-of-now consequences and formal vs. material consequences. While the same terms are generally used by different medieval authors, they often mean different things and have dissimilar criteria differentiating one kind from another. However, the natural vs. accidental distinction, while very important in early theories of consequence, is to be found only in Burley in the 14^{th} century (cf. [Green-Pedersen, 1984, 286; Pozzi, 1978, 58], so we shall not spend any time on it here. It is really the *ut nunc* vs. simple distinction and, even

[43]The paradoxical case put forward by Pseudo-Scotus is the following: 'God exists, hence this argument is invalid'. If this consequence is valid, then it has a necessary antecedent and a false consequent (since the consequent says that it is invalid). But then it is invalid. In sum, if it is valid, it is invalid, thus by *reductio ad absurdum* it is invalid. But if it is invalid, it is necessarily so, since the premise is a necessary proposition; therefore, we have a consequence with a necessary consequent, thus satisfying the modal criterion, but which is bluntly invalid, thus violating the modal criterion. See [Read, 2001] for an account of Pseudo-Scotus 'paradox', where it is also shown that this so-called paradox did not obtain the desired effect of violating the modal criterion.

more so, the formal vs. material distinction that are crucial for the understanding of 14th century theories of consequence. Besides these, there is also a distinction that is sometimes found explicitly stated but sometimes not, and which sometimes overlaps with the formal vs. material distinction but sometimes does not, namely the distinction *bona de forma* vs. *bona de materia*.

It is important to understand that these divisions are not necessarily meant to be sub-divisions of each another; they are also often understood as alternative, overlapping ways of dividing consequences. Ockham, for example, presents several such distinctions but does not present them as sub-divisions of one another (*Summa* III-3, Chapter 1). So let us now take a closer look at each of them.

Formal vs. material consequences. This distinction first appears in Ockham (cf. [Green-Pedersen, 1984, 287]), but afterwards it is to be found in virtually all treatises on consequence in the 14th century. However, what distinguished formal from material consequences varies per author.

For Ockham, this distinction is related to his doctrine of intrinsic and extrinsic middles. Ockham says that a consequence is formal when there is such middle, intrinsic or extrinsic, validating the consequence; otherwise, when there isn't such middle and the consequence holds only in virtue of its very terms (*Summa Logicae*, p. 589), it is a material consequence. As already mentioned, Ockham's extrinsic and intrinsic middles are reminiscent of the topical framework, but they are extremely modified versions of topical concepts, as shown by Green-Pedersen [1984, chapter E].

But what are intrinsic and extrinsic middles? An extrinsic middle is a proposition not containing the terms that form the antecedent and the consequent of the putative consequence, but which is a general rule describing the fact (ontological, logical or other) that warrants the passage from the antecedent to the consequent. An intrinsic middle, by contrast, is formed by the very terms that form the antecedent and the consequent of the putative consequence. Ockham says that some formal consequences hold only in virtue of an extrinsic middle, while others need an intrinsic as well as an extrinsic middle to hold. Syllogisms, for example, hold only in virtue of extrinsic middles; a consequence such as 'Only a man is a donkey, therefore every donkey is a man' holds in virtue of the following extrinsic middle: 'an exclusive and a universal with transposed terms signify the same and are convertible'. But a consequence such as 'Socrates is not running, therefore a man is not running' requires that the intrinsic middle 'Socrates is a man' be true in order to hold (and it still requires an extrinsic middle to validate it mediately). (Ockham, *Summa Logicae*, p. 588).

What exactly a material consequence is for Ockham is still a matter of controversy among scholars, as the passage where this notion is explained is known to be corrupted (*Summa Logicae*, p. 589).[44] Ockham gives two examples of material consequences, one of the *ex impossibili* kind (from an impossible proposition anything follows) and one of the *ad necessarium* kind (a necessary proposition follows

[44] See [Schupp, 1993] on the corruption of the text.

from anything), and that has led many commentators to believe that these are the only kinds of material consequences that Ockham recognizes. This matter cannot be settled at this point, but some have also raised the hypothesis that Ockham's notion of material consequence would go beyond such specific cases (see [Schupp, 1993; Kaufmann, 1993]).

Buridan, by contrast, is crystal clear concerning his distinction between formal and material consequence. His very terms are: 'Formal consequence means that [the consequence] holds for all terms, retaining the form common to all. Or [...] a formal consequence is that which, for every proposition similar in form which might be formed, it would be a good consequence.' [Buridan, 1976, 22-23]. A material consequence is one that does not satisfy this criterion but only the modal criterion.

While it is not immediately obvious why Ockham chose this nomenclature for his distinction[45], in the case of Buridan the terminology espouses perfectly the traditional Aristotelian notions of form and matter: the matter of a proposition is defined by its categorematic terms, while its form is defined by its syncategorematic terms. Thus, a formal consequence is one that holds in virtue of its form (the meaning of its syncategorematic terms); a material consequence is one that does not hold in virtue of its form alone but also in virtue of its matter (the meaning of its categorematic terms). Another way of describing Buridan's criterion is with the notion of substitutivity: a formal consequence is one that holds in all substitutional instances of its categorematic terms.

Buridan's use of the substitutional criterion is remarkable in that it is immune to much of the recent criticism against this criterion, most notably in [Etchemendy, 1990]. This is so because the substitutional criterion is applied only to consequences which already satisfy the modal criterion of incompatibility between the truth of the premise and the falsity of the conclusion. Under purely substitutional accounts of (logical) consequence, a clearly invalid logical consequence such as 'Bill Clinton was a president of the USA, thus Bill Clinton was male' comes out as valid if 'Bill Clinton' is seen as the only non-logical term of the consequence (as it happens to be so that, thus far, all presidents of the USA have been male, and thus all substitutional instances for 'Bill Clinton' will also validate the consequent). But in Buridan's account, this putative consequence would not be considered as a valid consequence in the first place, since it is not incompatible for somebody to be the president of the USA and not be a male (in fact this may happen even in the foreseeable future); therefore, it cannot be a formal consequence because it is not a consequence to start with. Buridan's account is best seen as what Shapiro [1998] has coined the 'hybrid' notion of formal (logical) consequence (which he presents as the most accurate conceptual characterization of logical consequence), that is,

[45] Ockham's choice of the term 'formal consequence' seems to be related to John Duns Scotus' notion of 'formal distinction'. Moreover, references to *consequentia formalis* can be found in Scotus' writings, for example in his *Quaestiones super libros Elenchorum* (p. 77 of the Vivès edition of *Opera Omnia*). On the relation between Scotus and Ockham on this matter, see [Martin, 2004].

the notion according to which a formal (logical) consequence must satisfy both criteria, the modal one and the substitutional one (see [Dutilh Novaes, 2005b].

This criterion for differentiating consequences had been proposed before by Abelard, who distinguished perfect from imperfect consequences [Abelard, 1956, 253-4]: perfect consequences received their warrant from their structure (*complexio*) alone, while imperfect inferences needed external warrant (for which Abelard turned to the Topics — cf. [Abelard, 1956, 256-7]). The same criterion was later rediscovered by Bolzano [1973] and further developed by Tarski [2002].

Intuitive though as it may seem to the modern reader, the definition of formal consequence based on the substitutional criterion was not widely accepted in the 14^{th} century. As already mentioned, while the treatises on consequence influenced by Buridan did maintain this criterion (Pseudo-Scotus, Albert of Saxony, Marsilius of Inghen), the majority of treatises followed a different notion of formal consequence. This alternative notion of formal consequence is what we could call the containment notion: a consequence is formal iff the consequent is contained in the antecedent, in such a way that whoever understands the antecedent necessarily understands the consequent.[46]

Here is Lavenham's formulation (as quoted in [King, 2001, 133]): 'A consequence is formal when the consequent necessary belongs to the understanding of the antecedent, as it is in the case of syllogistic consequence, and in many enthymematic consequences'. Strode's similar formulation is: 'A consequence said to be formally valid is one of which if it is understood to be as is adequately signified through the antecedent then it is understood to be just as is adequately signified through the consequent. For if someone understands you to be a man then he understands you to be an animal.' (as quoted in [Normore, 1993, 449]). Many other authors held similar definitions, such as Billingham [2003, 80] and Fland [1976].

Besides the fact that this is notably an epistemic notion of formal consequence (as opposed to Buridan's substitutional notion), for the authors adopting the containment notion, the extension of the concept of formal consequence is usually wider than that of those adopting the substitutional notion of formal consequence. Lavenham explicitly says that some enthymematic consequences are formal consequences — that is, consequences that, with the addition of an extra premise, acquire a syllogistic form, such as 'Socrates is a man, thus Socrates is an animal', which acquires a syllogistically valid form with the addition of 'Every man is an animal'. For Buridan, however, enthymematic consequences are not formal consequences because they do not satisfy the substitutional criterion, e.g. this particular example is not valid for all substitutional instances of 'man' and 'animal'. They do become formal consequences with the addition of the missing premise, but before that occurs they are merely material consequences (cf. [Buridan, 1976, 23]).

Another different criterion for formal consequences is presented at the very end of the 14^{th} century by Paul of Venice; he characterizes formal consequences as those

[46] The containment notion of (formal) consequence is not a 14^{th} century invention. For Abelard, it was a necessary condition for all valid consequences; in the 13^{th} century, it was held by authors such as Faversham and Kilwardby.

'in which the opposite of the consequent is repugnant formally to the antecedent', by which he means that the antecedent and the opposite of the consequent cannot even be conceived together. His example of a material consequence is 'God is not, therefore no man is': even though it is (or so he thinks) metaphysically impossible for men to be without God being, it is not inconceivable.

Consequences *bona de materia* vs. *bona de forma*. A criterion differentiating consequences that is intimately related to but not always identical to the formal vs. material distinction is the distinction *bona de forma* vs. *bona de materia*. For those who adopt the substitutional criterion to define formal consequences, this distinction is usually equivalent to the formal vs. material distinction; but for those who follow different criteria for formal consequences, there is indeed a point in recognizing that some consequences are valid no matter what their categorematic terms are, if their syncategorematic terms are retained. Ockham, for example, while using the rather idiosyncratic notion of 'middles' to define his formal consequences, also recognizes that some consequences are valid '*de forma*' (cf. Summa III-1, cap. 13, 32-35). Similarly for Paul of Venice; next to his 'conceivability' distinction between formal and material consequences, he adds the distinction between consequences '*bona de forma*' and '*bona de materia*' [Paul of Venice, 1984, 168]. Confusingly, though, he says that 'man runs, therefore animal runs' is a consequence '*bona de forma*', while it clearly does not satisfy the substitutional criterion that is usually associated with the notion of a consequence '*bona de forma*'.

Absolute vs. as-of-now consequences. The other important distinction of consequences for 14^{th} century authors is the one between absolute vs. as-of-now (*simplex vs. ut nunc*) consequences. Intuitively, the idea is that absolute consequences hold always and necessarily, while as-of-now consequences hold at a specific time or under specific assumptions (in particular at the moment indicated by the verbs in the consequence).

A clear account of this distinction is to be found in Pseudo-Scotus [2001]: for him, the absolute vs. as-of-now distinction applies only to material consequences (recall that for Ps.-Scotus a formal consequence is one that satisfies the substitutional criterion), and amounts to the modal value of the missing premise that can be added in order to turn the (enthymematic) consequence into a formal one. That is, if the missing premise is a necessary proposition, then the consequence is an absolute one (it always holds, since the condition for it to hold, namely the truth of the 'missing premise', always obtains); if the missing premise is a contingent truth (it has to be true with respect to the time indicated by the verbs of the consequence, otherwise the original material consequence does not hold), then the original material consequence holds only in some situations, namely the situations in which the contingent proposition happens to be true, and is thus an as-of-now consequence.

Although there may be slightly different formulations of the absolute vs. as-of-now distinction, these differences are in fact conceptually immaterial if compared to the very different formulations of the distinction formal vs. material consequence. An important difference, however, is that, while for some authors (Pseudo-Scotus, Buridan) the absolute vs. as-of-now distinction is posterior to the formal vs. material distinction and applies only to material consequences, there are also authors (e.g. Peter of Mantua — cf. [Pozzi, 1978, 61]) who present the absolute vs. as-of-now distinction as primary, and who see the formal vs. material distinction as applying only to absolute consequences (as-of-now consequences are always material consequences). In the latter case, clearly the distinction can no longer be cast in terms of the modal value of the missing premise, since at least in some cases of absolute formal consequences (e.g. valid syllogisms) there is in fact no missing premise. Alternatively, for Peter of Mantua, an as-of-now consequence is a consequence in which the contradictory of the consequent can indeed be true at the same time as the antecedent, but not at the time indicated by the copula or verb in question (the present if the verb is present-tense, the past if it is past-tense etc.).

For a while, modern commentators have been particularly interested in as-of-now consequences (in particular with respect to Ockham — cf. Mullik 1971), as some of them (cf. [Bohener, 1951]) saw these consequences as possible forerunners of modern material implication. The first problem with this association, as already argued, is that it seems unfitting to view medieval consequences as conditional sentences/implications (material or otherwise). Moreover, as-of-now consequences are contingent only insofar as the truth of the 'missing premise' is contingent. For the rest, the logical relation of necessity between the propositions involved is just as tight as with other consequences, provided that the missing premise is true — if the missing premise is true, then the truth of the antecedent is incompatible with the falsity of the consequent. In other words, as-of-now consequences display a stronger logical relation than mere truth-functional material implication.

The modern reader may be wondering: what is the point in distinguishing these different kinds of consequences? Is it yet one of those futile exercises that logicians of all times tend to be fond of, but with no practical application? Not so; the reason for such distinctions is in fact very practical: the different rules of consequences (to be presented in the next section) apply to specific kinds of consequences. That is, some rules apply only to formal or absolute consequences; others apply to material or as-of-now consequences. And such rules are extremely useful for the purposes of construing an argumentation; in fact, it seems that construing valid arguments is really the ultimate purpose of theories of consequences. But for a sound application of such rules, it is essential to identify the kind of a consequence in question, whence the importance of these criteria being as clearly formulated and effective as possible.

3.4 Rules of inference recognized by the medieval authors

Let us now look at the logical rules of consequence recognized by our 14th century authors. Some of them had already been identified by earlier authors[47] (for example the rules of opposition, equipollence and conversion for categorical propositions described in Aristotle's *De Interpretatione*); but what is remarkable in at least some of the medieval treatises is how they attempt at a systematization of these rules, such that from primary rules secondary rules are derived (see for example the first chapters of Burley's *De Puritate*, The Shorter Treatise — [Burley, 2000, 3-26]). Granted, other treatises are really no more than rather unsophisticated lists of rules, with no attempt to link the logical properties of each of them together. But quite a few of them present what we could view as the first stages of a proof theory.

Here, I present some of these rules making use of a notation inspired by Gentzen-style sequent calculus. The list of rules presented here is not exhaustive in that not all the rules studied and recognized by the medievals will be presented; the purpose here is to give the reader an idea of the level of logical sophistication attained by medieval treatises on consequences. A more thorough and extremely useful listing of the rules recognized by the medieval authors can be found in [Pozzi, 1978, 69-73].[48]

Burley, for example, lists ten main rules and several other rules that follow from these main rules. The first four rules are indeed easily rendered within the conceptual framework of propositional calculus, while the other six rely heavily on the properties of terms as well (and this is why it is inaccurate to say that medieval theories of consequences are purely propositional in nature; logical properties of terms still play a prominent role). Boh [1982, 312-314] presents a neat reconstruction of main rules 1 to 4, plus their derived rules, but the problem with his reconstruction is that it implies the view that consequences are conditionals, a view that, as already said, is rejected here.

Burley's rule 2, for example, states that 'whatever follows from a consequent follows from the antecedent', or alternatively, 'whatever is antecedent to the antecedent is antecedent to the consequent' [Burley, 2000, 4]. This is basically a formulation of the Cut-rule in sequent calculus (with the difference that no mention is made to the contextual propositional variables that are included in sequent calculus for the sake of generality).

$$\text{Rule 2} \quad \frac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$$

[47]It is also often said that the Stoics are the genuine pioneers of propositional logic; however, there is as of yet no evidence of direct or even of indirect influence from Stoic logic on the development of medieval theories of consequences. That is, even if many of such rules of consequence had already been recognized by the Stoics, it all seems to indicate that the medievals re-discovered them independently.

[48]Pozzi's study is based on the treatises on consequences of the following authors: Ockham, Burley, Pseudo-Scotus, Buridan, Albert of Saxony, Ralph Strode, Peter of Mantua, and Richard Ferrybridge.

From this he derives a few other rules, among which: (2′) 'whatever follows from a consequent and from its antecedent follows from the antecedent by itself' [Burley, 2000, 6] and (2″) 'whatever follows from a consequent with something added follows from the antecedent with the same thing added' [Burley, 2000, 7].

$$\text{Rule } 2' \quad \frac{A \Rightarrow B \quad A, B \Rightarrow C}{A \Rightarrow C}$$

$$\text{Rule } 2'' \quad \frac{A \Rightarrow B \quad B, C \Rightarrow D}{A, C \Rightarrow D}$$

Burley derives (2′) from Rule 2 plus what he takes to be a logical fact, which the modern reader may recognize as a special case of right-weakening: 'every proposition implies itself together with its consequent'; similarly, he derives (2″) from Rule 2 plus simultaneous applications of special cases of right-weakening and left-weakening: 'an antecedent together with something added implies the consequent with the same thing added'. His arguments can be reconstructed as follows:

$$(2') \qquad \cfrac{\cfrac{A \Rightarrow B}{A \Rightarrow A, B}\,\text{WR} \qquad A, B \Rightarrow C(\text{hyp.})}{A \Rightarrow C}\,\text{Rule 2}$$

$$(2'') \qquad \cfrac{\cfrac{A \Rightarrow B}{A, C \Rightarrow B, C}\,\text{WR, LR} \qquad B, C \rightarrow D(\text{hyp.})}{A, C \Rightarrow D}\,\text{Rule 2}$$

After having shown that the medievals did know the procedure of deriving rules from primitive rules, I now present a few other rules recognized by them (taken from Pozzi's very useful list, unless otherwise stated), not paying specific attention to the deductive structure between these rules:

'From the impossible anything follows.'

$$\frac{}{\perp \Rightarrow A}$$

In the case of material as-of-now consequences the requirement is weaker: from a false proposition anything follows.

'The necessary follows from anything.'

$$\frac{}{A \Rightarrow T}$$

Similarly, in the case of material as-of-now consequences the requirement is weaker: a true proposition follows from anything (these two weaker formulations for as-of-now consequences are to be found in Buridan and Pseudo-Scotus (cf. [Pozzi, 1978, 69]).

'If the antecedent must be conceded, so must be the consequent.'

$$\frac{\Rightarrow A \quad A \Rightarrow B}{\Rightarrow B}$$

'If the consequent must be denied, so must be the antecedent.'

$$\frac{\Rightarrow \neg B \quad A \Rightarrow B}{\Rightarrow \neg A}$$

'From the contradictory of the consequent the contradictory of the antecedent follows': contraposition [Ockham, 1974, 728].

$$\frac{A \Rightarrow B}{\neg B \Rightarrow \neg A}$$

'Whatever follows from the contradictory of the antecedent follows from the contradictory of the consequent.'

$$\frac{A \Rightarrow B \quad \neg A \Rightarrow C}{\neg B \Rightarrow C}$$

'Whatever is antecedent to the contradictory of the consequent is antecedent to the contradictory of the antecedent.'

$$\frac{A \Rightarrow B \quad C \Rightarrow \neg B}{C \Rightarrow \neg A}$$

'From a conjunction to one of its parts constitutes a valid consequence.'

$$\frac{}{A \& B \Rightarrow A}$$

'From one of its parts to the whole disjunction constitutes a valid consequence.'

$$\frac{}{A \Rightarrow A \vee B}$$

'From a conditional with its antecedent to its consequent constitutes a valid consequence.'[49]

$$\frac{}{A \to B, A \Rightarrow B}$$

Now something perhaps slightly surprising for the modern reader: rules for consequences that do not obtain (which I represent by '=/>'). In modern systems

[49]Here again it is clear that our authors were very much aware of the distinction between conditionals and consequences.

this is not necessary, as the enumeration of the valid rules is supposed to be exhaustive, and all patterns not falling within the patters of these valid rules are immediately false. The medievals, however, did not have the ambition of presenting exhaustive lists of valid rules, and therefore it was also useful for them to know how to identify when a consequence did not hold.[50]

'Whatever does not follow from the antecedent, does not follow from the consequent.'

$$\frac{A \Rightarrow B \quad C \not\Rightarrow B}{C \not\Rightarrow A}$$

'Of that from which the consequent does not follow, the antecedent does not follow from it either'

$$\frac{A \Rightarrow B \quad C \not\Rightarrow B}{C \not\Rightarrow A}$$

They also discussed rules involving modalities, such as 'from the necessary the contingent does not follow' or 'from the possible does not follow the impossible'.[51]

The rules presented here are those that can be easily formulated within a purely propositional framework. However, as already noted, treatises on consequences contained many more rules which, by contrast, were based on properties of terms; these rules were just as significant, and the only reason why they are not treated here is because their formulation presupposes concepts from elsewhere (such as supposition theory — for reasons of space, it is not possible to go into such details here). But just to illustrate the point, here is an example, Burley's Rule 7 in the shorter version of *De Puritate*: 'an inference holds from a distributed superior to an inferior taken either with distribution or without distribution. But an inference does not hold from an inferior to a superior with distribution. For it follows: 'Every animal runs, therefore, every man runs, and a man runs', but not conversely.' [Burley, 2000, 16] 'Animal' is the superior of 'man', and from a proposition where 'animal' is distributed follow both the proposition where its inferior 'man' is distributed and the one where it is not.

3.5 Dissident voices

So far I have treated medieval theories of consequences as if they were homogeneous with respect to the rules of consequence accepted by their authors. While not all authors stated all these rules explicitly, they are in any case all compatible with one another. However, a small minority rejected two of the very basic rules of the notion of consequence presented so far, namely the *ex impossibili* and the *ad necessarium* rules. These rules follow naturally from the modal definition of

[50]In the same manner that theories of fallacies — that is, theories of apparently sound but in fact unsound reasoning — were crucial for medieval logicians.

[51]More on modal inferences can be found in the chapter on modalities in the present volume.

consequence (if it is seen as a sufficient condition), such that those who accept this definition as a sufficient condition for a consequence must admit the validity of these two principles. This is so because the definition states that B is a consequence of A if it is impossible for A to be true while B is false. If it is impossible for A to be true *tout court*, the definition is satisfied *a fortiori* for any B whatsoever, and hence *ex impossibili* must hold. Similarly, if *ex impossibili* holds, by contraposition *ad necessarium* must hold.

But in several periods of the history of logic, some have seen these two principles as highly counterintuitive[52] given that, according to them, propositions that are otherwise not related by meaning or logic in any way whatsoever are in a relation of consequence with one another: 'It is raining and it is not raining, therefore I am God'; 'The cat is on the mat, therefore it is raining or it is not raining' would be examples of such counterintuitive inferences/consequences. On a semantic level, to deny the validity of these rules amounts to denying the modal definition of consequence as a sufficient criterion; on a syntactic level, modifications of the usual rules are required, since the derivation of any proposition from a contradiction can be obtained very easily from the usual rules of deduction (as shown by the famous Lewis argument, which may have been known already in the 12th century — cf. [Martin, 1986]): from P and not-P follows P; from P follows P or Q; but from P and not-P not-P also follows, and hence by disjunctive syllogism Q follows from P or Q and not-P. To block this derivation, at least one of these otherwise very natural rules must be discarded.

Prior to the 14th century, illustrious logicians such as Abelard and Kilwardby had already restricted their notion of consequence, not accepting the modal definition as a sufficient condition. Abelard, for example, required that the consequent be contained in the antecedent for a consequence to hold [Abelard, 1956, 253]. (As we have seen, 14th century logicians also made use of this criterion, but to define a sub-class of the valid consequences and not as a necessary and sufficient condition for all consequences.) With this move, *ex impossibili* and *ad necessarium* no longer hold.

In the 14th century we know of at least a few dissident voices. A certain Nicolaus Drukken of Dacia, writing in Paris in the 1340s, proposes a revision of the sufficient criterion of a valid consequence such that the 'total significate of the consequent be signified by the antecedent' [Read, 1993, 241][53] in his commentary to the *Prior Analytics* (edited by [Green-Pedersen, 1981]). Richard Ferrybridge also rejects *ex impossibili* and *ad necessarium* if the impossible and necessary propositions in question are impertinent to the other proposition in a putative consequence, precisely because he requires there to be a relation of relevance between antecedent and consequent (cf. [Pozzi, 1978, 60]. In practice, what such authors seem to be proposing is that the criterion of containment of the consequent in the antecedent be used as a necessary and sufficient criterion for all valid consequences, and not

[52]Recent examples of such dissident voices are paraconsistent and relevant logicians.

[53]Read (1993, 251) mentions that the early 16th century philosopher Domingo de Soto was another dissident voice with respect to *ex impossibili*.

only for the formal ones as in the case of Billinham, Strode *et al.*

3.6 Conclusion

14$^{\text{th}}$ century theories of consequences are without a doubt among the most important and most interesting developments in the logic of this century. Given the considerable length of the literature on the topic, and for reasons of space, here I had to focus on its main lines of development and disregard some of the secondary points and details that are nonetheless very interesting (such as Buridan's reformulation of the modal criterion in order to accommodate his token-commitment, Pseudo-Scotus analysis of a Curryian paradox, among others). Indeed, my purpose here was to give the reader a hint of the richness of this material, and encourage him/her to go look further.

4 OBLIGATIONS

4.1 Introduction

Obligationes were a regimented form of oral disputation. It consisted of two participants, Opponent and Respondent; Opponent would put forward several propositions, and Respondent was expected to accept, deny or doubt them on the basis of specific rules (the discussion of which will constitute the core of this chapter). It is without a doubt one of the main logical genres of the 14$^{\text{th}}$ century: a brief look at the list of authors and texts in Appendix A immediately shows that virtually every important author of this period wrote on *obligationes*. However, contrary to what is sometimes thought, *obligationes* are not a 14$^{\text{th}}$ century invention; interesting 13$^{\text{th}}$ century Parisian treatises[54] indicate that the genre was already quite developed at that time and place.[55] Even though the contents of these treatises fall out of the scope of the present investigation, it may be added that the theory of *obligationes* presented in them is very much in the spirit of the earliest of such treatises in the 14$^{\text{th}}$ century, namely Burley, indicating thus that Burley was most probably inspired by this early tradition.

With *obligationes* we have a phenomenon that falls in the context of the OX-INAT theory (cf. [Ebbesen, 1985]) previously mentioned, according to which the typical logical topics of the 14$^{\text{th}}$ century were re-introduced into Paris as British import, after a period of modistic predominance. Indeed, after these 13$^{\text{th}}$ century Parisian treatises on *obligationes*, few continental treatises on the topic were written in the 14$^{\text{th}}$ century; the fact is that in the 14$^{\text{th}}$ century, *obligationes* was an overwhelmingly British genre.

The 14$^{\text{th}}$ century British tradition on obligations begins, as already said, with Burley; his treatise seems to have been written in the very first years of the century (cf. [Braakhuis, 1993, 323]). Then, for at least two decades, nothing much seems

[54] Such as those edited in [Braakhuis, 1998] and [de Rijk, 1974; 1975; 1976].

[55] See also [Martin, 2001] for very early (12$^{\text{th}}$ century) developments in the genre.

to have been written on *obligationes* (in any case, we have not been able to unearth anything so far) until Kilvington's *Sophismata* [Kretzmann and Kretzmann, 1990], written before 1325), which contains interesting remarks on the genre (without actually presenting a full-fledged theory of *obligationes* — see [Spade, 1982]). Kilvington's remarks seem to have sparked renewed interest in the genre, especially among the Oxford Calculators, who then began to write prolifically on *obligationes*. The first of such treatises is Roger Swyneshed's treatise (≅ 1330-1335) [Spade, 1977], which indeed inaugurated a new trend within the genre, later to be named '*nova responsio*' by Robert Fland in his treatise (≅ 1350) [Spade, 1980c], as opposed to the '*antiqua responsio*' represented by the Burley-style form of *obligationes*. As for what exactly differentiates the *antiqua* from the *nova responsio*, this will be the core of the conceptual discussion to follow.[56]

Other British authors having written on *obligationes* in the decades following Swyneshed's treatises are (for the dates, I follow [Braakhuis, 1993]): Billingham (≅ 1350s) (cf. [Ashworth, 1985]), Martinus Anglicus (≅ 1350s), Richard Brinkley (≅ 1350s) [Spade, 1995], Ralph Strode (≅ 1360s) (cf. [Ashworth, 1993; Dutilh Novaes, 2006b]), an anonymous Mertonian [Kretzmann and Stump, 1985], and Richard Lavenham (later 14^{th} century) [Spade, 1978]. Paul of Venice, who for our present purposes is counted among 'British' authors, has a long treatise on *obligationes* in the *Logica Magna* [Paul of Venice, 1988], which is heavily inspired by Strode's treatise, and a short chapter on *obligationes* in the *Logica Parva*. Among these, Martinus Anglicus, Richard Lavenham and, to some extent, Robert Fland, follow Swyneshed's *nova responsio* style of *obligationes*; the others remain by and large faithful to the *antiqua responsio*.

As for continental authors, currently we only know of six continental authors who wrote *obligationes* treatises: Albert of Saxony (≅ 1350) (cf. Braakhuis 1993), John of Wesel (≅ 1350) (cf. [Spade, 1996b]), William Buser (≅ 1355) (cf. [Keepkens, 1982; 1993; Pozzi, 1990] for the edited text), Marsilius of Inghen (just before 1360) (cf. [Keepkens, 1982, 159-160]), John of Holland (just after 1360) (cf. John of Holland 1985) and Peter of Candia (very end of 14^{th} century) (cf. [Keepkens, 1982, 154]). The last two are thought to have studied in England, and therefore their exposure to the British *obligationes* literature can be taken for granted. As for the others: it has been argued convincingly that Albert of Saxony would have drawn significantly from Billingham's treatise, or in any case from the chapter on *obligationes* of the general manual *Logica oxoniensis*, which in turn is basically Billingham's text (see [Ashworth, 1985; Braakhuis, 1993]). As for William Buser and Marsilius of Inghen, it is certain that they would have had direct contact with Albert and his *obligationes*; Buser's text resembles Albert's in many aspects, and Marsilius's text in turn is visibly inspired by his master Buser's text (cf. [Keep-

[56]Notice though that these terms, '*antiqui*' and '*moderni*', are not consistently used by our authors; given the natural flow of generations, those that are referred to as '*moderni*' often end up being referred to as '*antiqui*' in subsequent generations (cf. [Spade, 1980, 42; Pozzi, 1990, 17, fn. 25]). In any case, in this section I will use the term '*nova responsio*' to refer to Swyneshed's style and '*antiqua responsio*' to refer to Burley's style of *obligationes*.

kens, 1982]). Concerning John of Wesel (in Paris in 1344-1353), of whom little is known, Spade argues that his text shows 'a close familiarity with the writings of Oxford logicians from the first half of the fourteenth century, in particular with those of Roger Swyneshed' [Spade, 1996b, 3]). Therefore, we may conclude that all these continental *obligationes* treatises of the 14^{th} century were directly or indirectly under the influence of the British literature on the topic.

And as for the *antiqua responsio* vs. *nova responsio* dichotomy with respect to the continental authors, it seems that, while all of them were aware of the innovations introduced by Swyneshed, they were mostly critical of them. John of Wesel seems to be sympathetic to some of Swyneshed's views (cf. [Spade, 1996b]); as for Peter of Candia, in spite of the attribution by some commentators of the qualification of *modernus* to him (cf. [Keepkens, 1982, 154]), from the description of his text in [Pozzi, 1990, 55] it becomes apparent that he was not a *modernus* with respect to relevance, which is the main aspect considered here.[57] All the other authors clearly side with the *antiqui*.

In what follows, I concentrate in Burley's and Swyneshed's treatises, as they are the most famous treatises of each of these trends (it is in fact a condensed version of [Dutilh Novaes, 2005a; 2006a]). Moreover, Burley's treatise has received a partial translation into English (Burley 1988), therefore it seems reasonable to focus on a text that can be consulted also by those who do not read Latin.

In his treatise, Burley describes six kinds of *obligationes*: *petitio, sit verum, institutio, positio*, *depositio* and *dubitatio*. Swyneshed, by contrast, recognizes only three kinds: *positio*, *depositio* and *impositio* (Burley's *institutio*). We shall be focusing on *positio*, as it is arguably the most representative form of *obligationes* (even though *impositio* in particular also offers the opportunity for very interesting semantic analysis).

4.2 Burley's treatise: antiqua responsio

The disputation has two participants, Opponent and Respondent. In the case of *positio*, the game starts with Opponent putting forward a proposition, called *positum*, which Respondent must accept as true for the sake of the disputation, unless it is self-contradictory. Opponent then puts forward propositions (the *proposita*), one at a time, which Respondent must either concede, deny or doubt, on the basis of inferential relations with the previously accepted or denied propositions, or, in case there is none (and these are called irrelevant or impertinent propositions[58]), on the basis of the common knowledge shared by those who are present. Respondent loses the disputation if he concedes a contradictory set of propositions. At

[57]Unfortunately, this text is still only available in the form of manuscripts (S. Brown is reportedly preparing and edition of the text), and I have not had the occasion of examining the manuscripts myself.

[58]Throughout the text, I use the terms 'relevant' and 'pertinent' as synonymous, as much as 'irrelevant' and 'impertinent'. The terms in Latin are '*pertinens'* and '*impertinens'*, but they are often translated as 'relevant' and 'irrelevant', for example in the translation of Burley's treatise.

the end, Opponent and possibly a jury determine whether Respondent responded well, in particular if he was able to keep consistency

I have argued elsewhere that Burley's *obligationes* is best seen as a logical game of consistency maintenance (cf. [Dutilh Novaes, 2005a]): here, I will follow this basic idea.[59]

4.2.1 Reconstruction

DEFINITION 1 (The obligational game (Burley)).

$$\boldsymbol{Ob} = \langle K_C, \Phi, \Gamma, R(\phi_n)\rangle$$

$\mathbf{K}_C$ is the common state of knowledge of those present at the disputation complemented by the *casus.* The *casus* was usually a proposition to be assumed as true, often to make it explicit that the *positum* was false. An example from Burley: "Suppose Socrates is black, and suppose it is posited that Socrates is white" [Burley, 1988, 378]. That Socrates is black is the *casus*, a proposition which all the participants are to assume to be true, and 'Socrates is white' is the *propositum.*[60]

$\mathbf{K}_C$ is an incomplete model, since some propositions do not receive a truth-value in it: for some propositions, it is not known whether they are true or false, although it may be known that they are true-or-false (these must be doubted — cf. [Burley, 1988, 381]). So, the state of common knowledge is a state of imperfect information: it includes all information that is considered common sense (that the pope is in Rome, all religious dogmas etc.), plus information circumstantially available, due to the pragmatics of the disputational situation, plus the *casus.*

Φ is an ordered set of propositions. It is the set of propositions actually put forward by O(pponent) during an obligation. Each element of Φ is denoted by 'ϕ_n', where n is a natural number, denoting the place of $\phi_n$in the ordering. The order corresponds to the order in which the propositions are put forward by O, starting with ϕ_0 (the *positum*).

Γ is an ordered set of sets of propositions, which are formed by R(espondent)'s responses to the various ϕ_n. How each Γ_n is formed will be explained below. The ordering is such that Γ_n is contained in Γ_{n+1}.

$\mathbf{R}(\phi_n)$ is a function from propositions to the values 1, 0, and ?. This function corresponds to the rules R must apply to respond to each proposition ϕ_n. 1 corresponds to his accepting ϕ_n, 0 to his denying ϕ_n and ? to his doubting ϕ_n (cf. [Burley, 1988, 381]).

The **procedural rules** of the game are quite simple: O first puts forward a proposition. If R accepts it (according to R(ϕ_0) defined below), then the game

[59] In fact, even though my game-interpretation is to my mind the one that best explains the data, how to interpret obligationes in terms of modern concepts is still an on-going issue. Other proposals have been put forward, such as *obligationes* as a logic of counterfactuals, *obligationes* as thought-experiments, *obligationes* as belief-revision, among others. I have discussed each of these proposals in [Dutilh Novaes, 2005a, 3.2].

[60] For more on the notion of *casus*, see [Yrjönsuuri, 1993].

begins. Then O puts forward a further proposition, R responds to it according to R(ϕ_n), and this procedure is repeated until the end of the game.

The **logical rules** of the game are defined by R(ϕ_n), in the following way:

DEFINITION 2 (Rules for *positum*).

$$R(\phi_0) = 0 \text{ iff } \phi_0 \Vdash {}^{61} \bot$$
$$R(\phi_0) = 1 \text{ iff } \phi_0 \nVdash \bot$$

The rule defining the response that R should give to ϕ_0 (the *positum* — [Burley, 1988, 378]) has interesting consequences for the idea that *obligationes* are games of consistency maintenance. If R is obliged to accept at the beginning a proposition that entails a contradiction — for example, any paradoxical proposition such as Liar sentences and the like — then there is no possible winning strategy for R. He cannot maintain the consistency of a set of propositions that, from the outset, contains a contradictory proposition. So the rules of the game stipulate that there always be a winning strategy for R, starting from this restriction upon the *positum*. Burley expresses this clause by saying that it must be in the Respondent's power to satisfy the requirement (of not falling in contradiction) (cf. [Burley, 1988, 376]).

DEFINITION 3 (Rules for proposita).

$$R(\varphi_n) = 1 \text{ iff} \left.\begin{array}{l} \Gamma_{n-1} \Vdash \phi_n, \text{ or} \\ \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n \text{ and } {K_c}^{62} \Vdash \phi_n \end{array}\right.$$

$$R(\phi_n) = 0 \text{ iff} \left.\begin{array}{l} \Gamma_{n-1} \Vdash \neg\phi_n, \text{ or} \\ \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n \text{ and } K_C \Vdash \neg\phi_n \end{array}\right.$$

$$R(\phi)n) =? \text{ iff } \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n, K_C \nVdash \phi_n, K_C \nVdash \neg\phi_n$$

[63]In case of K_C, it is not so much that K_C semantically implies a proposition ϕ_n, but rather that ϕ_n is contained in K_C (therefore a fortiori K_C also implies ϕ_n). For the sake of simplicity, I use only the forcing turnstyle.

That is, if Respondent fails to recognize inferential relations and if he does not respond to a proposition according to its truth-value within common knowledge, then he responds badly (cf. [Burley, 1988, 381]).

Formation of Γ_n. The different sets of propositions accepted by R (i.e., the propositions to which R has committed himself in the disputation) are formed in the following way:

DEFINITION 4 (The sets Γ_n).

If $R(\phi_n) = 1$, then $\Gamma_n = \Gamma_{n-1} \cup \{\phi_n\}$

[61]I us the forcing turnstyle $\Vdash$ throughout to express the relation of semantic implication between propositions. That is, within obligationes the relation of 'following' is not defined syntactically or proof-theoretically, but rather semantically.

If $R(\phi_n) = 0$, then $\Gamma_n = \Gamma_{n-1} \cup \{\neg\phi_n\}$
If $R(\phi_n) = ?$, then $\Gamma_n = \Gamma_{n-1}$
In particular, if $\mathrm{R}(\phi_0) = 1$, then $\Gamma_0 = \{\phi_0\}$. If $\mathrm{R}(\phi_0) = 0$ or $\mathrm{R}(\phi_0) = ?$, then there is no disputation.

These rules mirror closely the clauses of Lindenbaüm's lemma, the main idea being that propositions are gradually added to a set of propositions (which starts with one single element, the *positum*), while consistency is also maintained. There is a significant difference, though, in that, in the construction of a maximal consistent set according to this lemma, if the set $\Gamma_n = \Gamma_{n-1} \cup \{\phi_n\}$ formed is inconsistent, then the construction simply continues with Γ_{n-1}, i.e. the so far largest consistent set built. In the *obligationes* framework, however, if an inconsistent set is constructed, the procedure comes to a halt, as respondent has responded badly and thus lost the game.

Outcome. O wins the game if it is recognized that $\Gamma_n \Vdash \perp$; that is, if R has conceded a contradictory set of propositions. R wins the game if, when the disputation is declared to be over, it is recognized that $\Gamma_n \nVdash \perp$. The clause about the stipulated time concerns the feasibility of the game: the construction of maximal-consistent sets of propositions is not feasible within human time, therefore respondent is expected to keep consistency only during a certain time.

4.2.2 Can respondent always win?

The rules of the obligational game as defined guarantee that there always be a winning strategy for R. This is due to two facts: one is a stipulated rule of the game and the other is a general logical fact.[64] The relevant rule of the game is: a paradoxical *positum* should not be accepted. As stated by Burley himself, the point of this clause is exactly to guarantee that R stands a chance to win. Therefore, R always starts out with a consistent set of propositions.

It is a general principle of logic (and also the backbone of Lindenbaüm's lemma) that any consistent set of propositions can always be consistently expanded with one of the propositions of a contradictory pair $\phi_n$and $\neg\phi_n$.[65] R starts with a consistent set of propositions (the set composed of the *positum*); so at each move, there is in theory at least one 'correct' way of answering, i.e. either accepting or denying ϕ_n, which maintains the set of accepted and denied propositions consistent.

4.2.3 Why does R not always win?

But why does the game remain hard? It is a fact that R often makes wrong choices and is not able to keep consistency, even though to keep consistency is

[64] This fact has already been noticed by J. Ashworth: 'a certain kind of consistency was guaranteed for any correctly-handled disputation' (Ashworth 1981, 177).

[65] **Proof**: Assume that Γ is consistent. Assume that $\Gamma \cup \{\phi\}$ is inconsistent. Thus $\Gamma?\neg\phi$ (1). Moreover, assume that $\Gamma \cup \{\neg\phi\}$ is inconsistent. Thus $\Gamma?\phi$ (2). From (1) and (2), it follows that $\Gamma?\phi\&\neg\phi$, that is, that Γ is inconsistent, which contradicts the original assumption. The principle to be proven follows by contraposition.

always logically possible. If it were easy, then it would not fulfill its pedagogical and theoretical purposes. It seems that *obligationes* remains a difficult kind of disputation for Respondent for two basic reasons: Opponent makes use of intricacies (for example, the phenomena of synonymy and equivocation) of the language being used in the game to set up 'traps' for Respondent; and the game is essentially dynamic.

4.2.3.1 Intricacies of the language To have a glimpse of the kind of trap Opponent may set up, take a look at the hypothetical disputation represented in the scheme below (it is not an example taken from Burley's text, but it is very much in the spirit of the examples he proposes).

Proposition	Calculation	Verdict	Outcome
ϕ_0: **You are in Rome or you are the Pope.**	Possible	Conceded	$\Gamma_0 = \{\phi_0\}$
ϕ_1: **You are in Rome.**	$\Gamma_0 \nVdash \phi_1$, $\Gamma_0 \nVdash \neg\phi_1$, $K_C \Vdash \neg\phi_1$	Denied	$\Gamma_1 = \{\phi_0, \neg\phi_1\}$
ϕ_2: **The pope is in Rome.**	$\Gamma_1 \nVdash \phi_2$, $\Gamma_1 \nVdash \neg\phi_2$, $K_C \Vdash \phi_2$	Conceded	$\Gamma_2 = \{\phi_0, \neg\phi_1, \phi_2\}$
ϕ_3: **You are the pope.**	$\Gamma_2 \Vdash \phi_3$ (from ϕ_0 and $\neg\phi_1$)	Conceded	$\Gamma_3 = \{\phi_0, \neg\phi_1, \phi_2, \phi_3\}$
ϕ_4: **You are in Rome.**	$\Gamma_3 \Vdash \phi_4$ (from ϕ_2 and ϕ_3)	Conceded	$\Gamma_4 = \{\phi_0, \neg\phi_1, \phi_2, \phi_3, \phi_4\}$

But $\phi_1 = \phi_4$. So $\Gamma_4 \Vdash \perp$.

Where did it go wrong? Why was Respondent forced both to accept and to deny ϕ_4 in the last round? Could he have avoided the trap? A closer inspection of the propositions shows that ϕ_2 is not irrelevant (i.e., with no inferential connections to the previously accepted or denied propositions), as it might seem at first. Actually, from ϕ_0 and $\neg\phi_1, \neg\phi_2$ follows: by negating the first disjunct, Respondent has already (logically) committed himself to the second disjunct, that he is the Pope. So if he is not in Rome and he is the Pope, then the Pope cannot be in Rome. So Respondent should deny ϕ_2, instead of accepting it as a proposition irrelevant and true according to K_C, even though he has not explicitly granted ϕ_3 yet.

4.2.3.2 The game is dynamic Another source of difficulty in this game is its dynamic character. This is related to the inclusion of irrelevant propositions, accepted or denied according to K_C, in the set of propositions that will be used to respond to each *propositum* still to come. Burley himself attracts the reader's attention to this point: "One must pay special attention to the order [of the propositions]." [Burley, 1988, 385]

This means that, during a disputation, it may occur that (1) $\phi_0, \phi_1 \Vdash \phi_2$ but $\phi_0, \phi_2 \nVdash \phi_1$, or else that (2) $\phi_0, \phi_1 \Vdash \phi_2$ but $\phi_0 \nVdash \phi_2$. (1) is related to the obvious asymmetric character of implication, and (2) to the dynamic nature of the game, what I shall call the 'expansion of the informational base Γ_n'. This can be best seen if we examine what happens in terms of models during an obligational disputation. For that, here are some definitions:

DEFINITION 5. Γ_n= Informational base, i.e. a set of propositions.

DEFINITION 6. $\boldsymbol{UM}_n$ = The class of models that satisfy informational base Γ_n.

DEFINITION 7. $\boldsymbol{UM}\phi_n$ = The class of models that satisfy ϕ_n.

DEFINITION 8. $\boldsymbol{UM}_n \Vdash \Gamma_n$ iff $\boldsymbol{UM}_n \Vdash$ P for all P in Γ_n.

A model that satisfies a set of propositions satisfies each of them (i.e. they are all true in this model). It is clear that, if $\Gamma_k = \{\phi_n\} \cup \{\phi_m\}$, then $\boldsymbol{UM}_k = \boldsymbol{UM}\phi_n \cap \boldsymbol{UM}\phi_m$. So, the set of models that satisfy Γ_k is the intersection of all the models that satisfy each of the elements of Γ_k. Similarly, if $\Gamma_{n+1} = \Gamma_n \mathrm{U} \{\phi_{n+1}\}$, then $\boldsymbol{UM}_{n+1} = \boldsymbol{UM}_n \cap \boldsymbol{UM}\phi_{n+1}$.

THEOREM 9. *If $\Gamma_n \Vdash \phi_{n+1}$ and ϕ_{n+1} is accepted, then $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$.*

Assume that, at a given state of the game, $\Gamma_n \nVdash \phi_{n+1}$. According to $\mathrm{R}(\phi_n)$, ϕ_{n+1} must be accepted, forming $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$. Now take $\boldsymbol{UM}_n$, that is, all the models that satisfy Γ_n. According to the model-theoretic definition of implication (i.e. P $\Vdash Q$ iff Q is true in all models where P is true, that is, if $\boldsymbol{UM}_P \Vdash Q$), if $\boldsymbol{UM}_n \Vdash \Gamma_n$ and $\Gamma_n \Vdash \phi_{n+1}$, then $\boldsymbol{UM}_n \Vdash \phi_{n+1}$. Since $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$, $\boldsymbol{UM}_n \Vdash \Gamma_n$ and $\boldsymbol{UM}_n \Vdash \phi_{n+1}$, then $\boldsymbol{UM}_n \Vdash \Gamma_{n+1}$. It is defined that $\boldsymbol{UM}_{n+1} \Vdash \Gamma_{n+1}$, so $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$.

Thus, all the models that satisfy Γ_n also satisfy Γ_{n+1}.

THEOREM 10. *If $\Gamma_n \nVdash \phi_{n+1}$ and ϕ_{n+1} is accepted, then $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$.*

Assume that, at a given state of the game, $\Gamma_n \nVdash \phi_{n+1}$ and $\mathrm{K}_C \Vdash \phi_{n+1}$. According to $\mathrm{R}(\phi_n)$, ϕ_{n+1} must be accepted, forming $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$. $\boldsymbol{UM}_{n+1}$ is the intersection of $\boldsymbol{UM}_n$ and $\boldsymbol{UM}\phi_{n+1}$ ($\boldsymbol{UM}_{n+1} = \boldsymbol{UM}_n \cap \boldsymbol{UM}\phi_{n+1}$). But because $\Gamma_n \nVdash \phi_{n+1}$, $\boldsymbol{UM}_n \nVdash \phi_{n+1}$. So not all models that satisfy $\Gamma_n$also satisfy ϕ_{n+1}. Since $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$, not all models that satisfy Γ_n also satisfy Γ_{n+1}. Thus $\boldsymbol{UM}_{n+1} \neq \boldsymbol{UM}_n$. But Γ_n is contained in Γ_{n+1}, so all models that satisfy Γ_{n+1} also satisfy Γ_n – $\boldsymbol{UM}_{n+1} \Vdash \Gamma_n$. So $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$.

Thus, all the models that satisfy Γ_{n+1} are contained in the set of models that satisfy Γ_n.

Summing up; in an obligational game, $\boldsymbol{UM}_{n+1} \subseteq \boldsymbol{UM}_n$. If $\Gamma_n \Vdash \phi_{n+1}$, $\Gamma_n \Vdash \neg\phi_{n+1}$ or $\mathrm{R}(\phi_{n+1}) = ?$, then $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$, otherwise $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$. That is, the larger the informational base, the fewer models will satisfy it, and greater the constraints on the choice between $\neg\phi_n$ and ϕ_n will be (a model-theoretic way to see why a larger base implies that more propositions will have inferential relations with Γ_n). Clearly, the base is expanded (and therefore the range of models that satisfy it is reduced) only by inclusion of 'irrelevant' propositions.

4.3 Swyneshed's treatise: nova responsio

The main modification introduced by Swyneshed concerns the notion of pertinent propositions: while for Burley whether a *propositum* was pertinent was determined on the basis of all the previously accepted or denied propositions (*positum* and *proposita*), for Swyneshed being pertinent is a property of a *propositum* only with respect to the *positum*.[66] There were other aspects that were thought to characterize the *nova responsio* (such as the treatment of pragmatically inconsistent *posita* — cf. [Braakhuis, 1993, 334]), but here I focus on the redefinition of the notion of pertinent proposition and its corollaries (such as the behavior of conjunctions and disjunctions). As a result, while the main purpose of Burley's *obligationes* seems to be consistency maintenance, Swyneshed's *obligationes* seems to have as its main purpose that of inference recognition.[67]

4.3.1 Reconstruction

In Swyneshed's version, an obligation corresponds to the following quadruple:

DEFINITION 11 (The obligational game (Swyneshed)).

$$\boldsymbol{Ob} = \langle \Sigma, \Phi, I, R(\phi_n) \rangle$$

Σ is an ordered set of states of knowledge S_n. This is the first significant difference with respect to Burley's theory. In the latter, all irrelevant propositions were supposed to be answered to according to the static state of common knowledge K_C. Changes in things during the time of the disputation were not supposed to affect the response to (irrelevant) propositions, all the more since, once proposed and accepted or denied, these were included in the 'informational base' of the disputation. So, in Burley's theory, if, at a certain point, 'You are seated' is proposed to Respondent, and Respondent is indeed seated, he should accept the proposition. Subsequently, if Respondent stands up, and Opponent proposes 'You are not seated', Respondent should deny it, because it contradicts the set of previously accepted/denied propositions, and this logical relation has priority over reality.

In Swyneshed's theory, since irrelevant accepted or denied propositions are not included in the informational base of the disputation, as we shall see, the state of knowledge is not required to be static. So the response to irrelevant propositions, according to Swyneshed's theory, should take into account the changes in things during the time of the disputation; therefore, what we have is a series of states of knowledge S_n, ordered according to their index n, which is a natural number and corresponds to the stage of the disputation in which the state of common

[66]This fact has been acknowledged by virtually all studies on medieval *obligationes*, including [Stump, 1981; Ashworth, 1981; 1993; Spade, 1982b; Keffer, 2001] etc.

[67]In fact, from this point of view, the disagreement between Burley and Swyneshed may be viewed as a disagreement concerning the very purpose of *obligationes*, in terms of a student's training.

knowledge must come into play.[68]

Φ is an ordered set of propositions ϕ_n. (No difference here with respect to Burley's theory.)

I is an ordered set of responses $\iota_n = [\phi_n;\ \gamma]$. Responses are ordered pairs of propositions and one of the replies 1, 0 or ?, corresponding to Respondent's response to proposition ϕ_n.[69] Note that the index of the response need not be (but usually is) the same as the index of the proposition, in case the same proposition is proposed twice, in different moments of the disputation (in which case, for convenience, it is referred to by the index it received in the first time it was proposed).

$\mathbf{R}(\phi)$ is a function from propositions to the values 1, 0, and ?. This definition is identical to the definition of $\mathbf{R}(\phi)$ in the reconstruction of Burley's theory, but the **function** corresponding to the rules of Swyneshed's theory is different from the function of Burley's theory, since the rules are different.

4.3.2 Rules of the game

Swyneshed's **procedural rules** are quite simple (cf. [Spade, 1978, §72][70]), and identical to the procedural rules in Burley's theory. By contrast, the logical rules are quite different from Burley's.

Swyneshed's analysis of the requirements for a proposition to be accepted as *obligatum* (that is, the first proposition proposed, named *positum* in the specific case of *positio*) is less extensive than Burley's. Since an inconsistent *positum* gives no chance of success for Respondent, Burley clearly says that the *positum* mustn't be inconsistent. Swyneshed does not follow the same line of argumentation; rather, he requires that a proposition be contingent to be a *positum* (§ 73). This excludes impossible propositions — always false — and necessary propositions – always true –, and that is a necessary requirement in view of the *ex impossibili sequitur quodlibet* rule: if Swyneshed's rules of *obligationes* are indeed meant to test Respondent's abilities to recognize inferential relations, an impossible *obligatum* would make the game trivial (any proposition would follow).[71] Moreover, from a necessary proposition only necessary propositions follow, so if the *obligatum* is a necessary proposition, then the game becomes that of recognizing necessary

[68]But why use states of knowledge, and not simply states of affairs? Because (both in Burley's and Swyneshed's theories) proposed propositions whose truth-value is unknown to the participants of the disputation — for example, 'The Pope is sitting now' — should be accordingly doubted. We are dealing here with imperfect states of information.

[69]In my reconstruction of Burley's theory, responses were not primitive constituents of the game. But to express some of the interesting properties of Swyneshed's theory, the notion of responses is crucial.

[70]All references are to Spade's edition of the text. For the relevant passages and translations, see [Dutilh Novaes, 2006a].

[71]Notice that Swyneshed's reason for excluding impossible propositions is different from Burley's trivialization of the game *versus* absence of a winning strategy for Respondent. Keffer [2001] has also remarked that impossible (and true) *posita* have a *Trivialisierungseffekt* on both kinds of responses, but for different reasons (pp. 158-164).

propositions, i.e. a deviation from its (presumed) original purpose.

So the rule for accepting the *positum* could be formulated as:

DEFINITION 12 (Rules for *positum*).

$R(\phi_0) = 0$ iff, for all moments n and m, and for one reply γ, $\iota_n = [\phi_0; \gamma]$ and $\iota_m = [\phi_0; \gamma]$.

$R(\phi_0) = 1$ iff, for some moments n and m, for two replies γ and κ, $\gamma \neq \kappa$, $\iota_n = [\phi_0; \gamma]$ and $\iota_m = [\phi_0; \kappa]$

Moreover, Swyneshed also gives instructions as to how to respond to the *positum* if it is **posited again** during the disputation (§§ 62-64). A *positum* which is re-proposed must be accepted, except in the cases of a *positum* which is inconsistent with the very act of positing, admitting and responding in an obligational context. The paradigmatic example is 'Nothing is posited to you': it should be accepted as a *positum*, according to the rules above, but if it is again proposed during the same disputation, it should be responded to as if it were an irrelevant proposition. In this case, it would be denied, even though it had been accepted as *positum*. Burley, by contrast, would probably not accept such pragmatically inconsistent propositions as *posita* in the first place (cf. [Braakhuis, 1993, 330]).

In effect, from the start, the set of all propositions (not only those put forward during the disputation, which constitute Φ) is divided in two sub-sets, namely the set of propositions that are **pertinent** with respect to the *positum* ϕ_0(§4, §7) — denoted $\Delta_{\phi 0}$- and the set of those that are **impertinent** with respect to the *positum* ϕ_0(§8) – denoted $\Pi_{\phi 0}$. The sets are defined as follows:

DEFINITION 13 (Pertinent and impertinent propositions).

$$\Delta_{\phi 0} = \{\phi_n \varepsilon \Delta_{\phi 0} : \phi_0 \Vdash \phi_n \text{ or } \phi_0 \Vdash \neg\phi_n\}$$
$$\Pi_{\phi 0} = \{\phi_n \varepsilon \Pi_{\phi 0} : \phi_0 \nVdash \phi_n \text{ and } \phi_0 \nVdash \neg\phi_n\}$$

Assuming that any proposition implies itself, the *positum* $\phi_0$belongs to $\Delta_{\phi 0}$.[72] E. Stump [1981, 167] mentions the possibility of allowing for a second *positum* at any given moment of the disputation. In this case, obviously the two sets defined above must be revised, and the set of pertinent propositions is defined by the conjunction of the two (or more) *posita*.

4.3.2.1 Proposita The rules for responding to proposed propositions other than the *positum* are better formulated in two steps, first for the pertinent, then for the impertinent propositions, as this division is in fact the decisive aspect of the game in Swyneshed's version.

DEFINITION 14 (Rules for *proposita*).

[72]Except for the *posita* that are (pragmatically) repugnant to the act of positing (cf. §64); according to Swyneshed, those should be answered to as if they were impertinent, thus belonging to $\Pi_{\phi 0}$.

Pertinent propositions ($\phi_n \neq \phi_0$, $\phi_n \ \varepsilon \ \Delta_{\phi 0}$) ((§24, §67, §68).

$$\mathbf{R}(\phi_n) = 1 \text{ iff } \phi_0 \Vdash \phi_n{}^{73}\mathbf{R}(\phi_n) = 0 \text{ iff } \phi_0 \Vdash \neg\phi_n$$

Impertinent propositions ($\phi_n \ \varepsilon \Pi_{\phi 0}$) (§ 26, §69)

$$\mathbf{R}(\phi_n) = 1 \text{ iff } S_n \Vdash \phi_n$$
$$\mathbf{R}(\phi_n) = 0 \text{ iff } S_n \Vdash \neg\phi_n$$
$$\mathbf{R}(\phi_n) = ? \text{ iff } S_n \nVdash \phi_n \text{ and } S_n \nVdash \neg\phi_n$$

The game ends when Opponent says '*Cedat tempus obligationis'*. From Swyneshed's text, it seems that Opponent can say it at any time; he will say it when Respondent has made a bad move, and thus has lost the game (§98), but he may say it when he is satisfied with the performance of Respondent, who until then has not made any bad move, and therefore has 'won' the game.

4.3.3 Logical properties of nova responsio

Now I discuss some of the noteworthy logical properties of *nova responsio*, in particular those related to the reformulation of the notion of impertinent proposition.

4.3.3.1 The game is fully determined It is clear that, in Swyneshed's *obligationes*, only one answer to each proposition is correct at a given point. That this is the case is seen from the fact that R(ϕ) really is a function, assigning exactly one value to each argument of its domain (the class of propositions). Swyneshed's rules divide the class of propositions in two sets and in five sub-sets: pertinent propositions — 1. repugnant to or 2. following from the *positum* — and impertinent propositions — 3. which are known to be true; 4. which are known to be false; 5. which are not known to be true and are not known to be false. These five subsets exhaust the class of propositions, and for each of them there is a defined correct answer. In other words, at each stage of the disputation, Respondent's moves are totally determined by the rules of the game.

4.3.3.2 The game is not dynamic The game played according to the *antiqua responsio* is, as we have seen, dynamic in that the player must take into account all previous moves of the game in their corresponding order. By contrast, the game played according to the *nova responsio* is 'static': the response to a proposition is entirely independent of the order in which it occurs during the disputation, as it is entirely independent of all previous moves except for the first one, relative to the *positum*. In effect, for any proposition ϕ_n, at any round n of the disputation, the reply to ϕ_n is always the same.

[73] Clearly, if the introduction of extra *posita* occurs, then this definition holds for the set of *posita*, instead of for the first *positum* only.

Indeed, the great difference with respect to Burley's theory is that, in Swyneshed's version, the game is totally determined already once the *positum* has been posited, from the start, and not only at each move. All Respondent has to do is to determine correctly the two sets of pertinent and impertinent propositions from the outset. Opponent can do nothing to interfere with Respondent's winning strategy, as it simply consists of assessing correctly the presence or absence of relations of inference between the *positum* and the proposed propositions. Once more, the fact that the game is totally determined from the moment the *positum* is posited means that the order of presentation of the *proposita* does not matter, and that Opponent cannot do much to make the game harder for Respondent. Moreover, it also means that, during a disputation, only one response is the right one for a given proposition, independent of when it is proposed. In Burley's game, it can happen that a proposition is first doubted (as impertinent and unknown) and then accepted or denied (it has become pertinent in the meantime, given the expansion of the informational base). This cannot occur in Swyneshed's game.

There is one exception to this rule: impertinent propositions whose truth-values change during the course of the disputation. Swyneshed says that these propositions should be responded to according to the state of knowledge of that moment, and therefore the response depends on the moment in which they are proposed — but not on the moment **within the disputation** in which they are proposed (their relative position with respect to other propositions). Similarly, if such propositions are proposed twice during the same disputation, they may receive different answers, as a consequence of a change in things.

4.3.3.3 Two disputations with the same *positum* will prompt the same answers, except for variations in things. This is perhaps the main motivation for the changes introduced by Swyneshed to the obligational game. In many passages, he emphasizes that the response to impertinent propositions must vary only in virtue of changes in things, and not in virtue of other previously accepted/denied propositions. Indeed, the crucial element of a winning strategy for Swyneshed's game is the accurate definition of the two sets of propositions relative to a *positum* (the set of pertinent propositions and the set of impertinent ones).

So, if the game is defined once the *positum* is posited, then any two disputations with the same *positum* have **the same winning strategy**, that is, the establishment of the same two sets of pertinent and impertinent propositions. Since the propositions proposed by Opponent may vary, two disputations with the same *positum* will not necessarily be identical. But any given proposition proposed in both disputations will belong to the same set of propositions — either pertinent or impertinent — in both cases.

Again, the dissimilarity with Burley's theory is striking. In Burley's version of the game, the *positum* was merely one of the propositions constituting the set according to which a proposed proposition was to be evaluated as pertinent or impertinent (the others being the previously accepted/denied propositions). So in two disputations having in common only the *positum*, a given proposition proposed in each of them was most likely bound to receive different responses.

4.3.3.4 Responses do not follow the usual properties of the connectives

One of the most discussed aspects of the *nova responsio*, not only among medieval authors but also among modern commentators, is the non-observance of the usual behavior of some sentential connectives, in particular conjunction and disjunction. This is a corollary of the basic rules of the *nova responsio*, and it was thought to be one of its distinctive traits.[74]

Apparently, these two corollaries have struck some of Swyneshed's contemporaries as very odd, and were for them sufficient reason to reject the *nova responsio* as a whole. However, in careful inspection, it is only in appearance that two of the most fundamental laws of logic — the truth-conditions of conjunction and disjunction – are being challenged. As Yrjönsuuri suggested [Yrjönsuuri, 1993, 317], it is as if the bookkeeping of a Swyneshed-style obligational disputation featured two columns of responses, one for pertinent propositions and one for impertinent propositions. **Within each column**, the laws for conjunction and disjunction are in effect observed. So, if in one of the columns two propositions have been correctly granted, then their conjunction will also be granted (disregarding changes in things); similarly, if in one of the columns a disjunction has been correctly granted, then at least one of the disjuncts will have to be granted too.[75] This fact only emphasizes the idea that the crucial aspect of playing a Swyneshed-style game of *obligationes* is the correct division between pertinent and impertinent propositions.[76]

4.3.3.5 The set of accepted/denied propositions can be inconsistent

Perhaps the most surprising feature of Swyneshed's *obligationes* is the little importance attributed to consistency maintenance. That is, if one takes the set of all propositions granted and the contradictories of all propositions denied during a disputation, this set is very likely to be **inconsistent**, and this feature struck many medieval authors as very odd (cf. [Keffer, 2001, pp. 164-166]). There are two main sources of inconsistency in Swyneshed's game: the most obvious one is the case of impertinent propositions which receive two different responses in different times of the disputation (in particular if they are first denied and then accepted or vice-versa), in virtue of changes that occurred in things during the time of the disputation. The second source of inconsistency for this set is the behavior of conjunctions and disjunctions explained above (§101).

But again, the bookkeeping metaphor implies that this corollary is not as awkward as it seems. Since the set of propositions that follow from a proposition is

[74] Cf. [Stump, 1981, 139]. For formal proofs of these properties, see [Dutilh Novaes, 2006a; Keffer, 2001, pp.176-178].

[75] For simplicity, I am disregarding impertinent propositions whose truth-value may change during the disputation.

[76] For a discussion of the apparent conflict arising in cases in which a conjunction or disjunction is formed by propositions taken from both columns, see (Dutilh Novaes 2006a).

always consistent, the column for pertinent propositions will always be consistent, — for each contradictory pair of propositions (A, $\neg$A), a given proposition B implies either one of them (B $\rightarrow$A), or the other (B$\rightarrow$ $\neg$A), or none, but never both contradictory propositions. Therefore, it will never be the case that a *positum* forces the granting of a proposition A and of its contradictory $\neg$A. By contrast, the column for impertinent propositions can very well be inconsistent, in the case of impertinent propositions whose truth-value changes during the disputation and which are in fact proposed twice (and receive different responses).

These considerations indicate thus that Swyneshed has no interest whatsoever in the set formed by all granted and denied propositions during a disputation, and that he is perfectly willing to accept its inconsistency. For Burley, on the contrary, the ultimate goal of the *obligationes* game is to keep this very set consistent. So the differences between the two versions of the game do not only regard the rules governing them, but seemingly the very motivations for playing the game.

4.4 Conclusion

This section is not intended to give a comprehensive picture of the *obligationes* genre; the topic is much more complex than what can be covered in just a few pages. My goal was just to outline some of the interesting logical properties of this genre so as to inspire the reader to go look for further literature on the topic. Some significant aspects of *obligationes* had to be left out for reasons of space; in particular, the sophisms (i.e. logical puzzles) treated in the *obligationes* literature are particularly interesting, especially those related to self-reference and to pragmatic inconsistencies — see for example [Ashworth, 1993; Pironet, 2001]

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