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Recall:

- *Conditional Probability*: Knowledge about whether or not event A has occurred may affect the calculation of the probability of event B. We *define*

$$P(B|A) \equiv \frac{P(AB)}{P(A)}.$$

- *Independence*: If A and B are independent, then $P(A \cap B) = P(A)P(B)$.

Example: An electronic assembly consists of two components: A and B. The probability that A fails is 0.20, the probability that B fails alone is 0.15, and the probability that A and B both fail is 0.15. What is the probability that A fails given that B has failed? What is the probability that A fails alone?

Discrete Random Variables and Their Distributions

Example: You will toss a fair coin until it comes up Heads. What is the probability that Heads will show up for the first time on the

1. First toss?

2. Second toss?

3. Third toss?

4. Fourth toss?

5. i^{th} toss?

It is often useful to assign numerical values to the outcomes in a sample space. For example, given the two sequences arising from coin tosses

$$H\ T\ H\ H\ T\ T\ H\ H\ T\ H\ T\ H$$

and

$$H\ H\ H\ H\ H\ T\ T\ H\ T\ T\ T\ H$$

we might only be concerned with the number of heads (if, for instance, heads pay $5). In this case we would consider the two sequences to be identical and only care about the probability of k heads on a 12 toss sequence.

<u>Definition</u>
Given a sample space S, we define a random variable, X, to be a function which maps elements of the sample space, s, to real numbers, $X(s)$.

Notice that a random variable *induces* probabilities on subsets of the real line. For example, consider the toss of a fair coin with 4 tosses. What is the sample space?

Now define a function X on this sample space as

$$X(s) = \textit{number heads} - \textit{number tails}.$$

What are possible values for $X(s)$? What are the probabilities associated with these values?

<u>Definition</u>
Given a random variable X, define X to be discrete if X may assume a finite or a countably infinite number of values $x = X(s)$. Also, define a function $p(x) = P(X = x)$. $p(x)$ is called the probability function of X or the probability mass function of X.

We have the following:

1. $p(x) \geq 0 \; \forall \; x$

2. $\sum_x p(x) = 1$

Example: You are playing the following simple game: A fair die is rolled once. Experimentally determine an empirical probability mass function. On average, what do you obtain for a face value?

Cumulative Distribution Function The CDF of a discrete random variable X, denoted $F(x)$, is defined for each real number x as

$$F(x) \equiv P(X \leq x) = \sum_{y \leq x} p(y)$$

Determine the CDF for the example above, rolling a fair die.

We will define the formula for the expectation of a random variable in a manner analogous to the formula you obtain by treating the example above, $X(s) = \textit{number heads} - \textit{number tails.}$, as a set of grouped data:

$$ave = \frac{1}{\sum_i freq(x_i)} \sum_i x_i freq(x_i) = \frac{1}{\sum_{i=0}^{4} freq(i)} \sum_{i=0}^{4} i freq(i)$$

Now let probabilities take the place of relative frequencies $\frac{freq(x_i)}{\sum freq(x_i)}$ and define the *expected value*, written as E, (also called the *mean* and written μ), of a random variable in a natural way as

$$E(X) \equiv \mu \equiv \sum_x x f(x)$$

when the sum exists.

Example: Consider a sequence, s, of 4 tosses with a fair coin, eg. s = HTTH. Define a random variable $X(s) =$ *number of heads in s.*

The sample space and random variable defined by this problem:

s	number of heads
HHHH	4
HHHT	3
HHTH	3
HHTT	2
HTHH	3
HTHT	2
HTTH	2
HTTT	1
THHH	3
THHT	2
THTH	2
THTT	1
TTHH	2
TTHT	1
TTTH	1
TTTT	0

The sample space may be summarized as

number of heads	frequency
4	1
3	4
2	6
1	4
0	1

Calculate the expected value of this random variable.