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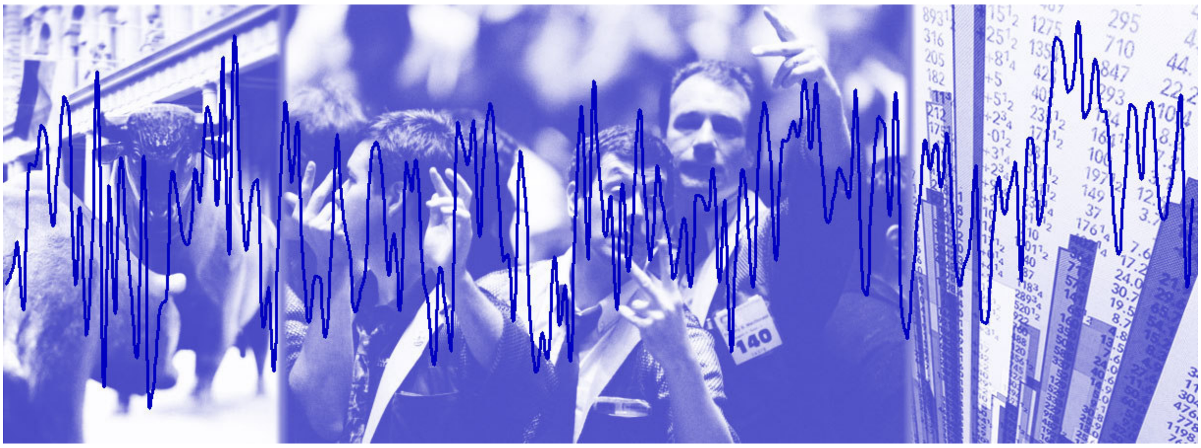

In the Mood

Investor Sentiment, Stock Returns and Volatility in Germany

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Marco van Daele (i079006)

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Supervisor: T. Lehnert

“I'd be a bum on the street with a tin cup
if the markets were efficient”

Warren Buffett

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1. Introduction

Warren Buffet, Chairman of Berkshire Hathaway, is one of the richest men of all times. He became that way through a simple strategy: identify valuable companies and invest in them at a price that leaves room for improvement. While the strategy sounds simplistic, it has obviously paid off, in the sense of the word.

Financial economists in turn are not exactly reputed for their enormous wealth. Maybe they think that money is not easy to be made in stock markets or, in the words of Milton Friedman, that "there is no free lunch". They used to have a seemingly well-understood and proven argument for this assumption: market efficiency.

The Efficient Markets Hypothesis, or EMH, was one of the most influential postulates in financial economics. Some argue that the whole field of academic finance emerged to question and test the implications of the statement that "prices are right". At first, most empirical evidence strongly supported the theory. Over time though, several phenomena were observed that are hard to explain with the classical tools: Closed-end funds persistently trade at a discount to their net asset value. Stocks of companies that are added to a widely-followed index experience a significant excess return around the time of inclusion. Equity carve-outs are valued at multiples of their original values, and prices of listed companies that have nothing in common except for a similar ticker symbol move in unison. Finally, stock prices seem to be at least partly predictable. The last blow came on October 19, 1987, when the Dow Jones collapsed by some 20% on a single day, without any news that could only marginally justify such a crash.

After that incident, the then still young field of Behavioural Finance experienced a surge in interest. New theories were developed on the basis of a new insight: at least some investors behave irrationally, and they do have an influence. Academic work thus concentrated on two distinct areas: First, the nature of their irrationality, and second, the kind and amount of influence they have on markets. To distinguish the two, the term Behavioural Finance and Noise Trader theory will be used, respectively. This thesis will concentrate on the latter.

1.1 Motivation

Numerous empirical analyses on Noise Trader theory have been carried out over the last two decades, yet up to now only a few studies have tested all the important implications, especially for expected returns and volatility. Additionally, the most appropriate methodological approach is not yet found. The first dispute is mainly about how to account for noise traders' irrational perceptions (sentiment). Several proxies and indicators have been used, only some of which have yielded significant results. Next, most researchers have focused on finding a direct relationship between sentiment and expected returns, which is only part of the original noise trading theory. The creation of an additional, systematic risk lies at the core of the theory. Changed perceptions of risk, or volatility, are an important channel through which sentiment presumably affects returns. Both returns and volatility should consequently be taken into account. Another important shortfall of previous studies is that most of them assume that noise traders equal individual investors. A frequent outcome of studies on sentiment was however that measures of *institutional* sentiment seemed sometimes to be more significantly related to returns than individual sentiment. Finally, nearly all empirical studies have been performed for U.S. American equity markets, mostly due to lack of data on sentiment measures from other, especially German markets. This has changed recently, which provides an opportunity to test the robustness of previous findings.

1.2 Problem Statement and Research Questions

The following problem statement will serve as a guide for the remainder of this thesis:

Does investor sentiment affect expected returns for stocks?

In order to make answering this problem statement feasible, the following research questions are proposed to cover the different aspects:

1. How could investor sentiment theoretically affect expected returns?
2. Which measure of investor sentiment is the most appropriate?
3. What is the most promising methodology to investigate expected returns and conditional volatility?

4. Does investor sentiment actually affect returns directly and/or through changes in conditional volatility?
5. Does the influence of investor sentiment differ between large- and small-cap markets?
6. Does the sentiment of individual and institutional investors affect large- and small-cap markets differently?

1.3 Outline

For the rest of this thesis, the attempt will be made to adequately answer the problem statement and research questions. The proceeding is as follows:
Chapter 2 briefly reviews classical concepts of asset pricing and mentions some recently discovered challenges to conventional theory. Chapter 3 introduces the Noise Trader approach to asset pricing as an alternative to standard presumptions. The model of De Long, Shleifer, Summers and Waldmann (1990) is dealt with in detail, as it constitutes the theoretical basis for Noise Trader theory and provides hypotheses for further research. These hypotheses have been tested extensively in numerous studies, the most important of which are reviewed in Chapter 4. Chapter 5 contains the empirical analysis. Data and methodology are laid out, and the empirical results of the model interpreted. Chapter 6 concludes.

2. Classical Asset Pricing Theory

In the following sections, parts of the classical theory on financial economics and asset pricing will be reviewed. However, as this is only the very basis for the focal parts of this thesis, the overview will be kept as short as possible without omitting critical parts[1].

2.1 Risk and Return

For investors, risk and return form the basic elements in their investment decision. Every expected return on an investment comes at a cost: the risk associated with it. The more risk is borne by an individual investor, the higher is his expected return, as otherwise he would be foolish to take the risk. In the context of portfolio selection, Harry Markowitz (1952) used statistical techniques to formulate the optimal behaviour for investors trying to make the most out of their funds. His basic finding was that investors should not "put all their eggs in one basket", meaning that they should spread their funds over many investments in order to limit their risk exposure. This suggestion was drawn from the insight that the contribution of a single security's (idiosyncratic) risk to the total risk of the portfolio declines with its share in the portfolio. Practically, the total risk of the portfolio declines with each additional security until it reaches its lower bound. The individual risks (variances) tend to cancel out each other, and what remains is the part of risk that is not diversifiable, or systematic. Systematic risk is influenced by economic factors that affect all securities, though not necessarily equally. The individual security's contribution to total risk is measured in its covariance with other securities. If the covariance is positive and high, it adds risk as market prices then tend to swing in unison. In contrast, if covariance is low or even negative, adding the security actually reduces total risk. Logically, the more a security adds to the total, not diversifiable risk of the portfolio, the more incentive (return) an investor requires as compensation to actually hold that security. The first attempt to formalize the relationship between risk and return in capital markets has been made by Sharpe (1964). He presented the simple Capital Asset Pricing Model (CAPM), which allowed actually putting a price on risk for the first time. The model simply states that the required return of a stock is linearly related to the market premium. Differences across stocks are the result of varying exposure to market risk, as expressed in the stocks' betas, which in turn reflect the covariance with the market. For the following years, CAPM

[1] For a more complete review of classical financial theory, refer to textbooks such as Megginson (1997)

was used as a benchmark against which to test asset prices in capital markets. However, soon criticism arouse that questioned the CAPM on both theoretical and empirical bases (e.g. Gibbons, 1982; Roll, 1977). The search for an alternative model led to the foundation of Ross' (1976) Arbitrage Pricing Theory (APT), which states that each security's expected return is determined by its sensitivity to several economic factors, rather than only to the market portfolio's return. Unfortunately, he was not able to specify the economic dynamics affecting returns. Nonetheless, Chen, Roll and Ross (1986) managed to identify some of these economic factors. Specifically, they found industrial production, changes in the risk premium, the term structure of interest rates and expected inflation to be significant determinants of stock prices. However, together these factors only account for about 12% of stock price variation. Although the methodology seems most logical, the academic community has struggled to pinpoint more of these factors, which is why the CAPM, although largely inadequate, is still in use today.

Concluding, this section provided an introduction into the basic interrelations of risk and return in financial markets. The next section concerns the functioning of financial markets and the investors interacting in them.

2.2 Rationality and Arbitrage

The rationality of investors lies at the core of the classical understanding of financial markets and security prices. Investors are assumed to be Bayesian in forming fully rational expectations about future returns and risks, given the relevant information available to them. Consequently, they correctly price securities at their fundamental value. When new information arises, expectations are adapted quickly and correctly, and prices are bid to the correct level. However, even classical theory acknowledges that some investors might not be all that rational – they could e.g. trade on news that is not relevant (any more). Nevertheless, as long as their misperceptions are uncorrelated, they are likely to cancel out, and the only consequence would be increased trading volume but no shift in prices. Finally, even if misperceptions across irrational investors were correlated, i.e. systematic, the remaining rational investors would engage in arbitrage and bring prices back to fundamental value. This argument has been most forcefully made both by Friedman (1953) and Fama (1965). If e.g. a security is overpriced relative to its fundamental value, it would pay to sell (short) that security and simultaneously buy another security with the same or similar risk/return characteristics. In the ideal case, arbitrage guarantees then a nearly riskless profit, which attracts more arbitrageurs. Collectively they trade securities back to fair value. Additionally, irrational investors will be crowded out by arbitrageurs, as they tend to buy high and sell low, which has devastating wealth effects in the long run.

In sum, according to classical theory, market forces and rational investors competing for arbitrage profits will always guarantee that prices reflect fundamentals. These ideas have been integrated by Fama (1970) into his fundamental hypothesis of efficient markets, which is laid out in the following section.

2.3 Market Efficiency

Eugene Fama (1970) published one of the most influential economic papers, practically the null hypothesis against which all subsequent capital markets research had to prevail. In formulating his Efficient Market Hypothesis (EMH henceforth), he defined what constitutes an efficient capital market, where efficiency means the ability to adequately incorporate relevant information into security prices. Fama (1970) distinguished three cumulative degrees of market efficiency:

- Weak-form market efficiency holds when all relevant historical information is reflected in asset prices, so that no inferences from past share prices can be drawn for their future development. This implies that markets react only to new information, which means that market movements are unpredictable in the sense that they could not have been derived from past facts. Consequently, the market is said to follow a random walk, where the future movements of stock prices are purely stochastic, and not serially or cross-sectionally correlated.
- The semi-strong form, which is also the most widely supported by earlier empirical evidence, entails that *all* relevant publicly available information, including historical, is incorporated in security prices. This assumption bears considerable consequences: as soon as any information becomes available that affects a company's future cash flows or associated risks, its share price adjusts instantaneously and accurately so it remains fair.
- The strong version of market efficiency says that *really all* relevant information – public and private – is factored into share prices. As soon as information emerges, it

leaks into the market and prices adjust – before an official announcement has been made. Even insider trading would not pay off in this world.

Market efficiency basically renders both technical and fundamental analysis for stock picking superfluous. A catchy quote in this respect comes from Burton Malkiel, who wrote in his classic book on investment strategies: "A blindfolded monkey throwing darts at a newspaper's financial pages could select a portfolio that would do just as well as one carefully selected by the experts" (Malkiel, 1985, p. 16). Formally stated, expected return always equals the required return under market efficiency. If it would not, trading would adjust the security's price instantaneously so that it is again in equilibrium.

Summarizing, markets are said to be efficient in incorporating relevant information that either affects the amount, timing or riskiness of future cash flows into stock prices. Information is priced correctly and quickly, and the required return always equals the expected return. Up to the end of the 1970s, empirical evidence seemed to support the EMH, especially the weak and semi-strong forms. However, both theoretical and empirical challenges have arisen since, which led to the formation of the Behavioural Finance field. These challenges will be reviewed in the following section.

2.4 Challenges to the EMH

The most voluminous set of critique aims against the very foundation of the EMH, investors' rationality. Findings from psychology and other social sciences have revealed that investors are subject to numerous cognitive biases. Among the most influential in this respect were Daniel Kahneman and Amos Tversky[2]. They were first to prove that people deviate from the axioms of rationality, and that they do it systematically. Specifically, people are overly confident; they are too optimistic and adhere to wishful thinking. Next, once they formed an opinion, they tend to stick to it, regardless of new information. Most importantly, their estimation of probabilities is often far from rational. In the case of the Representativeness Heuristic, people exhibit "a tendency to judge likelihoods based upon naive comparison of characteristics of the event being predicted with characteristics of the observed sample" (Daniel, Hirshleifer, & Teoh, 2002, p. 145). Kahneman and Tversky (1979) also formulated Prospect Theory, as an alternative to the Expected Utility framework by Von Neumann and Morgenstern (1944). In contrast to the original theory, people seem to evaluate risky gambles, such as

[2] For an overview of their contributions, see Shefrin and Statman (2003)

investing in stock markets, in terms of gains and losses rather than absolute expected final wealth levels. Furthermore, their attitude towards risk changes over different wealth effects. In particular, people are risk-seeking over losses, and risk averse over gains. They overvalue small probabilities and are overly sensitive to changes in probabilities at higher probability levels. The list of cognitive errors and their effects on investor behaviour is long, but will not be continued here for lack of space and relevance[3].

As becomes obvious, empirical evidence against investor rationality is overwhelming. Many authors have documented serious deviations from basic rationality. These deviations are systematic in that they are correlated across individuals (Barber, Odean, & Zhu, 2004). Consequently, their effects on the market do not cancel out, in contrast to what has been postulated by the EMH. Arbitrage seems to be the last line of defence. But even this last argument for efficiency has seen a lot of critique, and many limitations have been documented (see Shleifer & Vishny, 1997). First and foremost, perfect substitutes for individual securities are very hard to find, especially for stocks. If no perfect substitute exists, fundamental risk comes into play that is idiosyncratic to one stock and thus cannot be compensated for by an opposite transaction in another stock. Fear of this risk will limit arbitrageurs' willingness to engage in large positions. Next, due to commissions and bid/ask spreads, arbitrage is often costly. The shorting of securities is legally restricted and requires the payment of fees, especially over longer horizons. Next to all of these costs and risks associated with arbitrage, another very significant source of risk has been identified. This source limits arbitrage even if perfect substitutes existed, so no fundamental risk was involved, and there were no costs associated with arbitrage. This risk is called noise trader risk, which will be examined in detail in the following chapter.

[3] For more complete overviews and further reference, see e.g. De Bondt (1998), Hirshleifer (2001), Barberis and Thaler (2003), and Stracca (2004)

3. The Noise Trader Approach

In his article, Fischer Black (1986) laid the foundation by emphasizing the role of noise on a wide range of economic activities, including financial markets. Black (1986) contrasts noise with information. In a financial context, information is relevant news about fundamentals of risk and return, whereas noise is irrelevant news. According to his predictions, trading on noise is a vital function that only makes financial markets possible. If there are only trades on information, “taking the other side’s information into account, is it still worth trading?” (Black, p. 531). Thus noise trading, defined as trading on noise as if it was information, provides liquidity to information traders[4], but it also makes stock prices noisy as they will reflect the opinions of noise traders alongside the fundamentals of risk and return. Consequently, prices will be less efficient, which encourages information traders to engage in the market, as they should be able to exploit their informational advantage. However, if their information is already priced, their trading would resemble noise trading, which makes the distinction between both groups blurry. If unsophisticated traders act on noise as if it was value-relevant information, it becomes possible for events such as index inclusions to influence prices and push prices away from value. Volatility of prices will thus be greater than volatility of value, and it will change over time. In sum, “noise creates the opportunity to trade profitably, but at the same time makes it difficult to trade profitably” (Black, p. 534). Summers (1986) supported these theoretical predictions with empirical facts, when he showed that statistical tests are not very powerful at determining asset mispricing due to the joint hypothesis problem referred to by Fama (1970). Consequently, effectively detecting and correcting for price inefficiencies would be just as hard for rational arbitrageurs. From there, Shleifer and Summers (1990) managed to describe a whole new “Noise Trader approach to finance”. However, it took them another paper to present a formal model that incorporated these ideas (De Long *et al.*, 1990). This paper will be discussed in the following section and remain the theoretical basis for further discussions in this thesis.

3.1 The Logic Behind De Long *et al.*

De Long, Shleifer, Summers and Waldmann (1990, DSSW henceforth) formulated a theoretical model that demonstrates the effect of noise traders participating in financial markets, and

[4] Synonyms include: arbitrageurs, rational investors, sophisticated investors, rational agents, smart money

which implications their misperceptions have for expected returns and volatility. In their model, two classes of investors interact: rational investors and noise traders. The former hold fully rational expectations about future stock returns, and the latter trade on an external, noisy signal that differs from information. This signal is sentiment, which is abstract for expectations that are formed subject to behavioural and cognitive biases. While these biases are not explicitly specified in their article, DSSW model them as noise traders' misperceptions about intrinsic value evolving over time. Misperceptions may induce noise traders to take irrational positions, thus driving prices away from value. Normally, short-term deviations are theoretically quickly corrected for by rational arbitrageurs who ensure that "prices are right" (Fama, 1965; 1970). However, if arbitrageurs have finite horizons and are risk averse (see Shleifer & Vishny, 1997), stochastic noise trader sentiment creates a risk, as prices may deviate even more from value in the short run. Arbitrageurs concerned with liquidating their positions in the foreseeable future are likely to perceive this as an additional risk and limit their positions. Consequently, even in the absence of fundamental risk, noise trader sentiment, if systematic, creates a risk that limits arbitrage and thus allows prices to persistently differ from value. In the words of DSSW, "noise traders create their own space […], as arbitrage does not eliminate the effects of noise because noise itself creates risk" (p. 705). Furthermore, noise trader risk lowers asset prices as arbitrageurs pull out. If noise traders' opinions then entices themselves to invest in these assets, their expected return may be higher than the arbitrageurs'. Consequently, through their mere existence, "noise traders can earn higher relative expected returns solely by bearing more of the risk they themselves create" (p. 706). It is important to realise that noise traders do not necessarily have to be pessimistic about a security in order to drive its price down – underpricing is simply an adjustment for the risk created by the variability of sentiment. These results have severe implications for the functioning of markets, but also bear consequences for asset price behaviour and expected returns. DSSW distinguish four effects:

- First, there is the "create space" effect. If the variability of noise trader sentiment increases, risk becomes higher. Risk averse arbitrageurs tend to limit their bets against noise traders, who can thus earn higher expected returns. This effect becomes stronger the more noise traders in relation to arbitrageurs are trading in a particular asset or market, and the more volatile their misperceptions are.
- The "Friedman" effect means that noise traders have the worst possible market timing as they tend to herd. When noise traders buy when others buy, they buy high and sell

low. This of course has an adverse impact on their returns, which becomes worse the more volatile noise trader sentiment is.

- The "hold more" effect means that expected return to noise traders can be higher only if they are on average bullish about a particular (class of) stock. This is because their fluctuating sentiment is a risk which always leads to higher expected returns; however, only if noise traders are on average bullish do they invest in these securities, and thus actually reap the additional return.
- Finally, the "price pressure" effect sets in when increased bullishness leads noise traders to invest more in a risky asset, which increases demand and thus pushes up its price. Inevitably, higher prices mean lower expected returns, so that their trading activity actually hurts them. On the contrary, when they are bearish, they sell the security which prevents them from reaping future returns when prices bounce back.

Summarizing, the four effects affect expected returns through different channels, and in opposing directions. While the "hold more" and "price pressure" effects directly influence expected returns, the "Friedman" and "create space" effects are related to the variability of returns, which in turn poses a priced risk and thus has an indirect influence on returns. The "hold more" and "create space" effects work in favour of noise traders' returns, while the "Friedman" and "price pressure" effects are rather harmful. These relationships are depicted for clarity in Figure 1 below.

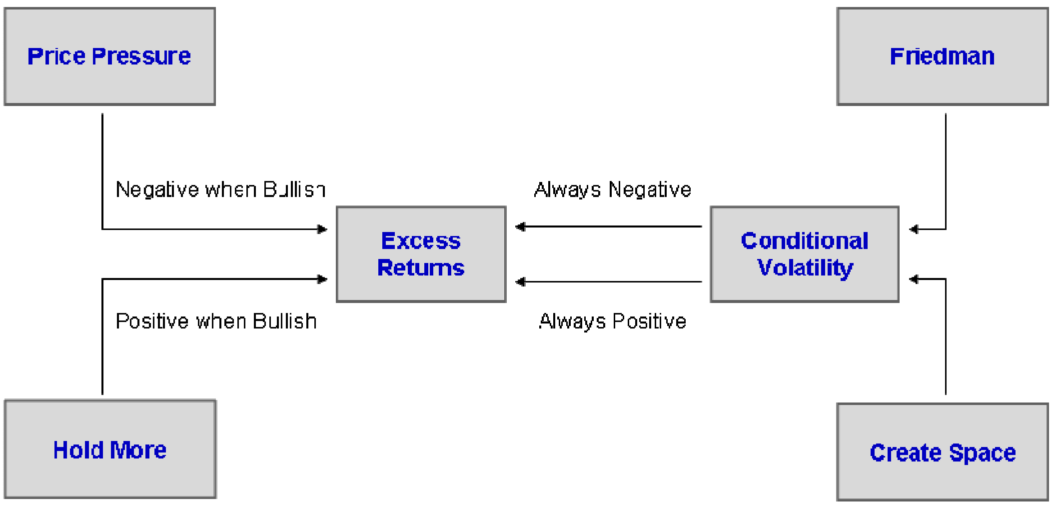

Figure 1: Impact of four effects on conditional volatility and returns

While the DSSW theory is quite parsimonious, it helps to shed some light on several problems in standard finance. Two previously observed phenomena can be explained by the implications of the noise trader model. First and foremost, if noise trading creates a systematic risk, this risk is priced in equilibrium. Consequently, securities affected by noise trader sentiment should earn higher expected returns, which implies that they are underpriced relative to their fundamental value. The persistent discount on closed-end funds is a prominent example (see C. M. C. Lee, Shleifer, & Thaler, 1991). Secondly, if stocks are more affected by noise trader sentiment than bonds, which seems likely, the same logic holds and provides a possible explanation for the persistent excess risk premium on stocks known as the Equity Premium puzzle (see Mehra & Prescott, 1985).

Concluding, the Noise Trader approach to finance and the DSSW model have some profound implications for the understanding of financial markets. More importantly, they provide testable hypotheses that stand in contrast to the EMH. The empirical evidence on these hypotheses will be reviewed in the following section.

4. Previous Empirical Evidence

The implications of Noise Trading theory have been tested by a host of authors. Several approaches can be distinguished: First, the implications of DSSW for the behaviour of closed-end fund discounts have been examined empirically. Secondly, tests have been performed on the direct relationship of investor sentiment and stock returns, using different measure of sentiment. Finally, a few authors have factored volatility into the equation. The previous evidence will be reviewed separately to provide an overview of the impact of noise trader sentiment on stock returns and volatility[5]. An important note is that mostly noise traders are equated with individual investors, which seems logical at first sight. Individual investors undoubtedly have less means to gather information at their disposal, they are comparatively inexperienced, and show significant psychological biases. Consequently, they are commonly assumed to be noise traders, and institutional traders play the role of arbitrageurs. The validity of this assumption will be discussed later.

4.1 Investor Sentiment and Closed-End Funds

As already explicitly outlined by DSSW, the noise trader approach serves as a possible explanation for the puzzle of persistent closed-end fund discounts. Closed-end funds are, like their open-end cousins, mutual funds that invest in other traded securities. Holders of open-end fund shares can sell (redeem) their shares to the fund at any point in time, as the number of shares in the fund issued is flexible. In contrast to that, closed-end funds issue a fixed number of shares, so that someone looking to divest has to find a buyer for his shares in the market. As the supply of shares is inelastic, the price of closed-end fund shares can be influenced by the changing demand for them. But mainly, the price of these funds is determined by the net asset value (NAV) per share, which is the market value of the shares the fund is invested in. As those are also traded, their value is readily observable. This poses a rare chance to test the implications of noise trader theory, as normally intrinsic value is hard to determine. In the U.S., where most of the empirical work on closed-end funds has been done, closed-end funds are mostly held by individual investors, who are suspected to be more prone to reacting to noise. Now, according to classical financial theory, price should equal value.

[5] Other financial instruments such as futures are not the focus of this thesis. While noise trading-related evidence exists that applies to non-equity markets, it will not be discussed here.

However, a persistent discount to the NAV per share on the shares of closed-end funds has been observed for decades. The discounts fluctuate over time, and can also turn into premia occasionally, but on average they remain discounts. This obvious and persistent mispricing poses challenges for explanation within the EMH framework, whereas DSSW explain the discount as an adjustment for the noise trader risk the fund shares are subject to. Lee, Shleifer and Thaler (1991) test this theory by examining monthly discounts of closed-end funds in the period between 1956 and 1985. Their findings support the noise trading explanation of closed-end fund discounts. First, discounts on closed-end funds are correlated, which implies that investor sentiment is not specific to particular funds (it poses systematic rather than idiosyncratic risk). Secondly, new closed-end funds get started when seasoned funds are selling at low discounts or premia. This means that sentiment actually triggers the formation of closed-end funds. Third, if noise trader risk is systematic, unrelated securities should be affected alongside the prices of closed-end fund shares, and in the same direction (a phenomenon known as *comovement*). Lee *et al.* (1991) confirm this relationship with the returns on small stocks, which are assumed to be held by individual investors and thus subject to their sentiment, just as closed-end funds. The authors conclude that an index of the discounts on close-end funds represents an indicator of individual investor sentiment. They find that this indicator adds explanatory power to the difference in small and large-stock returns, and is independent of other macroeconomic factors commonly used to explain stock returns. Thus, the widely known small firm effect can partly be explained by individual investor sentiment. While Lee *et al.* come to their conclusions mostly through theoretical reasoning and simple correlation statistics, they do not provide an estimate of how much of the variation in closed-end fund discounts is actually due to investor sentiment. Brauer (1993) fills this gap by using the signal extraction technique introduced by French and Roll (1986). His conclusion, based on this methodology, is that only 7% of the variation in discounts of closed-end funds can be explained by noise trading. However, signal extraction is not very straightforward, and it remains unclear whether it is an appropriate method to this end. In the same year, the discussion about the validity of using closed-end fund discounts as a sentiment indicator gained momentum when Chen, Kan and Miller (1993a) published their critique to the conclusions of Lee *et al.*, seemingly invalidating their findings by pointing out methodological mistakes. Several reactions to and renewals of the criticism followed (Chen, Kan, & Miller, 1993b; Chopra, Lee, Shleifer, & Thaler, 1993a, 1993b). In the end, the dispute was about methodological and interpretational matters, which left the original findings of Lee *et al.* in doubt, but not discarded.

Building on the findings of DSSW and Lee *et al.*, Sias, Starks and Tiniç (2001) test the hypothesis that investors in closed-end funds (mainly individuals) earn a higher return that investors in the underlying portfolios (mainly institutions), as is implied by noise trader theory. They prove that closed-end funds are more exposed to noise trader risk, as they exhibit greater volatility and mean reversion than the underlying portfolios. However, although in their sample closed-end funds sell on average at a discount, Sias *et al.* (2001), after accounting for fund expenses, could not find evidence that returns for closed-end funds are higher than for the underlying portfolios.

Many more empirical studies on the issue have been performed, which Dimson and Minio-Kozerski (1999) have compiled in their survey. Overall, the groundbreaking work of Lee *et al.* has provided some explanation for the phenomenon of closed-end fund discounts. Others have followed them and presented partial empirical evidence for the investor sentiment theory. Although there are still some disputes about the basic assumptions of this approach, the sentiment hypothesis remains the theory closest to an explanation to date. Meanwhile, Lee *et al.*'s main conclusion, namely that closed-end fund discounts are an indicator of individual investor sentiment, remained subject to inspection in further empirical tests, which will be reviewed in the following section.

4.2 Investor Sentiment and Stock Returns

After the special case of closed-end fund discounts had been dealt with academically, researchers turned to a more general application of the noise trader approach: the influence of noise trader sentiment on stock returns. The analysis of this relationship is inherently more complex, in comparison to the closed-end funds discount phenomenon. There is nothing like an objective measure for the intrinsic value of a stock, just its approximation in the form of the market price. The resulting dilemma is called the joint-hypothesis problem (Fama, 1970): In order to specify whether information is correctly priced, an accurate pricing model is needed to calculate the underlying fair value. When tests for market efficiency are conducted, the used pricing model is always assessed simultaneously. Statistical results are thus not clearly attributable to either the specification of the pricing model or the actual model to be tested. Consequently, misvaluations are very hard to determine objectively. As a result, most studies focus on return patterns that occur after a mispricing, rather than trying to determine the mispricing directly. If e.g. the price of a stock has been unreasonably pushed above its value by high investor sentiment, this stock should perform comparatively worse subsequently, when its price approaches fair value eventually.

In a first attempt to test the influence of noise traders on stock returns, Kelly (1997) examines which consequences the extent of noise trader participation has for returns. He conjectures that higher participation of noise traders is a negative predictor of stock returns, as noise traders tend to buy high and sell low (DSSW's "Friedman" effect)[6]. He assumes that the likelihood of a person to be a noise trader diminishes with household income, while the opposite is true for the chance of being smart money. Investors from intermediate-income households are assumed to be passive investors who do not have a direct bearing on the variation of stock market prices. His empirical results, after analysing data on U.S. dividend income tax from 1947-1980, support the theoretical predictions. Higher market participation by noise traders is a negative predictor of stock returns, while the opposite holds for smart money participation. The share of intermediate-income investors had no predictive power. Kelly thus managed to first pinpoint the influence of noise traders on stock market prices. However, he did not venture into making predictions about the nature of investor sentiment's influence on the variation in stock market prices. Consequently, researchers after that started running tests of how some measure of sentiment is statistically related to the movement in stock prices. In this field, two general approaches are to be distinguished, regarding the nature of the sentiment variable. The first is to formulate proxies of investor sentiment that are somewhat justified by financial theory. This is done by taking observable, objective variables that implicitly indicate investor sentiment and use them for statistical analyses. The second approach is to rely on explicit, mostly survey-based measures that try to capture the mood of the market more directly. Both approaches have their advantages and drawbacks. Objectively observable variables are more reliable with respect to their generation process; however their theoretical link to investor sentiment may be debatable. On the other hand, while survey-based measures are more direct in that respect, they often lack the sample size and statistical representativeness. Empirical studies employing both approaches will be reviewed separately in the following.

4.2.1 Implicit Measures of Investor Sentiment

Swaminathan (1996) examines the predictive power of individual investor sentiment for the excess expected returns on small firms. In accordance with Lee *et al.* (1991), he uses an index

[6] Kelly refers to expected returns *conditional on the participation level of noise traders*, in contrast to DSSW who find that noise traders on average earn higher *unconditional* expected returns than sophisticated investors

of closed-end fund discounts to proxy for sentiment. His empirical findings are quite unambiguous. Individual investor sentiment, as reflected in closed-end fund discounts, is able to forecast small firm returns. The information contained in the sentiment index is independent from other macro-economic variables such as the dividend yield on the market and term spread. Closed-end fund discounts only forecast the small firm factor, but in that they are exclusive. These results seem to support the hypothesis of Lee *et al.* that, as individual investors are the major shareholders in small firm and close-end fund shares, the fluctuating discounts should reflect their irrational sentiment and forecast small firm returns. However, as his closed-end fund discount index also seems to be correlated with expectations of future earnings growth and expected inflation, Swaminathan suggests that closed-end fund discounts reflect investors' *rational* expectations, rather than irrational sentiment. Nevertheless, in his study closed-end fund discounts were able to forecast returns. In contrast to that, Elton, Gruber and Busse (1998) present evidence against the theory of Lee *et al.* They show that an index of closed-end fund discounts enters the return-generating process of small firms not more often than expected by chance and even less than purely non-fundamental industry-indices consisting of large, institutionally-held firms. The incorporation of the closed-end fund index into an asset pricing model does not yield support for the hypothesis that sentiment is priced either. Doukas and Milonas (2004) come to the same conclusion after extending the work of Elton *et al.* (1998) to an out-of-sample dataset of Greek closed-end funds and stock market returns. The latest paper on the subject by Wang (2004) however again supports the use of closed-end fund discounts to proxy for sentiment. He constructs portfolios dependent on the exposition to closed-end fund discounts and finds significant excess returns than can not be explained by traditional financial models such as the one by Fama and French (1993). The discussion seems to remain whether closed-end fund discounts are the appropriate indicator of investor sentiment.

Next to closed-end fund discounts, Neal and Wheatley (1998) test two more measures of investor sentiment: the odd-lot[7] balance and net mutual fund redemptions. The first measure describes the ratio of odd-lot stock sales to purchases. The theory is that only individual investors trade in these small quantities. So when they are selling more than they are buying, they are bearish, which is when the odd-lot balance indicator rises to signal a potential buying opportunity for rational investors[8]. The same kind of logic applies to net mutual fund redemptions. If investors redeem more shares of mutual funds than others buy, their sentiment is as-

[7] Odd-lot: a stock transaction with volume of less than 100 stocks bought or sold at a time
[8] This line of argumentation goes back to Hardy (1939)

sumed to be low. As small investors hold a high stake in mutual funds, this measure is also seen as a proxy for *individual* investor sentiment[9]. The findings of Neal and Wheatley (1998) suggest that both closed-end fund discounts and net mutual fund redemptions bear explanatory power for the small firm premium, while the odd-lot balance does not seem to have a meaning in this respect. However, once the so-called Keim-Stambaugh factor (the cross-sectional average share price of small firms) is included in the analysis, only net mutual fund redemptions remain a statistically significant factor in explaining small firm excess returns. Along the same lines of argumentation, S. J. Brown, Goetzmann, Hiraki, Shiraishi and Watanabe (2002) confirm the applicability of mutual fund flows. They construct a new index from mutual fund flow data, and validate that it is priced, both for the U.S. and Japan.

The approach to draw inferences about investor sentiment from trading statistics such as the odd-lot balance has recently been taken to the next level by Kumar and Lee (2003), Jackson (2003a), and Kaniel, Saar and Titman (2005). Kumar and Lee (2003) examine a dataset from a major discount broker. They find that the broker's clients – individual investors – seem to trade systematically, in that their trading activity is correlated. This is the prerequisite in order for their activities to influence market prices. Their measure of investor sentiment – basically the ratio of share sales to purchases – is also highly correlated with recommendations from investment newsletters. Individual investors seem to adhere to this "expert" advice. Next, Kumar and Lee discover that their measure of retail investor sentiment "has incremental explanatory power for [returns of] small stocks, value stocks, stocks with low institutional ownership, and stocks with lower prices" (p. 4f). Their statistics show that when investors are bullish, these stocks earn higher excess returns. Finally, to further explore the significance of their sentiment measure, they relate it to observed seasonal patterns such as the January effect and the Day-of-the-Week effect. While they find strong evidence that retail investor sentiment partly explains the January effect, fluctuations in sentiment do not seem to have a special impact on particular weekdays.

The analysis by Jackson (2003a) is similar, but comes to different conclusions. He uses an extensive dataset of some 40 million trades with several Australian retail brokers. Analogous to Kumar and Lee, he finds that trading is significantly correlated on the market and cross-sectional level, and both within and across broker firms. Investors exhibit patterns of negative feedback trading, i.e. they buy after losses and sell after gains, a behaviour normally not associated with individual investors. Consequently, trading patterns by individual investors

[9] Malkiel (1977) first suggested that net mutual fund redemptions reflect "general investor sentiment" (p. 856)

positively forecast short-term subsequent returns, which is at odds with the general assumption of small investor irrationality.

Kaniel *et al.* (2005) analyse another dataset on individual investor trades, this time from the New York Stock Exchange. Their analysis shows two things. First, individual investors seem to be quite good at predicting short-term returns. Stocks that experience a surge in individual buying for one week show an average excess return of 1.4% for the following 20 days. However, the contrary is not evident: stocks that individual investors sell do not perform worse than average in the following 20 days. Secondly, and quite surprisingly, Kaniel *et al.* discover that individual investors seem to follow contrarian investment strategies: "The mean market-adjusted returns in the 20 days prior to a week of intense individual selling is 3.97%, while prior to a week of intense individual buying it is –2.54%" (p. 3). These results mirror those of Jackson (2003a). In contrast to Kumar and Lee (2003) and Jackson, however, they can not confirm that the trading activity of individual investors is correlated across stocks, so that sentiment does not pose a systematic factor that should be priced in equilibrium.

Summarizing, closed-end fund discounts do not seem to be able to explain subsequent stock returns, while net mutual fund flows could be more a promising statistic. Detailed data on the trading behaviour of individual investors yields surprising implications. While the trading of individual investors can explain part of the subsequent variation in stock prices, this relationship seems to be positive. When sentiment is high, subsequent returns tend to be more positive. Surprisingly, investors seem to follow contrarian strategies themselves – they buy after low returns, and sell after gains. Individual trading seems to be highly correlated.

After closed-end fund discounts and other implicit measures of investor sentiment had been studied extensively, researchers turned to explicit, i.e. survey-based, measures in order to confirm their conclusions with data presumed to reflect sentiment more directly. These studies will be treated in the following section.

4.2.2 Explicit Measures of Investor Sentiment

Fisher and Statman (2000) were among the first to include survey-based measures of investor sentiment into their research on opinions of different classes of investors. They identified three groups: small (individual) investors, medium investors (newsletter writers), and large, institutional investors (Wall Street strategists). First, to measure the sentiment of small investors, they drew on the weekly surveys by the American Association of Individual Investors (AAII), which has collected weekly data since 1987. Members of AAII simply classify themselves as bullish, bearish or neutral for the following period. Fisher and Statman (2000) use the percentage of bearish investors as their sentiment indicator. Second, in order to capture the expectations of newsletter writers, the service of Investors Intelligence (II) has been employed. Also on a weekly basis, II classifies opinions published in newsletters as bullish, bearish or waiting for a correction since 1964. Again, Fisher and Statman take the percentage of bearish newsletter writers as their measure of sentiment. Finally, data compiled by Merrill Lynch quantifies institutional sentiment. The investment bank determines the share of stock in portfolios recommended by up to 20 investment strategists on a monthly basis since 1985. Drawing on these time-series, Fisher and Statman arrive at several findings. First, while sentiment of individuals and newsletter writers is significantly (though not perfectly) correlated, the investment bankers did not seem to follow any of the two. Fisher and Statman then turn to the predictive power of the sentiment measures for next-month stock returns, both for the S&P 500 and an index of small stocks. While all regressions yield negative coefficients for the sentiment variables, suggesting their potential use as a contrary indicator, the only significant relationship is between small investor sentiment and next-month S&P 500 returns. Subsequently, the issue of the influence of returns on sentiment is examined, so that the variables in the regressions are basically flipped. The results are quite clear. Both small investors and newsletter writers are strongly influenced by past returns. After periods of positive market developments, their sentiment rises significantly. Surprisingly, and in contrast to the assumptions of e.g. Lee, Shleifer and Thaler (1991), individual investors seem follow the development of the S&P 500 more closely than small-cap stocks. The same is true for newsletter writers. In contrast, Wall Street strategists' opinions are not that easily influenced by past returns, which again is in line with the general assumption of their relative rationality. Finally, individual investors' allocation of funds to stocks is examined through AAII fund allocation survey data. Luckily for them, small investors do not seem to act on their sentiment. While surges in their sentiment tend to be followed by negative S&P 500 returns, returns tend to be higher after small investors invest more of their funds in stocks, so they do not seem that irrational after all.

In another study, Solt and Statman (2001) dug deeper into the subject and investigated whether investment strategies based on sentiment indicators can be profitable. They concentrate on the Investors Intelligence index and relate it to Dow Jones Industrial Average (DJIA) returns. They conclude that "there is no statistically significant relation between the index and changes in the DJIA in the subsequent four-week periods", so that the "sentiment index is not useful as a contrary indicator" (Solt & Statman, 2001, p. 47).

Fisher and Statman (2004) investigated the expectations by individual investors and Wall Street strategists again, this time on the background of the millennium stock market bubble. They use postings on Yahoo message boards and results from both the Gallup/UBS and BusinessWeek surveys to gauge the opinion of investors during times of high gains and losses around the 2000 stock market bubble. Again, sentiment seems to follow returns, as reflected in a higher percentage of optimistic investors at the climax of the market compared to the trough in the middle of 2002. Wall Street investors however, according to BusinessWeek surveys, were less bullish after strong gains, but became more so after the crash of the market, in contrast to the bearish individual investors at the time. This is in line with their previous findings. They conclude that "stock price of the late 1990s were likely driven higher by the exuberance of investors about their favourite individual stocks, as captured on the Yahoo message boards, or by the combined drive of many investors, each with modest expectations" (Fisher & Statman, 2002, p. 20).

G. W. Brown and Michael Cliff perform two extensive studies on the relationship of measures of investor sentiment and stock returns. While in Brown and Cliff (2002) they examine the long-run effects of investor sentiment on stock returns, they concentrate on the near-term stock market in their later study (Brown & Cliff, 2004). While the former will be summarized here, the latter is dealt with in the following section.

In their analysis, Brown and Cliff (2002) test two basic hypotheses:

1. "Excessive optimism leads to periods of market overvaluation", and
2. "High current sentiment is followed by low cumulative long-run returns"

They focus on the long-run reversal of returns to fundamental value, as arbitrage that could force prices back to value might be effective in the short run, but limited in the long run (see discussion in Chapter 3). The first hypothesis is tested by relating sentiment as reflected in the aforementioned Investors Intelligence (II) surveys to S&P 500 pricing errors. They draw on results by Bakshi and Chen (2001) for the pricing errors, which were estimated using a sophisticated methodology that is however of course still subject to the joint-hypothesis problem. The second hypothesis is investigated by finding significant relationships between high levels of sentiment and subsequent returns on the Fama and French (1993) and other portfolios over differing horizons. In all their analyses Brown and Cliff use ten control variables to distinguish the rational part of sentiment from the irrational. After all, changes in sentiment may not be completely irrational, but also simply reflect changes in fundamental factors. Their results are coherent. First, investor sentiment is significant in explaining parts of stock market misvaluations, even in presence of control variables. Second, positive shocks (over one standard deviation) to investor sentiment are nearly always followed by reduced returns over horizons of six, twelve, 24 and 36 months.

Summarizing, survey-based measures help to better understand the formation of investor sentiment and its impact on the behaviour of stock markets. Sentiment seems to follow recent stock market developments. Especially individual investors seem to become more bullish after gains on the stock market. Wall Street strategists in contrast seem to be less enthusiastic in bull markets, however they do believe in faster recovery while still in a market trough. Optimistic investors are possibly able to influence market valuations and make them less efficient, especially over longer horizons. Finally, surges in survey-based sentiment indicators seem to be followed by negative returns, which would render them candidates for contrarian indicators.

After studies that employ implicit and explicit measures of sentiment have now been discussed separately, some which integrate several measures in their analysis will be summarized in the following.

4.2.3 Combined Im- and Explicit Measures of Investor Sentiment

Both implicit and explicit measures of investor sentiment have been employed which were partly able to add to the discussion on whether investor sentiment is an important determinant of stock returns. However, it has remained unclear which measures are actually most appropriate, and to which extent they represent the same informational content. To further clarify this issue, two studies have gone further and attempted to integrate several measures.

The aforementioned research by Brown and Cliff (2004) complements their study of long-term return patterns and sentiment in that they focus on short-term effects this time. They come to their conclusions in two steps. First, they explore the interrelationships of a wide range of implicit and explicit measures of sentiment. Second, the explanatory power of investor sentiment for stock returns is inspected. As others before, Brown and Cliff (2004) rely on the AAII and II survey data. For the implicit measures, they use a wide range from indicators of recent market performance (e.g. the ARMS index), trading activity (e.g. the odd-lot balance, short sales activity), derivatives variables (e.g. put/call ratio, implied volatility) to IPO data (number of offerings, first day returns) and others such as the known closed-end funds discounts and mutual fund flows[10]. Their first main result is that all these measures contain

[10] See Brown and Cliff (2004) for a complete and detailed overview of the used indicators as well as their common classification as bullish or bearish

similar information, which is why they use integration techniques such as the Kalman filter and principal components analysis in order to arrive at new, unobserved measures of sentiment. These measures correlate significantly with contemporaneous returns. Further, causal analysis reveals that while returns strongly affect subsequent swings in sentiment, "very little evidence suggests sentiment causes subsequent market returns" (Brown & Cliff, p. 3). Finally, and surprisingly, the strongest link between sentiment and returns appears to be between institutional sentiment and large stocks. Although statistically not significant, this relationship would counter the usual assumptions that individual investors are the ones to add noise to stock prices.

The second, extensive study that integrates several measures of investor sentiment has been presented by Baker and Wurgler (2004). While Brown and Cliff use marketwide return data, Baker and Wurgler (2004) discuss cross-sectional differences in the time-series of stock returns. They propose two distinct mechanisms through which the cross-sectional return differences might become evident. First, sentiment, which they define as "propensity to speculate" (p. 5), might differ across stocks. The value of stocks with certain characteristics might be more subjective than for others, which allows inexperienced investors to justify a range of valuations. Then, e.g. in a bull market when people want to invest, they buy these stocks because they expect prices to rise to the upper end of their subjective pricing range. Second, arbitrage possibilities might vary across stocks. As the values of stocks with certain characteristics are more subjective, arbitrageurs cannot objectively determine value either, which limits arbitrage for these stocks. In effect, both mechanisms lead to the same result: some stocks are more susceptible to misvaluations. If that was true, they would display distinct return pattern when prices and values converge eventually. In particular, Baker and Wurgler suppose that if changing sentiment caused mispricings, it would have a comparatively higher impact on young, small, highly volatile, unprofitable, non-dividend paying, high-growth and/or distressed stocks. Their methodology is simple: After splitting their records into high and low levels of marketwide beginning-of-period sentiment, they sort corresponding stock returns by every aforementioned characteristic, respectively. Just as Brown and Cliff, they employ a wide range of implicit sentiment measures, although they do not make use of survey data. Unlike Brown and Cliff, they orthogonalize these indicators with respect to several macroeconomic variables[11]. By that technique, Baker and Wurgler filter some rational content reflected in sentiment from their indicators in order to achieve cleaner measures for the

[11] See Baker and Wurgler (2004) for an overview of used indicators and variables

irrational mood swings of investors. Their results are clear: when initial sentiment is low, smaller, younger, higher-return volatility, more unprofitable, non-dividend paying, high-growth and distressed stocks earn higher returns in the following month than their counter-parts. When sentiment is high, the effects reverse. The empirical evidence thus supports the theoretical predictions that firms with these characteristics are more affected by swings in sentiment, so that when investors are bearish, they are undervalued and thus show above-average subsequent returns, and vice-versa.

Summarizing, the integration of several measures of sentiment has proven a fruitful approach to exploring the relationship of investor sentiment and stock returns. On a marketwide basis, integrated measures of sentiment are not yet able to forecast returns. For the cross-section of stocks, the opposite is true. Stocks with subjective valuations are influenced by swings in sentiment and show distinguishable return patterns.

Up to now, studies that focus solely on the impact of sentiment on returns have been reviewed. The next section will deal with those that include volatility in the empirical analysis.

4.3 Investor Sentiment, Volatility and Stock Returns

Only few researchers have ventured into the field of relating investor sentiment to volatility. Nevertheless, this seems appropriate, as "the most fundamental prediction of the noise trader-model is that irrational investors acting in concert on a noisy signal […] cause a risk" (Brown, 1999, p. 84) which can reasonably assumed to be volatility. Brown (1999) investigates this issue by relating AAII sentiment survey data to closed-end fund discount volatility. Through his parsimonious regression analysis, he reveals several things. First, deviations from mean levels of sentiment lead to statistically significant higher discount volatility, which supports noise trader theory as it suggests that sentiment poses a systematic risk. Second, Brown shows that extreme levels of investor sentiment affect discounts only during trading hours, which is a prerequisite for the theory that noise traders influence prices through their trading activity. The fact that the number of trades in closed-end funds is significantly higher in periods of extreme sentiment corroborates this finding. Finally, when sentiment is extreme, total trading volume does not increase, meaning that the average trade size decreases, so that "the larger traders actually give way, to some degree, to noise traders" (p. 88). This is in line with the implications of the DSSW model's "create space" effect. With his study, Brown laid the foundation for studying the impact of sentiment on volatility.

Extending the focus to a broader equity market context, Jackson (2003b) uses the same database as in his companion paper (Jackson, 2003a). He comes to surprising results. Larger trading share of individuals in certain stocks does not increase their subsequent volatility. Additionally, individual participation does not lead stock returns to be correlated with a small stock portfolio. However, the opposite is true for *institutional* participation, which increases conditional volatility and leads to a higher correlation with other, mostly institutionally held stocks. Finally, Jackson finds a significant noise trader risk factor that is priced in equilibrium. In line with his previous results, and in contrast to the common interpretation of noise trader theory, the source of this risk seems to be *institutional* traders. According to Jackson, "institutional frictions are a much more plausible source of non-fundamental demand shocks than is an individual sentiment effect" (p. 6).

The next step was to integrate all former work and conduct an empirical study that incorporates the elements of sentiment, returns, and risk (volatility). This is exactly what W.Y. Lee, Jiang and Indro (2002) have done. They start off by arguing that empirical models that analyse only the impact of sentiment on either expected returns or volatility are "mispecified and at best incomplete" (p. 2280). This holds as the DSSW theory, to which they closely adhere, states four effects that affect returns, both directly and through the effects of changing misperceptions on return volatility (see Section 3.1). Consequently, an empirical model should include both channels, and not focus merely on returns. They propose a generalized autoregressive conditional heteroskedasticity (GARCH) in-mean model. As returns, Lee *et al.* (2002) employ data from the Dow Jones Industrial Average, NASDAQ, and S&P 500 indices. To model investor sentiment, survey data from Investors Intelligence is used, just as in several studies before. The empirical results are consistent with the Noise Trader model. Lee *et al.* find that "investor sentiment is an important factor in explaining equity excess returns and changes in conditional volatility" (p. 2291). Specifically, changes in sentiment are significantly and positively related to excess returns. This means that the "hold more" effect dominates the adverse "price pressure" effect. Investors earn higher expected returns when they are bullish, because their optimism leads them to hold more of the risky asset, for which they are compensated. While the increased demand for the risky asset increases its price (and thus lowers its expected return), this does not offset the positive effect completely. In contrast, when investors turn bearish, their trading pushes prices down. They pull out of the risky asset so that they can not reap compensation for bearing noise trader risk. Returns are also found to be significantly influenced by volatility, which in turn is subject to the variability and magnitude of changes in sentiment. Bullish shifts in sentiment lead to subsequent down-

ward adjustment of future volatility, whereas bearish shifts lead to increased volatility. When investors are optimistic, the "create space" effect dominates the "Friedman" effect, and thus raises expected returns. However, this relationship turns with sentiment when it becomes bearish. Then the space noise traders create is not sufficient any more so that their poor market timing hurts them. Finally, while the supposed January and October effects generally turned out not to prevail in their analysis, Lee *et al.* find a significant (positive) seasonal pattern in the January NASDAQ returns. Their results are robust to the selection of sub-periods.
In sum, the inclusion of volatility as a channel through which sentiment might affect returns has yielded some important results. Swings in sentiment cause adjustments in the variability of closed-end fund discounts. Investors actually trade on their sentiment, and by that move prices. In stock markets, sentiment has an influence just as well. Upward movements in sentiment lowers expected volatility and raises expected returns, whereas bearish sentiment has the opposite influence.

4.4 Conclusion

Over the last fifteen years, a large body of empirical evidence on the implications of the Noise Trader theory has accrued. All of its facets add to the big picture of the interplay between investor sentiment and stock returns. After DSSW had put forward their theory of noise trader risk, researchers first explored the nature of closed-end fund discounts. These discounts seem to be correlated among each other and with small stocks, which implies that whatever moves them is not completely specific to individual funds or stocks. Investor sentiment, or their changing misperceptions, serves as a possible explanation. These pose a risk that, because it is systematic, is priced in equilibrium: closed-end funds trade at a discount. It was proposed that an index of closed-end fund discounts could actually serve as a sentiment indicator. Consequently, it has been used alongside other objectively observable variables to forecast stock returns, with partial success. It remains in doubt whether closed-end fund discounts are actually an appropriate measure of investor sentiment. Next to closed-end fund discounts, net mutual fund redemptions seem to have a significant relationship with subsequent (small firm) returns, while the odd-lot balance seems unimportant. Trading statistics from brokers helped to pinpoint the systematic character of individual investors' trading activities. Furthermore, sentiment based on these measures has incremental explanatory power for the stock returns. Survey-based measures then helped to shed some led on the determinants of sentiment: it mostly follows past returns. Additionally, they proved useful in explaining stock returns and volatility, especially when integrated and/or filtered to purely represent irrational swings in sentiment.

One issue has emerged several times that has contradicted researchers' initial assumptions. Many studies – implicitly or explicitly – assume that noise traders mostly consist of individual or small investors. As becomes clear, this notion can not generally be upheld. Individual investors actually seem to follow contrarian investment strategies, a strategy normally assigned to professional investors. UK closed-end funds behave mostly like their U.S. counterparts, although their ownership lies mostly in the hands of institutions. Measures of individual sentiment are sometimes more relevant in explaining future S&P500 rather than small stock returns. Institutional, rather than individual, participation in trading seem to better forecast volatility, comovement and returns. This unveils a potential shortfall in noise trading theory as it is interpreted by most researchers and has some implications for further research. Results are actually influenced by the choice of an investor group whose sentiment should be measured. Nevertheless, the original theory by DSSW does not state that noise traders equal individual investors. All theoretical conclusions of the noise trader model remain valid. However, research should be sensible as to which group of investors to label "noise traders".

5. Empirical Analysis

After the classical and the Noise Trader theory as well as related previous research have been reviewed, this chapter will provide the empirical analysis of this thesis. The focus will be on the influence of investor sentiment on stock returns and conditional volatility. The structure is as follows: first, data for investor sentiment, stock returns and implied volatility will be discussed. Second, the theory behind the focal methodology of this chapter, GARCH, will be briefly outlined. The principles of maximum likelihood estimation as well as model selection aspects will be touched upon in the process. Finally, models will be fitted and their estimated parameters interpreted.

5.1 Data and Descriptives

This section will provide a summary and some descriptive statistics of the data to be used in the further analyses. Specifically, there are time series for stock returns and investor sentiment, as well as an index of expected volatility. As mentioned before, the analyses will concentrate on the German stock market.

5.1.1 Investor Sentiment

This thesis will exclusively use survey-based sentiment indicators, as it is the most direct measure of investor sentiment. No vague relation between e.g. closed-end fund discounts and returns has to be assumed, so it seems likely that survey-based measures are more objective. Unlike in the U.S., survey-based sentiment indicators have only recently become available for the German stock market. They range from monthly surveys ("G-Mind") over monthly summaries of newsletter opinions ("Notes", similar to the Investor's Intelligence index) to weekly surveys. In this thesis, weekly data is more appropriate for the applied methodology. Therefore both the G-Mind and the Notes will not be used here. One weekly index, the AnimusX-sentiment, had to be excluded as well, due to a very small sample size as a result of data loss on the side of the issuer. This leaves two sentiment indices: the Sentix and the Bull/Bear index. For the GARCH model, changes in sentiment rather than absolute levels will capture innovations, because the sentiment indices are highly autocorrelated. As the sentiment indices are already in percentage form, the simple difference between this and last period's sentiment is taken as the change:

$$\Delta S_t = S_t - S_{t-1}$$

The two separate sentiment indices will be discussed in the following subsections.

5.1.1.1 Sentix DAX, TecDAX

The Sentix is a weekly, survey-based index issued and provided by Sentix Behavioral Indices GbR[12]. It has been published every Friday since 2/23/2001, except for the Fridays around the end of the year. For this thesis, data from the beginning of the series until 4/22/2005 is used. The survey method is Internet-based. Registered investors (who classify themselves as individual or institutional) are sent an email at the end of each week, in which they are asked to visit a certain Internet site that allows them to give their appraisal of the respective market for the following month[13]. Next to the German DAX and TecDAX indices, several other markets are treated as well, alongside a "special topic" such as the oil price etc. In total, approximately 1,800 investors are now registered with Sentix, some 80% of which are private investors. The weekly response rate has risen steadily from around 60 in the beginning to around 550 for recent months. More detailed statistics can be found in Table 1.

	DAX			TecDAX		
	Individual	Institutional	Total	Individual	Institutional	Total
Average N	228	66	294	223	62	285
Average %	77.6%	22.4%		78.2%	21.8%	

Table 1: Respondent structure Sentix

The respondents have three options: either they are bullish, bearish or neutral for the respective market. Alongside the combined sentiment of individual and institutional investors, their separate sentiments are provided as well. The original sentiment index as provided by Sentix is calculated as in:

$$S_t = \frac{N_{bullish} - N_{bearish}}{N_{total}}$$

12 www.sentix.de - All copyrights by Manfred Hübner
13 For an example, see http://www.surveymonkey.com/s.asp?u=75953292830

However, for this thesis the calculation will be adjusted. The reason is comparability with the Bull/Bear sentiment index, which is calculated differently. The index will not lose or change any of its meaning; only the values will range from 0% - 100%, rather than from -100% - +100% as before. The new formula for the index is:

$$S_t = \frac{N_{bullish}}{N_{total} - N_{neutral}}$$

Tables 2 and 3 provide some descriptive statistics for the different Sentix indices.

	Sentix Index, S			Change in Sentix Index, ΔS		
	Individual	Institutional	Total	Individual	Institutional	Total
Count	209	209	209	209	209	209
Mean	48.49%	56.27%	50.45%	-0.15%	-0.10%	-0.12%
Median	47.29%	57.89%	49.55%	-0.98%	-0.23%	-0.87%
Std. Deviation	16.89%	17.51%	16.37%	19.37%	21.91%	19.28%
Variance	0.0285	0.0307	0.0268	0.0375	0.0480	0.0372
Minimum	14.29%	12.50%	13.73%	-42.51%	-57.73%	-45.69%
Maximum	84.62%	100.00%	89.66%	54.89%	77.78%	62.38%

Table 2: Sentix DAX descriptive statistics

	Sentix Index, S			Change in Sentix Index, ΔS		
	Individual	Institutional	Total	Individual	Institutional	Total
Count	209	209	209	209	209	209
Mean	45.18%	51.13%	46.58%	-0.05%	-0.11%	-0.07%
Median	46.72%	52.00%	47.96%	-0.16%	0.91%	0.02%
Std. Deviation	17.64%	17.98%	17.09%	19.16%	22.68%	19.18%
Variance	0.0311	0.0323	0.0292	0.0367	0.0514	0.0368
Minimum	11.63%	6.67%	10.71%	-44.47%	-53.90%	-44.92%
Maximum	83.83%	90.00%	83.78%	58.15%	78.03%	65.78%

Table 3: Sentix TecDAX descriptive statistics

As can be seen, all Sentix indices have a mean of roughly 50%, around which they fluctuate with a standard deviation of around 17%. The mean change across all indices is always close to zero but slightly negative. Together with the considerable standard deviation of changes, this suggests larger but offsetting positive and negative changes. Partial averages support this assumption, as listed in Table 4.

	Changes in Sentix DAX			Changes in Sentix TecDAX		
	Individual	Institutional	Total	Individual	Institutional	Total
Average ΔS^-	-15.3%	-17.0%	-15.0%	-15.0%	-18.6%	-15.5%
Average ΔS^+	15.9%	17.2%	15.9%	15.2%	17.5%	15.1%

Table 4: Average positive and negative sentiment shifts

It might prove useful later in the analysis to split positive and negative shifts in sentiment in order to identify a possibly asymmetric impact on returns and volatility.
Next, some light will be shed on the relationship of the two subindices (Individual, Institutional) with the total sentiment index. The correlations in Table 5 provide some insight.

	Sentix DAX			Sentix TecDAX		
	Individual	Institutional	Total	Individual	Institutional	Total
Individual	1.00			1.00		
Institutional	0.78	1.00		0.78	1.00	
Total	0.98	0.87	1.00	0.99	0.87	1.00

Table 5: Sentix DAX, TecDAX correlations of individual, institutional and total sentiment

While individual sentiment is nearly perfectly correlated with the total index (0.98), institutional sentiment is less so (0.87). This is not surprising as the vast majority (78%) of respondents to the Sentix surveys are individual investors, so they make up most of the total index. However, the sentiment of institutionals and individuals are not perfectly correlated either, which reflects a tendency of differing opinions between the two groups. This suggests that analysing both individual and institutional sentiment separately might be useful in further analysis.

5.1.1.2 Bull/Bear DAX, TecDAX

The Bull/Bear[14] index is issued and provided by Deutsche Börse AG. The sentiment index for the DAX reflects the opinion of around 150 actively invested institutional investors that are sampled every Wednesday since 8/21/2002 via E-mail. The TecDAX Bull/Bear index includes an additional 150 individual investors, sampled the same way since 3/5/2003. Investors are asked whether they are bullish, bearish or neutral for the following month. The resulting index, calculated using the differing number of bearish and bullish opinions, fluctuates between 0 and 100% (see Appendix for descriptive statistics). Additionally, a not further specified “certain level of basic optimism” (www.deutsche-boerse.com) is deducted from the weekly figures. Unfortunately, Deutsche Börse does not publish the method of calculating the index. The Bull/Bear is not subdivided into individual and institutional sentiment.
For several reasons, the Bull/Bear index will only play a minor role in the analyses of section 5.3. First and foremost, the sample size is very limited. For the DAX, 138 weekly observations are available, for the TecDAX there are a mere 112. Estimated GARCH models under

[14] Bull/Bear Index is a registered trademark of Deutsche Börse AG

these circumstances are likely to be instable and not very reliable. The Sentix has been published for over 18 months longer, which produces a sample size of 209. This is still not overwhelming, but the estimated models of section 5.3 seem quite stable. Additionally, the calculation method of the Bull/Bear index is ambiguous, although the numbers suggest a method similar to the one proposed above for the Sentix. However, the Bull/Bear is significantly more stable over time, as reflected in lower standard deviation considerably narrower range. As Deutsche Börse refuses to publish the exact formula, the reasons for these differing characteristics remain subject to speculation.

5.1.2 Stock Returns

For the return data series, two widely followed German stock indices will be utilized: the DAX and TecDAX[15]. While the former is the leading German blue-chip index, the latter consists of comparatively smaller-capitalization, technology-related stocks that are more often traded by individuals, in contrast to the DAX stocks. The weekly closing prices on the sampling weekday of the corresponding sentiment index will give the continuously compounded return series:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Descriptive statistics for the weekly returns of both indices in time periods and intervals corresponding to the sentiment indices are summarized in Table 6.

	DAX		TecDAX	
	since 3/2/01	since 8/28/02	since 3/2/01	since 3/12/02
Frequency	weekly	weekly	weekly	weekly
Count	217	139	217	111
Mean	-0.168%	0.056%	-0.637%	0.354%
Median	0.185%	0.433%	-0.207%	0.555%
Std. Deviation	3.77%	3.97%	5.55%	3.64%
Variance	0.0014	0.0016	0.0031	0.0013
Kurtosis	1.224	4.435	1.363	1.206
Skewness	-0.0984	-0.0273	-0.1855	0.1923

Table 6: Descriptive statistics for return series

[15] DAX and TecDAX are registered trademarks of Deutsche Börse AG, who is also responsible for composition and publication of the indices. For more information, see http://www.deutsche-boerse.com

For further analysis, the return data has been matched to the day of the sentiment surveys (Friday and Wednesday for Sentix and Bull/Bear, respectively). The few single missing values for the sentiment indices (mostly around the end of the year) have been correspondingly deleted from the return series, and the next return calculated over the whole period. So when in one week no sentiment data is available, the following week's return is actually a two-week return. Alternatively, one could have interpolated the sentiment data to fill the missing values. Due to the high volatility in sentiment this however seemed more subjective and was thus discarded.

5.1.3 Implied Volatility

For the German stock market, Deutsche Börse AG publishes the VDAX, an index that indicates the expected DAX volatility. The index is calculated using implied volatilities from the options market. Specifically, it gives the implied expected volatility in percentage points for a time horizon of 45 days. For the sake of this analysis, the original index is divided by 100, in order to align the scale with that of the estimated conditional volatilities. Within the GARCH framework, it will be attempted to improve results by incorporating the VDAX into the model as an exogenous variable. The theoretical background for this will be more elaborated upon during the analysis in section 5.3.3

5.2 Methodology

As has been mentioned before, the methodological framework for the empirical part will be GARCH, which is short for Generalized Autoregressive Conditional Heteroskedasticity. In the following, the merits of this methodology will be discussed, alongside the means of parameter estimation and model selection.

5.2.1 The Theory Behind GARCH

GARCH is an extension of the ARCH model that was first conceived by Engle (1982). It was created out of a deficiency of the well-known Ordinary Least Squares (OLS) methodology. Several assumptions lie at the heart of OLS:

- There must be no exact linear relationship (multicollinearity) between the independent variables

- The independent variables must be uncorrelated with the error terms
- The errors are mutually independent
- Each error term is a random variable with expectation zero that follows a normal distribution
- The errors have a constant variance (homoskedasticity)

It is the last assumption that often does not hold, especially when financial time-series data is involved. Already intuitively it does not seem realistic to assume constant variance in, say, equity returns. Often “wild” periods are followed by calm, low-volatility periods, a phenomenon that Mandelbrot (1963) named “volatility clustering”. The (expected) variance, or volatility in this case, is a risk that has a direct impact on prices and returns, as it should be priced in equilibrium. When OLS is used to model financial time series data, the resulting problem is that, although the estimated parameters remain unbiased, OLS understates the standard errors and thus produces confidence intervals that are too narrow (Engle, 2001). Consequently, Engle tried to model the error terms themselves rather than accepting them as the result, or residual, of his models. His achievement was that for the first time volatility was actually modelled, rather than employing simplistic measures such as the rolling standard deviation. The basic ARCH model is concerned with both the conditional mean and conditional variance, meaning that future values depend on information subsumed in past values of the data. A simple ARCH model, applied to asset returns, looks like this:

$$r_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, h_t^2) \tag{5.1}$$

$$\text{and} \quad h_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \varepsilon_{t-1}^2 \tag{5.2}$$

where r_t : Return for period t
h^2: Estimated Variance
ε_t: Estimation error
p,m: Number of autoregressive/ARCH terms

Because this model had some limitations (such as potentially long lag structures), Bollerslev (1986) generalized it by including autoregressive terms in the variance equation:

$$h_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \gamma_i h_{t-1}^2 \tag{5.3}$$

This is called the GARCH(*p*,*q*) model, where *p* and *q* indicate the number of ARCH and GARCH parameters in the variance equation, respectively. By making current values of conditional volatility dependent on past values of itself, the volatility clustering feature now entered the model more directly. In turn, Engle *et al.* (1987) modified the mean equation in order to allow the changing conditional volatility to directly influence contemporaneous returns:

$$r_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i} + \delta h_t^2 + \varepsilon_t \tag{5.4}$$

Together, equations 5.3 and 5.4 constitute the GARCH(*p*,*q*) in-mean model, which is widely used in financial applications. One aspect that entered GARCH models later was the so-called leverage effect. The effect describes the fact that (extreme) news, or shocks, affect conditional variance depending on their sign. In particular, negative past shocks (as represented in the ARCH parameters in the variance equation) are expected to be followed by higher levels of conditional variance than positive shocks. The reasoning goes back to Black (1976) and argues that when negative news about a company arrives, its stock price drops, which causes a higher leverage ratio. Firms with higher leverage are perceived as more risky, so that investors raise their conditional volatility expectation. Although it is not exactly clear if the change in relative leverage is the only or most important reason, the name "leverage effect" has stuck. Engle and Ng (1995) provide a review of GARCH specifications that allow for this asymmetric effect and identify the simple implementation of Glosten, Jagannathan and Runkle (1993) as the best. They propose the following variance equation for a simple GARCH(1,1) model:

$$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 I_{t-1} \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 \tag{5.5}$$

where I_{t-1}: Dummy variable. Equals 1 when ε_{t-1} is positive, and 0 otherwise

The parameter β_2 is thus expected to be negative, so that positive past shocks cause a lower conditional variance that do negative ones. After that, many extensions and modifications have been found. However, they will not be discussed here both for brevity and because they are all derived from these basic models. The Glosten *et al.* (1993) modification however will enter the model building phase in section 5.3. The actual parameters of these kinds of models have to be estimated by maximum-likelihood techniques, which will be discussed now.

5.2.2 Maximum Likelihood Estimation

In the maximum likelihood approach, a model is estimated such that the joint probability of observing the data sample, given the parameters used, is maximized. To do so, for each observation a probability density function is calculated which, under the assumption of normality for the error term, equals

$$P(r_t \mid \theta) = \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{1}{2}\left(\frac{\varepsilon_t}{h_t}\right)^2}$$

where r_t: Return for period t

θ: Vector of model parameters

ε_t : Observed estimation error

h_t: Estimated standard deviation

(see Watsham & Parramore, 1997, pp. 267-8)

All parameters to be estimated are included in this function, either through the error term (determined by the mean equation parameters) or the estimated standard deviation (determined by the variance equation parameters). The product of these individual functions would yield the joint probability as represented in the likelihood function:

$$L(r) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{1}{2}\left(\frac{\varepsilon_i}{h_t}\right)^2}$$

For simplicity, the natural logarithm is taken, so that the log-likelihood function to be maximized becomes:

$$\ln L(r) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_t \ln(h_t) - \frac{1}{2}\sum_t \left(\frac{\varepsilon_t}{h_t}\right)^2$$

Because ln(L) is an increasing function of L, this transformation does not affect the final result of maximization, but only simplifies the estimation process. Numerical search procedures, here as implemented in Microsoft Excel's Solver, are used to maximize the likelihood function by finding optimal values for the parameters. The appropriate parameters are determined in the model selection phase, which is discussed in the following.

5.2.3 Model Selection Criteria

During the process of model selection, the actual form of the model is sought. Specifically, the optimal number of autoregressive terms in the mean equation and the number of ARCH and GARCH terms in the volatility equation will be determined. For models with the same number of observations, but different specifications, the absolute values of the maximum likelihood function could be directly compared. After all, a higher log-likelihood value should indicate a better model fit. However, the log-likelihood, much like the (unadjusted) R^2 in OLS models, typically increases with the number of parameters used. In order to avoid over-parameterization, more objective model selection criteria have to be employed. The two most commonly used are the Akaike and Schwartz information criteria (AIC and SIC, respectively). The purpose of model selection criteria is to formalize the trade-off between goodness of fit and model complexity. In this case, the highest maximum likelihood estimate should be achieved by using the lowest possible number of parameters. So while model fit is desired, complexity will be penalized. The basic difference between the various information criteria is the weight of the penalty for model complexity. Kuha (2004) for example suggests the following forms of AIC and SIC for the context of models estimated by maximum likelihood:

$$AIC = 2(MLE_F - MLE_P) - 2(p_F - p_P) \quad \text{and}$$
$$SIC = 2(MLE_F - MLE_P) - \ln(n)(p_F - p_P)$$

where MLE_F, MLE_P: Maximum likelihood estimates for the full and partial model, respectively
p_F, p_P: Number of parameters for the full and partial model
n: Number of observations

As can be seen, these forms involve the comparison of the respective model to the full model, which is the one with the full range of applicable parameters. The latter normally also yields the highest MLE, so the trade-off is evident. Both criteria indicate a comparably better model when the values are *negative*. The Schwartz criterion gives more preference to parsimonious models, especially with larger sample sizes. This or any other form of AIC and SIC are in widespread use when it comes to model selection. However, the criteria have to be modified in order to fit the heteroskedastic features of GARCH models (Brooks & Burke, 2003).

The proposed adjusted measures[16] are:

$$AIC = \sum_{t=1}^{n} \ln\left(h_t^2\right) + 2p \quad \text{and} \quad SIC = \sum_{t=1}^{n} \ln\left(h_t^2\right) + \ln(n)\, p$$

where n: Number of observations
h^2: Estimated variance
p: Number of model parameters

Although the resulting absolute values for the criteria are considerably different, the use is similar: a *lower* value for the criterion indicates a better model. Again, the Schwartz criterion is more restrictive concerning the use of parameters. The number of model parameters (p) is somewhat ambiguous, because it is not clearly defined whether to include the intercept terms from the mean and variance equation. In the following analysis, where the criteria will be used, p is defined as the number of parameters *excluding* intercepts.

5.3 Analysis

In this part, a GARCH in-mean model is fitted that best suits the data, and the results interpreted in the following. First, base models are sought that fit the weekly return data for the DAX and TecDAX well. Afterwards, the base models for the two stock indices are augmented with sentiment parameters. The estimated models are interpreted with respect to Noise Trader theory. Finally, the VDAX as a measure of implied volatility is integrated to reveal any improvements when options market information is included.

As mentioned before, the Bull/Bear index is only of limited use as a sentiment indicator in the context of GARCH models due to small sample size. While the analyses are performed with both indicators, only the results of the Sentix-related models are reported and discussed in this section. The corresponding results for the Bull/Bear models can be found in the appendix. Because they are probably quite unstable, no reliable conclusions can be drawn, so they are not further commented on.

[16] For derivation and discussion of the criteria see the appendix in Brooks and Burke (2003)

5.3.1 Base Model Selection

This section contains the determination of an optimal base model for the returns of both DAX and TecDAX. The analysis starts with the DAX index. For convenience, the AR(*m*)-GARCH(*p*,*q*) in-mean including the Glosten, Jagannathan and Runkle (1993) adaptation for asymmetric news impacts is repeated below:

$$r_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i} + \delta h_t^2 + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, h_t^2) \quad \text{and}$$

$$h_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \varepsilon_{t-i}^2 + \sum_{i=1}^{u} \psi_i I_{t-i} \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i h_{t-i}^2$$

where
- r: Weekly return for period t
- h^2: Estimated Variance
- ε: Estimation error
- I_{t-1}: Dummy variable. Equals 1 if $\varepsilon_{t-1} > 0$, and 0 otherwise
- m: Number of autoregressive terms
- p: Number of ARCH terms
- u: Number of Glosten *et al.* news asymmetry terms (if any, u equals p)
- q: Number of GARCH terms

Model building starts with a simple AR(1)-GARCH(1,1) in-mean model, without allowing for asymmetric effects (so u = 0). From there, the number of autoregressive terms for the mean equation is adjusted. Then, the optimal GARCH specification is found (adjust p and q). Finally, asymmetric news impact effects are included to discover whether this improves the model. For prioritizing the different models, the aforementioned Akaike and Schwartz information criteria are drawn on. The resulting criteria are listed together with the estimated parameters in Table 7.

	1: Determine number of AR			2: Determine GARCH			3: Add GJR asymmetry			
	1	2	3	(2,1)	(1,2)	(2,2)	(1,1)	(2,1)	(1,2)	(2,2)
α_0	-0.001	0.000	-0.001	0.000	-0.001	0.000	-0.001	0.000	-0.001	0.000
r_{t-1}	0.498	1.606	2.054	1.613	1.399	1.614	1.644	1.629	1.457	1.616
r_{t-2}		-0.982	-1.856	-0.976	-0.933	-0.980	-1.004	-0.987	-1.004	-0.981
r_{t-3}			0.613							
h^2	0.894	0.262	0.352	0.340	0.732	0.265	-0.009	0.175	0.167	0.264
β_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε^2_{t-1}	0.378	0.626	0.485	0.800	0.271	0.654	0.213	0.734	0.150	0.653
$I_{t-1}\varepsilon^2_{t-1}$							-0.459	-0.522	-0.332	-0.122
ε^2_{t-2}				-0.703		-0.611		-0.525		-0.612
$I_{t-2}\varepsilon^2_{t-2}$								0.222		0.081
h^2_{t-1}	0.798	0.655	0.738	0.948	1.274	1.312	0.996	0.948	0.970	1.319
h^2_{t-2}					-0.416	-0.337			0.025	-0.335
n	208	208	208	208	208	208	208	208	208	208
p	4	5	6	6	6	7	6	8	7	9
MLE	75.3	77.2	77.1	79.8	76.6	80.3	83.5	83.0	82.9	81.7
AIC	-1272.0	-1276.2	-1274.0	-1286.0	-1269.2	-1281.5	-1307.7	-1292.7	-1319.2	-1283.8
SIC	-1258.7	-1259.5	-1254.0	-1266.0	-1249.2	-1258.2	-1287.7	-1266.0	-1295.8	-1253.8

Table 7: Estimated model parameters for DAX base models

Note: Parameters have been estimated with Microsoft Excel's Solver function. During the estimation process, the conditional variances were restricted to remain positive at all times. Beginning values in the dataset for the expected return and conditional variance were taken as the observed mean return and observed variance of returns within the sample.

The model parameters are not exceptional. Both intercepts are very close to zero, contemporaneous volatility has an increasing effect on returns, and the GARCH parameters show that volatility is significantly serially related, which is in line with volatility clustering. In the first step, both model selection criteria are lowest for the AR(2) model, which is thus preferred. As not even MLE is gained by adding another AR parameter, the increased model complexity does obviously not lead anywhere. Now that the number of autoregressive terms for the mean equation (2) was found, the variance equation is modified. Bollerslev, Chou and Kroner (1992) point out that the simple GARCH(1,1) is well suitable for most applications. However, as expected, in step two the model with the highest number of parameters gains the highest MLE. Nevertheless, the selection criteria point consistently to the GARCH(2,1) specification. Finally, in step three, asymmetric news impacts are allowed for. While the number of AR parameters in the mean equation is held constant, all four GARCH variants are estimated again. Surprisingly, when asymmetric effects are included, the selection criteria prefer the GARCH(1,2) specification. This can be explained by the fact that when asymmetric effects are allowed for, for every ARCH term of the variance equation one additional parameter is added. As the model selection criteria penalize overuse of parameters, it becomes possible for another model to be more favourable. To conclude, the chosen base model for the DAX is as follows:

$$r_t = \alpha_0 + \alpha_1 r_{t-i} + \alpha_2 r_{t-2} + \delta h_t^2 + \varepsilon_t \qquad \text{with} \quad \varepsilon_t \sim N(0, h_t^2) \quad \text{and}$$

$$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \psi_1 I_{t-1} \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 + \gamma_2 h_{t-2}^2$$

For the TecDAX, the results as summarized in Table 8 differ slightly.

	1: Determine number of AR			2: Determine GARCH			3: Add GJR asymmetry			
	1	2	3	(2,1)	(1,2)	(2,2)	(1,1)	(2,1)	(1,2)	(2,2)
α_0	0.000	0.000	-0.001	0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.002
r_{t-1}	0.946	1.648	0.737	1.655	1.645	1.628	1.653	1.652	1.645	1.627
r_{t-2}		-0.988	0.730	-0.991	-0.987	-0.990	-0.990	-0.989	-0.986	-0.986
r_{t-3}			-1.018							
h^2	-0.060	-0.146	-0.096	-0.200	-0.129	0.086	-0.290	-0.189	-0.239	0.078
β_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε^2_{t-1}	0.345	0.382	0.265	0.516	0.344	0.406	0.360	0.397	0.264	0.639
$I_{t-1}\varepsilon^2_{t-1}$							-0.295	-0.005	-0.203	-0.132
ε^2_{t-2}				-0.256		-0.412		0.036		-0.624
$I_{t-2}\varepsilon^2_{t-2}$								-0.302		0.073
h^2_{t-1}	0.813	0.791	0.858	0.859	1.012	1.600	0.869	0.836	1.304	1.329
h^2_{t-2}					-0.200	-0.599			-0.405	-0.326
n	208	208	208	208	208	208	208	208	208	208
p	4	5	6	6	6	7	6	8	7	9
MLE	36.5	38.7	38.0	38.8	39.6	43.7	40.5	40.8	41.8	41.8
AIC	-1126.9	-1130.9	-1129.8	-1129.9	-1130.2	-1148.8	-1137.4	-1131.9	-1134.7	-1142.0
SIC	-1113.6	-1114.2	-1109.7	-1109.9	-1110.2	-1125.5	-1117.4	-1105.2	-1111.3	-1112.0

Table 8: Estimated model parameters for TecDAX base models

In step one, the preferred number of autoregressive parameters is two, just like before. Step two reveals a GARCH(2,2) model as the most suitable. It has the highest MLE and scores best both with respect to AIC and SIC. These results can not even be topped by asymmetric models. As said before, asymmetric models require even more parameters, so this is entirely possible. Additionally, it is possible that the TecDAX returns and volatilities do not exhibit asymmetric effects in the first place. Engle and Ng (1995) propose the news impact curve as a simple measure of identifying asymmetric effects in GARCH models. Basically, past estimation errors are plotted against current conditional volatilities, and the resulting graph is examined. Figure 2 gives an example for a news impact curve for the TecDAX GARCH(2,2) model identified above.

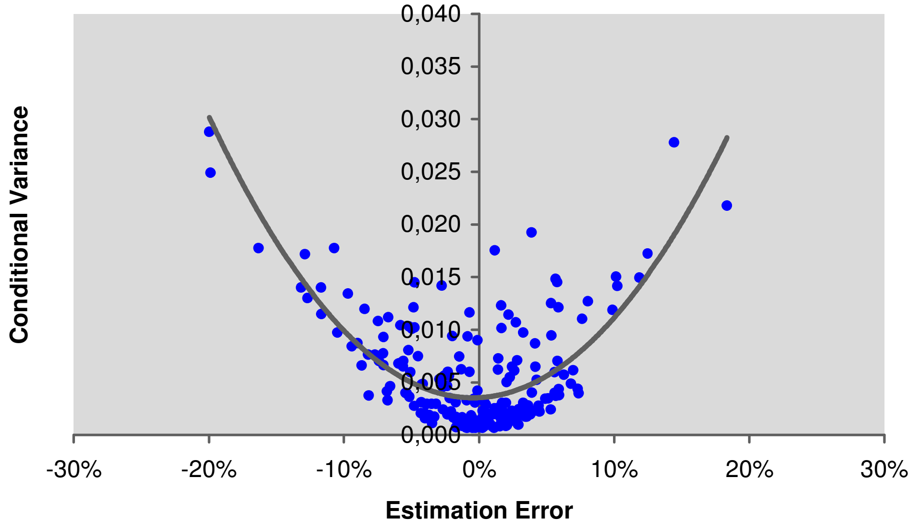

Figure 2: TecDAX News impact curve

As can be seen, neither negative nor positive past errors exert greater influence on conditional variances. The fitted line, which approximates the news impact curve, seems symmetric. A simple ANOVA analysis confirms the impression (see Table 9).

Groups	Count	Sum	Average	Variance
Error < 0	104	0.632	0.006	0.000
Error > 0	103	0.538	0.005	0.000

Source of Variation	SS	df	MS	F	P-value
Between Groups	0.0000	1	0.0000	1.3526	0.2462
Within Groups	0.0057	205	0.0000		
Total	0.0057	206			

Table 9. Single-Factor ANOVA analysis of TecDAX base model conditional variances

The null hypothesis of equal means for conditional volatilities after past positive and negative errors cannot be rejected at any relevant significance level.

One important fact of the displayed GARCH models for the TecDAX is that the volatility parameter in the mean equation mostly has a negative sign, which implies a negative relation of volatility and returns. This is at odds with classical capital asset pricing theory, where taking higher risks should be rewarded with excess expected returns. However, numerous researchers before have encountered and explained the same phenomenon, including Glosten, Jagannathan and Runkle (1993) and De Santis and Gerard (1997). According to them, this negative relation can occur in times of high savings rates and limited alternative investment opportunities, or in times of high inflation and downward sloping term structures (Boudoukh, Richardson, & Smith, 1993). A more econometric explanation has been provided by Backus and Gregory (1993), who find that the relationship of returns and conditional volatility of asset returns can be positive, negative or flat, depending on the preferences of the representative agent and other statistical features. They conclude that the validity of GARCH models is not restricted when atypical relationships of risk and return are found. In the GARCH(2,2) model that seems best for the TecDAX however the δ parameter in the mean equation is positive, which is the classical expectation. Consequently, this becomes the base model for the TecDAX analyses:

$$r_t = \alpha_0 + \alpha_1 r_{t-i} + \alpha_2 r_{t-2} + \delta h_t^2 + \varepsilon_t \qquad \text{with} \quad \varepsilon_t \sim N(0, h_t^2) \quad \text{and}$$

$$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \gamma_1 h_{t-1}^2 + \gamma_2 h_{t-2}^2$$

Now that the two base models have been identified, the next section will add parameters for investor sentiment.

5.3.2 Adding Sentiment Parameters

In this section, sentiment parameters are added to the previously specified base models for the DAX and TecDAX in turn. To this end, the sentiment indices described in section 5.1.1, or rather their changes, are employed.

5.3.2.1 DAX GARCH with Sentiment

Starting with the DAX, the previously specified AR(2)-GARCH(1,2) in-mean base model is augmented with parameters for changes in sentiment, so it becomes:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \delta h_t^2 + \eta_1 \Delta S_t + \eta_2 \Delta S_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, h_t^2)$$

$$\text{and} \qquad h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \psi_1 I_{t-1} \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 + \gamma_2 h_{t-2}^2 + \lambda \Delta S_{t-1}^2$$

where r_t : Return for period t

h^2: Estimated conditional variance

ε: Estimation error

ΔS: Change in sentiment

I_{t-1}: Dummy variable. Equals 1 if $\varepsilon_{t-1} > 0$, and 0 otherwise

The model is motivated by Noise Trader theory as laid out by De Long, Shleifer, Summers and Waldman (DSSW, 1990). As discussed in section 3.1, changes in noise trader sentiment affect returns through four effects. In short, the "price pressure" effect lowers expected returns when sentiment increases as prices are pushed up by trading activity. The "hold more" effect in contrast means that noise traders are induced to hold more of the risky asset when they are bullish, for which they are compensated with excess returns. When they turn bearish, noise traders sell the risky assets and cannot reap extra returns anymore. The "create space" effect describes the fact that when sentiment is more volatile, i.e. changes are of higher magnitude, the created irrational risk (volatility) crowds out professional traders so the benefits of holding the risky assets accrues proportionately more to noise traders. Finally, the "Friedman" effect is also concerned with the magnitude of shifts, but works in the opposite direction. Because noise traders buy (sell) when others buy (sell), they have the worst possible market timing. So the more volatile their sentiment is, the more they loose through their trading activities. Accordingly, the λ in the variance equation indicates whether the "create space" or "Friedman" effect dominates, while the ηs of the mean equation refer to the "price pressure" and "hold more" effects. It still has to be determined whether to use contemporaneous or past changes in sentiment in the mean equation, or even both. The options estimated with Sentix sentiment parameters are summarized for comparison together with the DAX base model in Table 10.

	DAX Base	S_t	S_{t-1}	$S_t + S_{t-1}$
α_0	-0.001	0.000	-0.003	0.000
r_{t-1}	1.457	0.622	0.413	1.046
r_{t-2}	-1.004	0.150	-0.048	-0.135
h^2	0.167	0.241	0.150	0.100
ΔS_t		0.120		0.112
ΔS_{t-1}			-0.003	-0.047
β_0	0.000	0.000	0.000	0.000
ε^2_{t-1}	0.150	0.118	0.247	0.229
$I_{t-1}\varepsilon^2_{t-1}$	-0.332	-0.338	-0.503	-0.627
h^2_{t-1}	0.970	0.777	0.797	0.666
h^2_{t-2}	0.025	0.202	0.207	0.315
ΔS^2_{t-1}		0.00042	-0.00003	0.00091
n	208	208	208	208
p	7	9	9	10
MLE	82.9	116.8	78.0	120.3
AIC	-1319.2	-1445.1	-1260.9	-1441.1
SIC	-1295.8	-1415.0	-1230.8	-1407.7

Table 10: DAX base model augmented with sentiment parameters

Considering the AIC and SIC, the first and most important result is that adding sentiment parameters significantly improves the model fit when contemporaneous shifts in sentiment are included in the mean equation. However, only including the change in sentiment of last period actually worsens the model. Including contemporaneous shifts is imperative, and including the lagged shift does not improve the model further. If this is done, the model is actually dramatically better, as indicated by MLE, AIC and SIC. Most other parameters remain comparable to those of the base model. The δ in the mean equation stays positive - volatility and return maintain their classical relationship. Conditional volatility is stronger serially correlated when sentiment is included (γ_2 is higher). Volatility clustering seems to be more prevalent than anticipated by the base model.

Interpreting the parameters for the sentiment measures, the results are in line with the DSSW theory. First, the relation of changes in sentiment with expected returns is positive. Bullish (bearish) shifts in contemporaneous sentiment increase (decrease) expected returns, while the opposite is true for past shifts, though with lower magnitude (parameters η_1 and η_2). Consequently, the "hold-more" effect dominates the adverse "price pressure" effect. When investors become more bullish, they are induced to invest more in the risky assets, for which they are compensated. The "price pressure" effect is obviously not able to completely offset this benefit. Furthermore, conditional volatility increases with the magnitude of shifts in sentiment (ΔS^2). The interpretation is that noise trader's misperceptions create a risk (volatility) that is priced. The increased risk crowds out rational investors ("create space" effect), which lets the benefits of holding risky assets accrue overproportionately to noise traders, so expected returns are raised.

The next step is to distinguish between negative and positive shifts in noise trader sentiment and assess their influence on conditional volatility. The data descriptives in Section 5.1.1 already suggested this step because of the comparably high standard deviation in changes. The variance equation is adjusted to allow for this as follows:

$$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \psi_1 I_{t-1} \varepsilon_{t-1}^2 + \gamma_1 h_{t-1}^2 + \gamma_2 h_{t-2}^2 + \lambda_1 \Delta S_{t-1}^2 D_{t-1} + \lambda_2 \Delta S_{t-1}^2 (1 - D_{t-1})$$

where D_{t-1} : Dummy variable. Equals 1 if $\Delta S_{t-1} > 0$, and 0 otherwise

The estimated parameters are listed in Table 11 below.

	DAX Base	S_t	$S_t + S_{t-1}$
α_0	-0.0008	-0.0001	0.0000
r_{t-1}	1.4569	0.6046	0.9086
r_{t-2}	-1.0043	0.1979	-0.0387
h^2	0.1671	-0.0017	-0.1074
ΔS_t		0.1245	0.1144
ΔS_{t-1}			-0.0334
β_0	0.0000	0.0000	0.0000
ε^2_{t-1}	0.1502	0.1612	0.1988
$I_{t-1}\varepsilon^2_{t-1}$	-0.3316	-0.4086	-0.4754
h^2_{t-1}	0.9696	0.7759	0.5245
h^2_{t-2}	0.0247	0.2165	0.4739
$\Delta S^2_{t-1}D_{t-1}$		-0.0016	-0.0014
$\Delta S^2_{t-1}(1-D_{t-1})$		0.0011	0.0014
n	208	208	208
p	7	10	11
MLE	82.9	117.6	118.9
AIC	-1319.2	-1427.8	-1439.7
SIC	-1295.8	-1394.5	-1403.0

Table 11: DAX base model with split sentiment parameters

This time, the model *with* lagged changes in sentiment in the mean equation seems to perform best. Note that the sign for the volatility parameter in the mean equation (δ) has flipped to negative. As explained before, this does not invalidate the GARCH model. Again, changes in sentiment are positively correlated with returns. In the mean equation, the ηs still indicate that the "hold more" effect dominates the "price pressure" effect. Now, the variance equation allows conclusions to be drawn about the interaction of the "Friedman" and "create space" effects. The negative λ_1 indicates that past positive changes in sentiment lead to a downward revision of conditional volatility, while negative changes cause the opposite. So when sentiment becomes more bullish (bearish), volatility goes down (up) which in turn raises (lowers) expected returns, as δ is negative. In the logic of DSSW, noise traders create their own space, but are only able to reap additional benefits when they are bullish and actually invest in the assets subject to noise trader risk. The influence of the magnitude of changes in sentiment can be positive or negative, which implies that the bad market timing ("Friedman" effect) eats up noise traders' returns when they turn bearish. They simply sell when everybody does, which is when prices are already low. On the other hand, noise traders are very irrational when they turn bullish, a phenomenon called *overconfidence*. This behaviour drives away rational investors. Expected returns then accrue to noise traders, despite their bad market timing. The following subsection will go through the same steps for the TecDAX.

5.3.2.2 TecDAX GARCH with Sentiment

The augmented model for the TecDAX is as follows:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \delta h_t^2 + \eta_1 \Delta S_t + \eta_2 \Delta S_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, h_t^2)$$

and $$h_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \gamma_1 h_{t-1}^2 + \gamma_2 h_{t-2}^2 + \lambda \Delta S_{t-1}^2$$

where r_t : Return for period t

h^2: Estimated conditional variance

ΔS_t: Change in sentiment

ε_t: Estimation error

From there, the same steps as for the DAX model are performed. First, only the magnitude of shifts in sentiment is included in the variance equation, while later positive and negative changes are accounted for separately. The results are summarized in Table 12.

	TecDAX Base	Lagged ΔS			Split ΔS²	
α_0	-0.0011	-0.0011	0.0000	-0.0011	0.0001	-0.0007
r_{t-1}	1.6280	0.8000	1.9253	0.8000	0.6038	0.8178
r_{t-2}	-0.9899	0.1000	-0.9442	0.1000	0.2476	-0.1271
h^2	0.0862	0.0862	-0.0151	0.0862	-0.2804	-0.0495
ΔS_t		0.1201		0.1201	0.1524	0.1453
ΔS_{t-1}			0.0031	-0.0123		-0.0336
β_0	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
ε^2_{t-1}	0.4057	0.4056	0.8275	0.4056	0.4358	0.5603
ε^2_{t-2}	-0.4118	-0.4118	0.4209	-0.4118	-0.4463	-0.5651
h^2_{t-1}	1.5997	1.5996	-0.0988	1.5996	1.5035	1.3912
h^2_{t-2}	-0.5989	-0.5990	0.4301	-0.5990	-0.5011	-0.3887
ΔS^2_{t-1}		-0.00012	0.01610	-0.00012		
$\Delta S^2_{t-1} D_{t-1}$					-0.0005	-0.0018
$\Delta S^2_{t-1}(1-D_{t-1})$					0.0003	0.0010
n	208	208	208	208	208	208
p	7	9	9	10	10	11
MLE	43.7	62.5	39.3	62.9	68.5	66.0
AIC	-1148.8	-1190.5	-1122.0	-1191.2	-1230.0	-1222.8
SIC	-1125.5	-1160.5	-1092.0	-1157.8	-1196.7	-1186.1

Table 12: TecDAX base model augmented with sentiment parameters

The findings are similar as for the DAX, although the overall model, even after adding sentiment parameters, is not as good as for the DAX. Adding sentiment parameters again definitely improves the model when contemporaneous changes in sentiment are included in the mean equation. The selection criteria do not agree whether the lagged change in sentiment in the mean equation does add explanatory power. The signs of the relevant sentiment parameters are also similar to the DAX. Expected returns rise with contemporaneous bullish shifts in sentiment – the "hold more" effect dominates. In the variance equation, the relationship between sentiment changes and conditional volatility is not clear. Sometimes "Friedman" prevails, sometimes "create space" is more powerful. Splitting changes in sentiment into positive and negative again yields further insight. In contrast to the DAX, the lagged change in sentiment in the mean equation still does not add explanatory power. However negative past changes in sentiment lead to an upward revision of conditional volatility, whereas positive sentiment calms the market. As the relation between conditional volatility and expected returns is again consistently negative, the interpretation in the context of Noise Trader theory is confirmed. The results are also in line with the findings of Lee, Jiang and Indro (2002), who performed a similar analysis. They did not however distinguish between institutional and individual sentiment, an aspect that will be dealt with in the following section.

5.3.3 Distinguishing Individual and Institutional Sentiment

The next question now is which kind of sentiment better explains DAX returns: individual or institutional sentiment. As the Sentix dataset distinguishes between individual and institutional answers, further insights can probably be gained from separate analysis.

Previous empirical research has mostly at least implicitly equated noise traders with individual investors. They are supposedly "less sophisticated" and thus more prone to behaving irrationally. As noise traders have to trade in order to benefit from eventual excess returns, it is assumed that their sentiment should affect rather small capitalization stocks, which are traded more by individuals. The DAX is Germany's blue chip index of stocks that are mainly traded by professionals. Individual sentiment should accordingly not explain the market behaviour very well, while it should be more helpful in explaining the movements in the TecDAX. The latter consists of rather small capitalization, mostly technology-related stocks. The valuation of these companies is relatively more subjective, so they should be more sensible to irrational swings in sentiment. This logic follows the argumentation in Baker and Wurgler (2004, see Section 4.2.3). Apart from the theoretical motivation, Section 5.1.1 identified less-than-perfect correlations between the subindices for individual and institutional sentiment, which also suggests differing results. The results for the DAX and TecDAX base models, augmented with both institutional and individual sentiment measures, are presented in Table 13.

	DAX			TecDAX		
	Total	**Individual**	**Institutional**	**Total**	**Individual**	**Institutional**
α_0	0.0000	-0.0001	0.0000	0.0001	0.0000	0.0004
r_{t-1}	0.9086	0.9086	0.9631	0.6038	0.5524	0.5524
r_{t-2}	-0.0387	-0.0387	-0.1506	0.2476	0.2352	0.2352
h^2	-0.1074	-0.1074	-0.0225	-0.2804	-0.3990	-0.3990
ΔS_t	0.1144	0.1143	0.0948	0.1524	0.1335	0.1335
ΔS_{t-1}	-0.0334	-0.0334	-0.0136	0.0000	0.0000	0.0000
β_0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ε^2_{t-1}	0.1988	0.1988	0.2064	0.4358	0.5579	0.5581
ε^2_{t-2}				-0.4463	-0.5661	-0.5659
$I_{t-1}\varepsilon^2_{t-1}$	-0.4754	-0.4754	-0.5306			
h^2_{t-1}	0.5245	0.5245	0.4652	1.5035	1.3743	1.3744
h^2_{t-2}	0.4739	0.4740	0.5218	-0.5011	-0.3697	-0.3703
$\Delta S^2_{t-1}D_{t-1}$	-0.0014	-0.0014	0.0003	-0.0005	-0.0021	-0.0014
$\Delta S^2_{t-1}(1-D_{t-1})$	0.0014	0.0014	0.0007	0.0003	0.0012	0.0008
n	208	208	208	208	208	208
p	11	11	11	10	10	10
MLE	118.9	116.8	110.6	68.5	66.1	60.1
AIC	-1439.7	-1423.8	-1417.4	-1230.0	-1230.7	-1200.2
SIC	-1403.0	-1387.1	-1380.7	-1196.7	-1197.3	-1166.8

Table 13: DAX and TecDAX sentiment models by investor class

For the DAX, total sentiment seems to have the most explanatory power as measured in MLE, AIC and SIC. This does not seem far-fetched because the DAX is the most widely followed German stock index, so it seems logical that it should be affected by swings in the mood of the general investor community. Secondly, and surprisingly, individual investor sentiment also performs well in explaining expected returns and conditional volatilities. According to classical interpretation of noise trading theory, expectations of small investor should affect only classes of assets they are proportionately heavily invested in. So the results start to question this understanding. Furthermore, although it scores worst of the three, institutional sentiment still has significant explanatory power. The sentiment of allegedly rational professionals obviously does have some impact. For the TecDAX, the total sentiment again yields the highest MLE, however the model selection criteria point to individual sentiment. This time, individual investors do have an influence, as was expected. However, the institutional investors do as well. This finding is consistent with Lee, Jiang and Indro (2002) and Brown and Cliff (2004), who also found relationships between individual sentiment and returns that counter the normal interpretation.

Concluding, contrary to equating individual investors with noise traders, it is very likely that noise traders exist in every investor class. The term "noise trader" has thus to be treated carefully and only to abstractly separate irrational from rational behaviour.

Now that the basic analyses concerning sentiment have been performed, the next section will turn to another aspect that might improve previous models: including implied volatilities from the options market.

5.3.4 Adding Implied Volatilities

As has been mentioned before, one purpose of GARCH models is to provide forecasts for conditional variances. These are extremely important for several financial applications, among the most important being option pricing. The standard model for pricing option contracts is the pathbreaking approach by Black and Scholes (1973). In a rather simple manner, they managed to predict option prices with significant accuracy. One input to their estimation technique is expected volatility, which at the time was mainly past volatility extrapolated into the future. Later, conditional variances derived from GARCH-type models served as more accurate inputs. Latane and Rendleman (1976) turned the logic around and used the Black-Scholes formula to calculate the expected volatility as implied in observed option prices by solving for different parameters. Day and Lewis (1995) then integrated these implied volatilities in a GARCH framework. So while first GARCH variances were used as exogenous variables for estimating option prices, later implied volatilities from observed option prices served as an input to GARCH models in order to estimate future returns. Day and Lewis found that implied volatilities, when included as an exogenous variable, may add information content to standard GARCH and EGARCH models. Accordingly, the very same approach will be attempted here. To avoid the tedious task of calculating implied volatilities from option prices, the VDAX, an implied volatility index issued by Deutsche Börse AG, will be utilized. As has already been mentioned in section 5.3.1, the VDAX only incorporates options on the DAX, so only the corresponding models will be reviewed here. As can be seen in Figure 3, the VDAX moves somewhat in line with the estimated conditional variances from the DAX base and sentiment model[17].

[17] The conditional variances have been restated in order to match the VDAX 45 day horizon and format (%). The transformation is as follows: $E(v)_t = \left(1+\sqrt{h_t^2}\right)^{\frac{45}{7}} - 1$, where h^2 is the original conditional variance

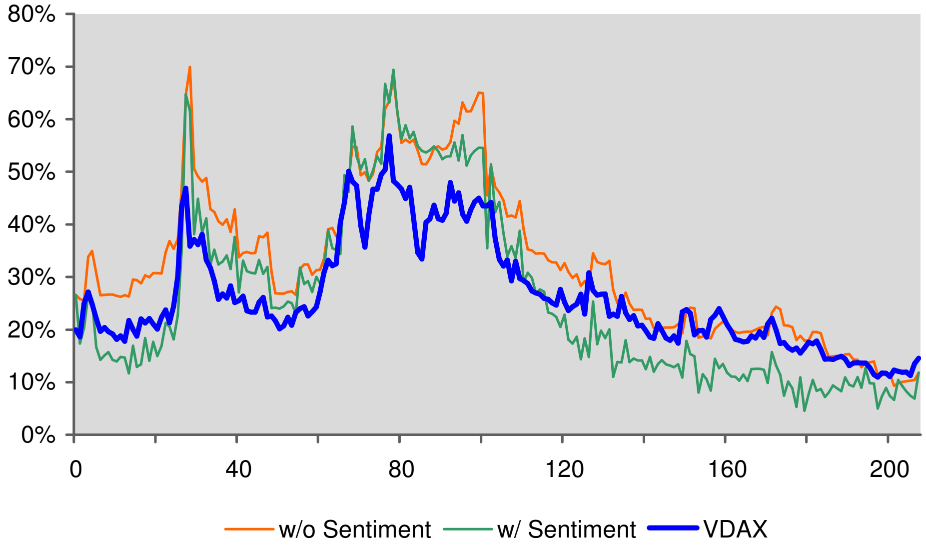

Figure 3: Time series of VDAX and estimated conditional volatilities

On the one hand, this shows that the model is quite well-specified, on the other it suggests that the VDAX might be useful to implement. Consequently, the base model for the DAX as well as its sentiment-augmented counterpart has been extended with an additional variable *VDAX* in the conditional variance equation. The results are presented in Table 14 below.

	DAX Base	With sentiment		
		Total	Individual	Institutional
α_0	-0.0005	0.0006	0.0001	0.0005
r_{t-1}	1.3945	-0.0521	0.2382	0.2081
r_{t-2}	-0.6776	0.4468	0.3341	0.4531
h^2	-0.1443	-0.0583	-0.2068	-0.0709
ΔS_t	0.0000	0.1066	0.1050	0.1029
ΔS_{t-1}	0.0000	0.0633	0.0270	0.0572
β_0	0.0000	0.0000	0.0000	0.0000
ε^2_{t-1}	0.2025	0.2575	0.2489	0.2012
$I_{t-1}\varepsilon^2_{t-1}$	-0.4091	-0.3020	-0.2958	-0.3850
h^2_{t-1}	0.9888	0.3119	0.6010	0.7306
h^2_{t-2}	0.0143	0.6186	0.3340	0.2310
$\Delta S^2_{t-1}D_{t-1}$	0.0000	-0.0018	-0.0023	-0.0021
$\Delta S^2_{t-1}(1-D_{t-1})$	0.0000	0.0025	0.0018	0.0018
$VDAX_{t-1}$	-0.0100	-0.0257	-0.0097	0.0046
n	208	208	208	208
p	8	12	12	12
MLE	82.2	113.9	113.6	107.7
AIC	-1305.5	-1434.6	-1420.1	-1382.4
SIC	-1278.8	-1394.5	-1380.1	-1342.3

Table 14: DAX sentiment models with additional implied volatility parameter

Apparently, adding implied volatilities does not add a lot goodness of fit. All model selection criteria, whether for base or sentiment models, prefer the original form. Additionally, the parameter values are very small, although the index has already been scaled by 100. This suggests that all information from past variances is already included in the GARCH terms. Consequently, rather than adding the *VDAX* parameter, the lagged conditional variances in the volatility equation will be substituted by lagged *VDAX* values. As Table 15 reveals, this does not help the model either.

	DAX Base	With sentiment		
		Total	Individual	Institutional
α_0	-0.0010	-0.0003	-0.0003	-0.0003
r_{t-1}	1.4587	1.1091	1.1636	0.8897
r_{t-2}	-1.0072	-0.2113	-0.2395	-0.0827
h^2	0.0398	0.1725	0.1280	0.2396
ΔS_t		0.1267	0.1232	0.1185
ΔS_{t-1}		-0.0599	-0.0685	-0.0052
β_0	0.0000	0.0000	0.0000	0.0000
ε^2_{t-1}	1.0491	1.1730	1.1375	1.2644
$I_{t-1}\varepsilon^2_{t-1}$	-0.9347	-1.0005	-1.0369	-0.8101
$VDAX_{t-1}$	0.0885	0.1278	0.4272	0.0561
$VDAX_{t-2}$	0.5621	0.1933	-0.0951	0.2814
$\Delta S^2_{t-1}D_{t-1}$		-0.0013	-0.0002	-0.0028
$\Delta S^2_{t-1}(1-D_{t-1})$		0.0033	0.0044	0.0013
n	208	208	208	208
p	7	11	11	11
MLE	79.6	112.8	110.3	108.3
AIC	-1276.2	-1400.9	-1391.1	-1389.0
SIC	-1278.8	-1394.5	-1380.1	-1342.3

Table 15: DAX sentiment models with substituted implied volatility parameters

To the contrary, this version actually performs worse than when the *VDAX* is added separately. Again, the parameters are very small, while those for the ARCH terms (lagged errors) have jumped significantly. Obviously, those factors now take over, while little influence is granted to the lagged VDAX values. The model has basically turned itself into an ARCH model. Summarizing, introducing implied volatilities from option markets in the ways proposed here does not seem to add explanatory power to the GARCH models.

6. Final Discussion and Conclusion

Classical theories of financial markets postulate the efficiency of markets with respect to their information processing abilities. Already for decades, the assumptions and implications of the Efficient Markets Hypothesis have been under scrutiny. Early research presented so-called puzzles that were hard to explain with classical reasoning. Two extreme market movements have intensified the need for new explanations: the stock market crash in October 19, 1987, and the millennium bubble. Both were extreme phenomena that could not be sufficiently justified with fundamental changes in risk/return characteristics. Noise Trader theory was founded by researchers as Fischer Black and Andrei Shleifer, among others, to provide a new paradigm in contrast to efficient markets. The basic approach is to allow for irrational investors to persistently influence financial markets. Through their irrationality, noise traders pose a new source of risk that is hard to arbitrage, especially for investors who are concerned with liquidating their funds in the foreseeable future. Limited arbitrage is thus both a prerequisite for and a result of noisy asset prices. De Long, Shleifer, Summers and Waldmann (1990) paved the way for empirical research on the topic in formulating their noise trading model. Subsequently, the influence of investors' mood, sentiment or irrational expectations on prices and returns has been examined. In the process, many studies have been performed that used different approaches and measures of sentiment.

Early researches focused on closed-end fund discounts as indicators of investor sentiment. Individual investors, who were assumed to be noise traders, overproportionately trade in closed-end funds, so their sentiment should affect fluctuations in the observed discounts. In turn, shifts in closed-end fund discounts were used as early sentiment indicators, along with other measures derived from observable market variables. These implicit measures of sentiment showed several things: first of all, trading is cross-sectionally correlated. Accordingly, the sentiment behind the trading activity can be assumed to be systematic as well. Furthermore, especially the small firm effect of excess returns seems to be at least partly explained by exposition to noise trader sentiment. While these findings seem quite robust, other studies have countered their argumentation and methodology. Specifically, the theoretical link of trading statistics and closed-end fund discounts with investor sentiment was questioned. New measures of sentiment were called for that could validate the results. Researchers thus turned to explicit, survey-based measures of sentiment. No theoretical link is to be assumed here: investors simply state what they think. Corresponding to the previously found positive relationship of investor sentiment and future small firm returns, sentiment seems to be negatively related with blue-chip returns. One could follow that while investors might have expectations for the widely followed blue-chip indices, they are not able to influence them significantly through trading. In contrast, small stocks are undervalued due to noise trader risk, which enlarges their expected returns.

After the workings of sentiment and returns had been explored, some researcher ventured into the relation of sentiment and conditional volatility. Again, closed-end fund discounts served as the starting point. Their volatility increases significantly after swings in sentiment, although trading volume does not. This is in line with Noise Trader theory: shifting sentiment poses a risk which drives away rational investors, so they make way to noise traders. Correspondingly, stock market volatility seems to be affected by sentiment just as well. This has been found by means of a methodology that is able to examine returns and volatility simultaneously: Generalized Autoregressive Conditional Heteroskedasticity. When integrated in a GARCH framework, the impact of sentiment becomes obvious: bullish shifts lead to increased expected returns, while bearish shifts cause the opposite. Additionally, when investors become more optimistic, conditional volatility is lower for the following periods. Due to a postulated inverse relationship of conditional volatility and expected returns, the beneficial effect of bullish shifts in sentiment is reinforced. Again, bearish shifts lower expected returns, also through their influence on conditional volatility.

As can be seen, some facts have emerged through previous empirical analysis. Survey-based measures of investor sentiment seem to provide good results that are straightforward to explain. They probably reflect the expectations of investors in the clearest way. Furthermore, studies in the context of Noise Trader theory should take both returns and volatility into account. GARCH is an obvious candidate for an appropriate methodology.

One aspect that is often mentioned but seldom looked into is the differential impact of institutional and individual sentiment. This question is linked to the interpretation of the term “noise trader”. While in the original theory it is used to abstractly distinguish rational from irrational agents, many have equated private investors with noise traders. In contrast to professional investors, they are assumed to be more subjective in their judgements and thus more prone to acting irrationally. However, many studies came to results that counter this – maybe irrational – intuition.

The empirical analysis of this thesis, presented in Chapter 5, has been performed on the basis of these findings. It has been decided to employ survey-based measures of investor sentiment within a GARCH framework. The aspect of differing individual and institutional sentiment has been given further examination.

At first, the data to be used has been outlined and examined by means of descriptive statistics. Two important hints have been discovered: positive and negative changes in sentiment could be analyzed separately, and splitting sentiment into individual and institutional might lead to further insights. Both intuitions have proven useful in the later analysis stage. The analysis phase started off with determining base models for the German DAX and TecDAX, derived from weekly return data. Rather than assuming equivalent specifications, two different models were selected. The DAX model was identified as a GARCH(1,2) in-mean specification including asymmetric effects of past error terms on conditional variances. For the TecDAX, a symmetric GARCH(2,2) in-mean specification seemed most appropriate. The particular model scored the highest selection criteria value, and further inspection of the news impact curve as well as a simple ANOVA analysis did not find obvious traces of asymmetry.

The proposal of Day and Lewis (1995) to include implied volatilities from option markets in GARCH models has been followed. In contrast to their findings, neither adding the VDAX as an exogenous variable nor substituting for the GARCH parameters did improve the previously estimated models.

Next, measures of sentiment have subsequently been added o the respective base models. The findings and interpretation across stock indices were coherent. First and foremost, sentiment has a significant impact on expected returns and conditional volatility. Specifically, contemporaneous changes in sentiment positively affect expected returns. In the context of Noise Trader theory, the "hold more" effect dominates the "price pressure" effect. When investors become more optimistic, they are inclined to buy stocks that are underpriced due to noise trader risk, so that expected returns rise. The fact that, in doing so, they push up prices again does not completely offset this advantageous effect. When investors turn bearish eventually, the aforementioned effects work in unison to diminish expected returns: noise traders sell stocks affected by noise trader risk, so they can't be compensated anymore. The magnitude of shifts in sentiment has a similar impact, due to revisions in conditional volatility. Larger bullish shifts in sentiment lead to additional expected returns, while the opposite is true for bearish shifts. In the logic of Noise Trader theory, noise traders create their own space in that they scare away arbitrageurs by their irrationality. As the latter pull out, they lower prices and leave the returns to noise traders. However this effect works only for noise traders in periods of exaggerated bullishness. The quicker noise traders turn pessimistic, the worse their market timing becomes, so that the "Friedman" effect eats up their returns.

Finally, distinguishing between institutional and individual investor sentiment suggested that individual investors as a group are sometimes falsely accused of equalling noise traders. In theory, assets that are mainly traded by individuals should be affected more by their shifting sentiment. This would implicate that the individual sentiment measure could explain the TecDAX movements better than the blue-chip DAX. The opposite is true. Furthermore, institutional sentiment also adds substantial explanatory power, regardless of the stock index. It does not however explain as much as individual sentiment. These findings point into a certain direction, although admittedly they do not provide a clear-cut answer. Most probably, neither individuals nor institutionals can be labelled “noise traders” per se. Just as researchers from the field of Behavioural Finance found psychological biases that affect people regardless of their profession, it seems only natural that there is a bit of a noise trader in every investor.

Warren Buffet was obviously not that wrong after all.

APPENDIX

A Descriptive statistics for Bull/Bear index

	Bull/Bear index		Change in Bull/Bear	
	DAX	TecDAX	DAX	TecDAX
Count	138	112	137	111
Mean	60.9%	64.7%	-0.04%	0.14%
Median	62.0%	65.7%	-1.1%	0.7%
Std. Deviation	8.7%	9.5%	8.7%	12.4%
Variance	0.0075	0.0090	0.0076	0.0154
Minimum	39.4%	40.7%	-23.9%	-31.5%
Maximum	77.5%	84.3%	21.1%	29.3%

Table 16: Bull/Bear descriptive statistics

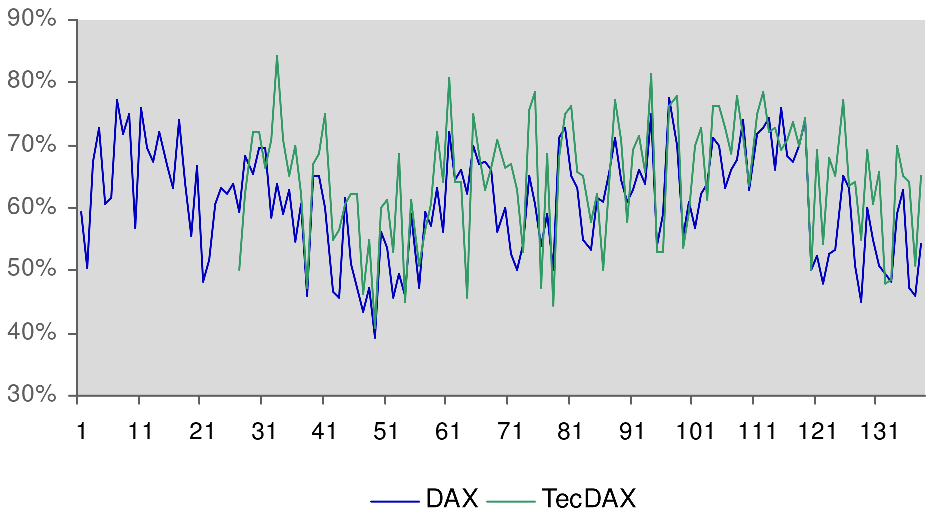

Figure 4: Bull/Bear DAX, TecDAX time series graph

B Bull/Bear estimated GARCH models

	1: Determine number of AR			2: Determine GARCH			3: Add GJR asymmetry			
	1	2	3	(2,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,1)	(2,2)
α_0	-0.0035	0.0068	0.0186	-0.0061	0.0017	0.0021	-0.0022	-0.0012	-0.0069	0.0039
r_{t-1}	-0.2535	-1.4448	-1.9512	-0.9083	-0.2759	-0.4308	0.8728	0.1631	-0.8738	-0.4265
r_{t-2}		-0.6557	-1.8104							
r_{t-3}			-0.6057							
h^2	4.0581	2.2839	0.0402	7.2485	3.6976	3.5396	0.9998	2.2454	6.7132	1.6506
β_0	0.0008	0.0008	0.0005	0.0017	0.0011	0.0010	0.0009	0.0013	0.0013	0.0011
ε^2_{t-1}	0.9027	0.9465	1.1714	0.5983	1.0685	1.3507	1.3773	1.5151	0.8362	1.1261
$I_{t-1}\varepsilon^2_{t-1}$							-0.9341	-0.9800	-0.2575	-0.9038
ε^2_{t-2}				0.6975		0.6709			0.8011	0.5119
$I_{t-2}\varepsilon^2_{t-2}$									-0.2926	-0.0315
h^2_{t-1}	0.2589	0.2230	0.2734	-0.4639	0.3594	0.0004	0.1939	0.0865	-0.2418	0.1123
h^2_{t-2}					-0.2281	-0.0649		-0.1122		-0.1976
n	111	111	111	111	111	111	111	111	111	111
p	4	5	6	5	5	6	5	6	7	8
MLE	38.2	38.8	39.6	39.9	40.0	40.3	39.1	39.1	40.5	41.1
AIC	-673.1	-673.3	-674.6	-688.7	-673.3	-666.8	-674.3	-680.6	-682.6	-695.6
SIC	-662.3	-659.8	-658.3	-675.1	-659.7	-650.6	-660.8	-664.4	-663.6	-673.9

Table 17: Estimated base models in period corresponding to Bull/Bear TecDAX index

	1: Determine number of AR			2: Determine GARCH			3: Add GJR asymmetry			
	1	2	3	(2,1)	(1,2)	(2,2)	(1,2)	(1,1)	(2,1)	(2,2)
α_0	-0.0027	0.0005	0.0004	-0.0003	0.0073	-0.0002	0.0073	-0.0048	0.0026	0.0054
r_{t-1}	-0.7212	0.7298	1.1129	1.4446	0.2539	1.5972	0.2531	0.1011	-0.2419	-0.2519
r_{t-2}		-1.1229	-1.3945	-0.8768	-0.8770	-0.9626	-0.8770	-0.9481	-0.4323	-0.9072
r_{t-3}			0.4192							
h^2	3.6152	0.0816	0.0965	0.6075	-0.8155	0.4037	-0.8153	-0.9517	-1.1896	-1.5801
β_0	0.0001	0.0000	0.0000	0.0001	0.0056	0.0000	0.0033	0.0038	0.0039	0.0006
ε^2_{t-1}	0.4205	0.4336	0.3698	0.3348	0.2122	0.3936	0.1977	0.4429	0.3018	1.5442
$I_{t-1}\varepsilon^2_{t-1}$							-0.1098	-0.1723	-0.3285	-1.5361
ε^2_{t-2}	0.0000	0.0000	0.0000	0.1997		-0.3560			0.1355	1.0667
$I_{t-2}\varepsilon^2_{t-2}$									-0.2476	-1.2494
h^2_{t-1}	0.7587	0.7740	0.8027	0.7005	-0.4302	1.3889	-0.4411	-0.5074	-0.6046	-0.0122
h^2_{t-2}					0.5781	-0.4180	0.5725			0.2224
n	137	137	137	137	137	137	137	137	137	137
p	4	5	6	6	6	7	7	6	8	9
MLE	51.4	52.4	52.4	51.6	39.5	53.3	42.7	42.4	42.8	56.9
AIC	-843.4	-845.6	-845.4	-840.2	-739.8	-850.1	-791.8	-802.8	-812.6	-852.4
SIC	-831.7	-831.0	-827.9	-822.7	-722.3	-829.7	-771.4	-785.3	-789.2	-826.1

Table 18: Estimated base models in period corresponding to Bull/Bear DAX index

	DAX			TecDAX		
	S_t	S_{t-1}	$S_t + S_{t-1}$	S_t	S_{t-1}	$S_t + S_{t-1}$
α_0	0.0012	0.0011	0.0049	0.0061	0.0058	0.0028
r_{t-1}	0.0083	0.2635	0.1908	-0.2577	-0.5531	0.0519
r_{t-2}	-0.4061	-0.6353	-0.6803			
h^2	0.4604	0.8762	0.3175	0.4966	0.6174	0.6934
ΔS_t	0.0629		0.1142	0.0696		0.1002
ΔS_{t-1}		-0.0447	0.0500		0.0161	0.0037
β_0	0.0000	0.0000	0.0000	0.0010	0.0008	0.0007
ε^2_{t-1}	0.2795	0.1695	0.2837	1.0991	1.0340	1.0702
$I_{t-1}\varepsilon^2_{t-1}$				-0.4795	-0.6315	-0.5626
ε^2_{t-2}	-0.2830	-0.1711	-0.2905	0.7237	0.6592	0.7941
$I_{t-2}\varepsilon^2_{t-2}$				-0.2746	-0.1093	-0.5160
h^2_{t-1}	1.6318	1.6879	1.6315	0.0901	0.1390	0.0701
h^2_{t-2}	-0.6336	-0.6909	-0.6300	-0.1499	-0.1583	-0.1187
ΔS^2_{t-1}	-0.0001	-0.0011	-0.0011	0.0181	0.0230	0.0168
n	137	137	137	111	111	111
p	9	9	10	10	10	11
MLE	55.4	54.9	54.8	43.6	40.4	45.2
AIC	-858.2	-858.6	-854.5	-677.8	-671.0	-694.9
SIC	-831.9	-832.3	-825.3	-650.7	-643.9	-665.1

Table 19: Base model augmented with Bull/Bear DAX, TecDAX respectively

	DAX			TecDAX		
	S_t	S_{t-1}	$S_t + S_{t-1}$	S_t	S_{t-1}	$S_t + S_{t-1}$
α_0	0.0001	0.0010	0.0008	-0.0001	-0.0022	0.0005
r_{t-1}	0.7003	0.7003	0.6001	0.8380	0.8518	0.6051
r_{t-2}	-0.1990	-0.1996	-0.1988	0.0000	0.0000	0.0000
h^2	0.8764	0.8764	0.8765	0.4119	0.9204	0.6602
ΔS_t	0.1946		0.1944	0.1347		0.1214
ΔS_{t-1}		-0.0447	0.0499		-0.0470	-0.0158
β_0	0.0000	0.0000	0.0000	0.0006	0.0014	0.0007
ε^2_{t-1}	0.1824	0.1859	0.3987	0.8906	0.8892	0.8753
$I_{t-1}\varepsilon^2_{t-1}$				0.0339	-0.1931	-0.2651
ε^2_{t-2}	-0.1885	-0.1855	-0.3814	0.5950	1.1386	0.7144
$I_{t-2}\varepsilon^2_{t-2}$				-0.6551	-1.0381	-0.6882
h^2_{t-1}	1.6876	1.6870	1.6863	0.0485	0.0773	0.0483
h^2_{t-2}	-0.6884	-0.6902	-0.6893	-0.0956	-0.1329	-0.0913
$\Delta S^2_{t-1}D_{t-1}$	0.0022	-0.0012	0.0163	0.0131	-0.0303	-0.0043
$\Delta S^2_{t-1}(1-D_{t-1})$	-0.0023	0.0006	-0.0226	0.0332	0.0050	0.0082
n	137	137	137	111	111	111
p	10	10	11	11	11	12
MLE	49.2	52.8	31.1	46.3	40.2	45.9
AIC	-843.1	-835.6	-676.7	-703.0	-675.0	-718.0
SIC	-813.9	-806.4	-644.6	-673.2	-645.2	-685.4

Table 20: Base models augmented with split Bull/Bear DAX, TecDAX respectively

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