Prof. Thistleton

Examples: The following examples show the types of problems we will be able to handle as the semester proceeds.

- Would you rather go to a bank which has 2 tellers, both of whom work at the same average rate, or a bank with 1 teller who works twice as fast?

- You have been told that a certain brand of light bulb will burn, on average, for 1000 hours. You have a bulb which has been burning for 900 hours. How much longer do you expect it to burn?

- You wish to determine whether or not a coin is fair. You toss it 1000 times and obtain 545 heads. Do you think the coin is fair?

- An employee of yours has tested positively for drug use. You know that the incidence of drug use for the general population is 10%. If the sensitivity of the test is 95% and the specificity of the test is 97%, do you believe the employee has been using drugs?

Preliminaries: There are certain mathematical prerequisites for successfully dealing with the material in our course. The following illustrate several of these techniques. Homework 1 will serve to further refresh/enhance your skills.

1. $\int_0^\infty e^{-x} dx$

2. $\int_0^\infty x e^{-x} dx$

3. $\int_0^\infty x e^{-x^2} dx$

4. $\int_0^\infty x e^{-ax} dx$

5. $\int_0^\infty x^n e^{-ax} dx$

6. $\sum_{k=0}^{n} a^k$

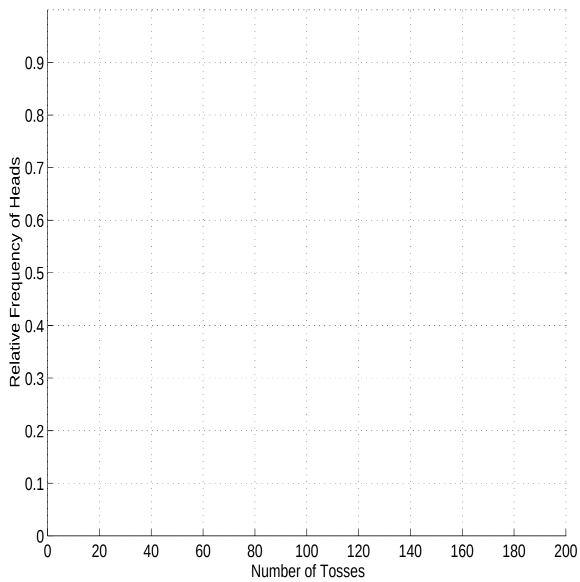
7. $\sum_{k=0}^{\infty} p^k$ where $-1 < p < 1$.

8. $\int_0^4 \int_0^2 \ x^2 \ y \ dxdy$

9. $\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} p^j$ where $-1 < p < 1$.

Basic Notions of Probability: We can consider our course to develop methods of quantifying and characterizing uncertainty in various situations. For example, consider a coin toss with a fair coin. We will toss the coin 20 times and, after each toss, calculate the *relative frequency* of heads.

Toss	Relative Frequency



Relative Frequency Notion of Probability: We will need a few terms.

- Experiment

- Outcome

- Event

Now, repeat an experiment many times. *(What does many mean?)* Let E be an event associated with the experiment. Perhaps the most intuitive way to define the probability of an event E, which we will denote as $P(E)$, is as follows.

$$P(E) \equiv \lim_{n\to\infty} \frac{Number\ of\ times\ E\ occurs}{n}$$

There are two key ideas here: On the one hand, we must be able to reproduce the experiment as often as we like, under unchanging conditions. We are also assuming that the limit defined above exists. If these conditions are not met, then we must reconsider what we mean by the word *probability.*

Classic Examples:

- Tossing a fair coin and observing H or T.
- Rolling a fair die and observing 1, 2, 3, 4, 5, or 6.
- Observing the lifetime of a light bulb.
- Observing the lifetime of someone who is currently 40 years old and a non-smoker.
- Counting the number of a certain type of microorganism in a 10 `ml`. sample from a lake.

Notice that we may not directly attach a probability to the statement *We will have a colony on the moon in the year 2100* in a meaningful way with the above notion of probability.

Famous Example Suppose you are rolling dice with a friend. She offers you \$ 1 if no 6 occurs on the next 4 rolls of a fair die. You must pay her \$ 1 if you obtain at least one 6. Is this a fair bet?

The following *MATLAB* code produced the result below.

```
%mere.m
numThrows = 1000000;
numTrials = 4;
gotOne = 0;
for i=1:numThrows
   data = unidrnd(6,1,numTrials);
   for j = 1:numTrials
      if ( data(j) == 6 )
         gotOne = gotOne + 1;
         break
      end
   end
end
gotOne/numThrows
```

RESULT = 0.517549