Recall:

- $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
- $X_1, \ldots, X_n$ *iid* with common mean μ and common variance σ^2, $\Rightarrow \frac{\bar{X}-\mu}{s/\sqrt{n}} \approx N(0,1)$ for large n, say $n \geq 30$.
- $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t(df = n-1)$

1. You have been told that SAT scores for the population of applicants to a certain university has an average value of 1250 combined score. You believe the average (mean) is lower and so you gather the following data:

   1156 1033 1212 1228 1085 1319 1318

   Set up and test an appropriate hypothesis if

   (a) You wish to use the traditional SAT standard deviation value of 100 points.

   (b) You are not certain that this standard deviation is appropriate for your particular population and so estimate the population standard deviation with s.

- Acceptance Region/Rejection Region
- Type I error and α
- Type II errors and the β's
- p value

Suppose you are testing the null hypothesis $H_0 : \mu = 100$ against the alternative $H_1 : \mu > 100$ and you take the critical x value separating the acceptance and rejection regions to be $x_{crit} = 110$. Suppose further that the population standard deviation is $\sigma = 20$ and that you are sampling with sample size $n = 16$.

1. What is the probability that you will commit a type I error?

2. Suppose that, in reality, $\mu = 104$. What is the probability, *in this case* that you will commit a type II error?

3. Fill int the following table:

μ	β
90	
95	
100	
105	
110	
115	

In general, when we plot the probability of **accepting** the null hypothesis (i.e. β) against the set of possible population means we refer to an *Operating Characteristic Curve*. When we plot the probability of **rejecting** the (false) null hypothesis (i.e. $1-\beta$) against the set of possible population means we refer to the *Power Function* of the statistical test.

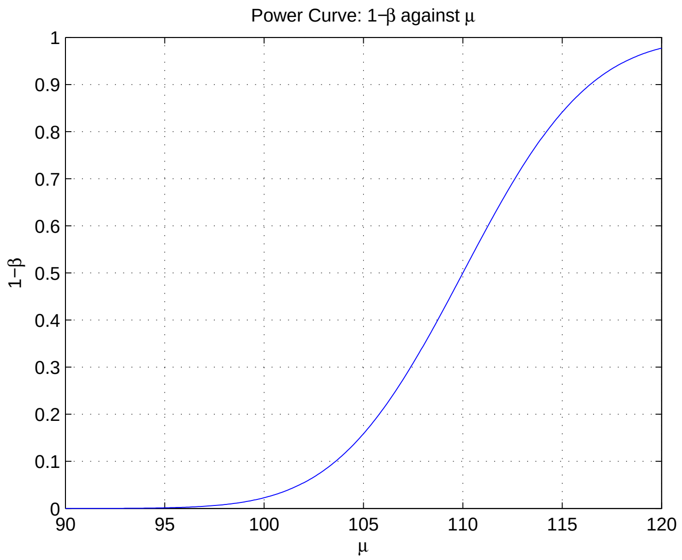
The following *MATLAB* code produced the accompanying plot.

```
clc;
clear;

n            = 16;
sigma        = 20;
xcritical    = 110;

i=0; for mu = 90:.1:120
    i=i+1;
    beta = normcdf(xcritical,mu,sigma/sqrt(n));
    power(i) = 1-beta;
end

plot(90:.1:120,power)
title('Power Curve: 1-\beta against \mu')
xlabel('\mu')
ylabel('1-\beta')
grid on
```



Develop a general formula for β as a function of μ given α, σ, and n.

1. $H_1 : \mu > \mu_0$

2. $H_1 : \mu < \mu_0$

3. $H_1 : \mu \neq \mu_0$

Develop a general formula for n as a function of $\mu,\ \alpha,\ \sigma,$ and β.

1. $H_1 : \mu > \mu_0$

2. $H_1 : \mu < \mu_0$

3. $H_1 : \mu \neq \mu_0$