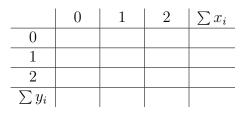
Applied Probability

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You will perform the following experiment: From a fair deck, deal two cards. Let the random variable X indicate the number of spades in your hand and let the random variable Y indicate the number of hearts.

We may construct the following table showing the possible counts in each of the categories shown.



Now construct the joint distribution of X and Y.

	0	1	2	$\sum f(x_i)$
0				
1				
2				
$\sum f(y_i)$				

Are X and Y independent?

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*Example* Now keep X, the number of spades, as above, but let Z indicate the number of face cards, i.e. Jack, Queen, King.

Construct the table showing the possible counts in each of the categories shown.

	0	1	2	$\sum x_i$
0				
1				
2				
$\sum z_i$				

Now construct the joint distribution of X and Z.

	0	1	2	$\sum f(x_i)$
0				
1				
2				
$\sum f(z_i)$				

Are X and Z independent?

Define the *correlation function* of two random variables X and Y as

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

In order to better understand what cov(X, Y) and  $\rho(X, Y)$  are designed to measure we present two important results.

**Result 1** Let X and Y be independent random variables. Then

cov(X, Y) = 0 and  $\rho(X, Y) = 0$ 

**Result 2** Let X and Y be random variables such that Y = aX + b. Then 1.

$$\mu_Y = a\mu_X + b$$

2.

$$E[XY] = aE[X^2] + bE[X]$$

3.

or

$$V(Y) = a^2 V(X)$$
$$\sigma(Y) = |a|\sigma(X)$$

From these, conclude that

4.

$$\rho(X,Y) = \pm 1$$

**Result 3** Suppose that X and Y are random variables, and define S = aX + bY. Calculate V(S).

Note the following special case.

**Result 4** If X and Y are independent, then V(X + Y) = V(X) + V(Y).

We may extend the above in a natural way as

$$V(\sum_{i} a_i X_i) = \sum_{i} a_i^2 V(X_i)$$