You will perform the following experiment: From a fair deck, deal two cards. Let the random variable X indicate the number of spades in your hand and let the random variable Y indicate the number of hearts.
We may construct the following table showing the possible counts in each of the categories shown.

	0	1	2	$\sum x_i$
0				
1				
2				
$\sum y_i$				

Now construct the joint distribution of X and Y.

	0	1	2	$\sum f(x_i)$
0				
1				
2				
$\sum f(y_i)$				

Are X and Y independent?

Example Now keep X, the number of spades, as above, but let Z indicate the number of face cards, i.e. Jack, Queen, King.

Construct the table showing the possible counts in each of the categories shown.

	0	1	2	$\sum x_i$
0				
1				
2				
$\sum z_i$				

Now construct the joint distribution of X and Z.

	0	1	2	$\sum f(x_i)$
0				
1				
2				
$\sum f(z_i)$				

Are X and Z independent?

Define the *correlation function* of two random variables X and Y as

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

In order to better understand what $cov(X,Y)$ and $\rho(X,Y)$ are designed to measure we present two important results.

Result 1 *Let X and Y be independent random variables. Then*

$$cov(X,Y) = 0 \text{ and } \rho(X,Y) = 0$$

Result 2 *Let X and Y be random variables such that $Y = aX + b$. Then*

1.

$$\mu_Y = a\mu_X + b$$

2.

$$E[XY] = aE[X^2] + bE[X]$$

3.

$$V(Y) = a^2V(X)$$

or

$$\sigma(Y) = |a|\sigma(X)$$

From these, conclude that

4.

$$\rho(X, Y) = \pm 1$$

Result 3 *Suppose that X and Y are random variables, and define $S = aX + bY$. Calculate $V(S)$.*

Note the following special case.

Result 4 *If X and Y are independent, then $V(X + Y) = V(X) + V(Y)$.*

We may extend the above in a natural way as

$$V(\sum_i a_i X_i) = \sum_i a_i^2 V(X_i)$$