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Working With Continuous Distributions Consider the following examples.

Define

$$f(x,y) = \begin{cases} ax(x+y) & 0 < x < 1,\ 0 < y < 2 \\ 0 & else \end{cases}$$

1. Calculate a to make this a legitimate PDF.
2. Calculate the joint CDF $F(x,y) = P(X < x, Y < y)$.
3. Calculate $P(X < .5, 1 < y < 1.5)$.

Define

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & x,y>0 \\ 0 & else \end{cases}$$

1. Calculate the joint PDF $f(x,y)$.
2. Calculate the marginal distributions.

Define

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 < x, y < 1 \\ 0 & else \end{cases}$$

1. Calculate the marginal distributions.
2. Calculate the conditional distribution $X|Y = y$.

Define

$$f(x,y) = \begin{cases} 4xy & 0 < x, y < 1 \\ 0 & else \end{cases}$$

1. Calculate the marginal distributions.
2. Calculate the conditional distributions $X|Y = y$ and $Y|X = x$.

Given a joint distribution $f(x, y)$, we say that X and Y are independent if

$$f(x, y) = f_X(x) f_Y(y)$$

Let X and Y be independent, identically distributed uniform random variables uniformly distributed on $(0, 1)$. (Note: we can write this as $X, Y \overset{iid}{\sim} unif(0, 1)$). Calculate the density of $Z = X + Y$ and verify your result numerically.

Let X and Y be independent, identically distributed random variables exponentially distributed with $\lambda = 1$. (Note: we can write this as $X, Y \overset{iid}{\sim} exp(1)$). Calculate the density of $Z = X + Y$ and verify your result numerically.