



# Satellite Orbital Decay Calculations



**Summary:** The decay of a satellite from low earth orbit is of interest to many people. The drag force that such a satellite experiences is due to its interaction with the few air molecules that are present at these altitudes. The density of the atmosphere at LEO heights is controlled by solar X-ray flux and particle precipitation from the magnetosphere and so varies with the current space weather conditions.

This article presents a simple model for atmospheric density as a function of space environmental parameters, and shows how this may be applied to calculate decay rates and orbital lifetimes of satellites in essentially circular orbits below 500 km altitude. A computer program in QBASIC is presented showing how the model may be implemented. Practical use and limitations of the program are discussed, and references are given to guide those interested in further study.

## 1. Introduction

Low Earth orbiting satellites experience orbital decay and have physical lifetimes determined almost entirely by their interaction with the atmosphere. Prediction of such lifetimes or of a re-entry date is of great interest to satellite planners, users, trackers, and frequently to the general public.

The prediction of satellite lifetimes depends upon a knowledge of the initial satellite orbital parameters, the satellite mass to cross-sectional area (in the direction of travel), and a knowledge of the upper atmospheric density and how this responds to space environmental parameters which must also be predicted.

Even with a complete atmospheric model describing variations with time, season, latitude and altitude, complete specification of orbital decay is not possible because of uncertainties in the prediction of satellite attitude (which affects the relevant cross-sectional area), and solar and geomagnetic indices (which substantially modify the atmospheric model).

Even when most of the quantities are known there appears to be an irreducible level below which it is not possible to predict. This level appears to be around 10% of the remaining satellite lifetime, irrespective of what that lifetime is. In other words, the error in predicting the decay of a satellite expected to remain aloft for about 10 years is one year, whereas the demise of a satellite expected to re-enter in 24 hours time is only accurate to about 2 hours! Note that these figures do not apply to a spacecraft such as the Space Shuttle that has a controlled re-entry into the Earth's atmosphere.

An appreciation of the uncertainty is shown by a NORAD prediction in April 1979 for the expected re-entry of the SKYLAB space station between 11 June and 1 July of that year. The actual re-entry occurred on July 11, outside the stated interval, a prediction error from mid-interval of around 15%. In the light of such errors in relatively short term predictions, there is a gross mismatch between the detail employed in the atmospheric models of large sophisticated prediction programs and the accuracy of the forecast space environmental parameters employed as input. In the following section, a very simple atmospheric model is described that appears more closely matched to the accuracy limitations imposed upon a forecast of satellite lifetime by the uncertainties in the other variables.

## 2. The Atmospheric Model

This atmospheric model, and thus the prediction scheme, have been confined to satellites with orbits totally below about 500 km altitude. Such orbits can be regarded as essentially circular, with the use of the semimajor axis in place of the orbital radius. The atmospheric density  $\rho$  is specified by a simple exponential with variable scale height  $H$ . For a fixed exospheric temperature  $T$ ,  $H$  is made to vary with altitude  $h$  through the use of an effective atmospheric molecular mass  $m$ . This  $m$  includes both the actual variation in molecular mass with height and a compensation term for the variation in temperature over the considered range from 180 to 500 km. The variation in density due to the space environment is introduced through  $T$  which is specified as a function of the solar radio flux  $F_{10.7}$  and the geomagnetic index  $A_p$ .

A brief discussion on the cause of atmospheric density variations is in order. The two terms which are used in the model describe the effects of different agencies, both of which originate from the Sun. The solar X-ray output incident upon the Earth is generally absorbed at the base of the thermosphere (around 120 km) and this gives rise to a direct heating effect which propagates itself upward from this level. The solar 10 cm radio flux is used as a surrogate for the total solar X-ray flux which produces this effect. This flux can vary from a low of about 65 to over 300 Solar Flux Units ( $1 \text{ SFU} = 10^{-22} \text{ W/m}^2/\text{Hz}$ ). The other agency is the precipitation of particles (mainly electrons and protons) from the magnetosphere down into the lower thermosphere. The energy dumped by this precipitation again acts to heat the atmosphere, which subsequently changes the atmospheric density. Most of these particles originate from the Sun. They are expelled in Coronal Mass Ejections, travel through the interplanetary medium, and eventually arrive at the Earth. Precipitation of such particles is well correlated with large variations in the geomagnetic field as measured at ground level, and quantified by a number of geomagnetic indices, including the

planetary A index used here. This index, computed every 24 hours, hovers just above zero in quiet times, but may rise to above 400 (no units) at times of major geomagnetic storms.

The set of defining equations for the model are given by:

$$\begin{aligned}
 T &= 900 + 2.5 ( F10.7 - 70 ) + 1.5 A_p && \text{(Kelvin)} \\
 m &= 27 - 0.012 ( h - 200 ) && 180 < h \text{ (km)} < 500 \\
 H &= T / m && \text{(km)} \\
 \rho &= 6 \times 10^{-10} \exp ( - ( h - 175 ) / H ) && \text{( kg m}^{-3} \text{ )}
 \end{aligned}$$

All constants were empirically derived to give an appropriate fit to the standard models. It should be noted that the only really valid output of this model is the density. The intermediate variables used in deriving this density in general do not correspond to true atmospheric values at any height within the considered range. The temperature may be regarded as the mean asymptotic value for the exosphere at large altitudes. The molecular mass might be regarded as an integrated mean value from the base of the thermosphere up to the specified height.

The solar 10 cm radio flux is generally used in an averaged form, and the average preferred is that of the last 90 days prior to the specified date. Sometimes a small correction is made to weight the current flux more strongly.

### 3. Satellite Drag

When a spacecraft travels through an atmosphere it experiences a drag force in a direction opposite to the direction of its motion. This drag force is given by the expression:

$$D = 1/2 \rho v^2 A C_d$$

where D is the drag force,  $\rho$  is the atmospheric density, v is the speed of the satellite, A is its cross-sectional area perpendicular to the direction of motion, and  $C_d$  is the drag coefficient. At the altitudes at which satellites orbit,  $C_d$  is generally assumed to be equal to two, although experiments have shown that this can vary widely. Because it is usually difficult to separate out independent variations in the cross-sectional area from the variations in the drag coefficient, we shall henceforth use an effective cross-sectional area  $A_e = A C_d$  for the rest of the model.

We can use this drag force in Newton's second law together with energy considerations of a circular orbit to derive an expression for the change in the orbital radius and period of the satellite with time.

For a circular orbit we have the following relation between period P and semimajor axis a :

$$P^2 G M_e = 4\pi^2 a^3$$

where G is the Universal Gravitational Constant and  $M_e$  is the mass of the Earth

The reduction in the period due to atmospheric drag is given by:

$$dP/dt = -3\pi a \rho (A_e/m)$$

The last two equations, together with the equations modelling the atmospheric density, can be iterated from the starting satellite altitude and time. In other words the satellite is flown around its orbit using appropriate past or forecast values for the space environment variables (F10.7 and  $A_p$ ). Re-entry is assumed to occur when the satellite has descended to an altitude of 180 km. In all but the heaviest satellites (those with a mass to area ratio well in excess of 100 kilogram per square metre), the actual lifetime from an altitude of less than 180 km is only a few hours.

### 4. A Simple Program

The following source code, in QuickBasic, is a simple implementation of the above model to illustrate the steps involved. This implementation does not allow for variations in the space environment, and as a result is only suitable for short time periods, or during longer times when solar and geomagnetic activity do not show significant variation. This generally only occurs around the years of solar minimum.

```

' SATELLITE ORBITAL DECAY
'get required input parameters from keyboard
INPUT "Satellite name "; N$
INPUT "Satellite mass (kg) "; M
INPUT "Satellite area (m^2) "; A
INPUT "Starting height (km) "; H
INPUT "Solar Radio Flux (SFU) "; F10
INPUT "Geomagnetic A index "; Ap
'print information to printer
LPRINT "SATELLITE ORBITAL DECAY - Model date/time "; DATE$; " @ "; TIME$
LPRINT : LPRINT "Satellite - "; N$
LPRINT : LPRINT USING "   Mass = #####.# kg"; M
LPRINT USING "   Area = #####.# m^2"; A
LPRINT USING "   Initial height = ###.# km"; H
LPRINT USING "   F10.7 = ###   Ap = ###"; F10; Ap: LPRINT
'print column headings to printer
LPRINT " TIME   HEIGHT   PERIOD   MEAN MOTION   DECAY"
LPRINT "(days) (km)     (mins)   (rev/day)   (rev/day^2)"
  f$ = "#####.#   #####.#   #####.#   ##.#####   ##.###^^^" 'print format
'define some values
Re = 6378000!: Me = 5.98E+24   'Earth radius and mass (all SI units)
G = 6.67E-11                 'Universal constant of gravitation
pi = 3.1416: T = 0: dT = .1   'time & time increment are in days
D9 = dT * 3600 * 24          'put time increment into seconds
H1 = 10: H2 = H              'H2=print height, H1=print height increment
R = Re + H * 1000           'R is orbital radius in metres
P = 2 * pi * SQR(R * R * R / Me / G) 'P is period in seconds
'now iterate satellite orbit with time
DO
  SH = (900 + 2.5 * (F10 - 70) + 1.5 * Ap) / (27 - .012 * (H - 200))
  DN = 6E-10 * EXP(-(H - 175) / SH)   'atmospheric density
  dP = 3 * pi * A / M * R * DN * D9   'decrement in orbital period
  IF H <= H2 THEN                     'test for print
    Pm = P / 60: MM = 1440 / Pm: nMM = 1440 / ((P - dP)) / 60 'print units
    Decay = dP / dT / P * MM          'rev/day/day
    LPRINT USING f$; T; H; P / 60; MM; Decay 'do print
    H2 = H2 - H1                     'decrement print height
  END IF                               'end of print routine
  IF H < 180 THEN EXIT DO             'are we finished? ie re-entry?
  P = P - dP: T = T + dT              'compute new values
  R = (G * Me * P * P / 4 / pi / pi) ^ .33333 'new orbital radius
  H = (R - Re) / 1000                 'new altitude (semimajor axis)
LOOP                                   'keep flying satellite
LPRINT
'now print estimated lifetime of satellite
LPRINT USING "Re-entry after #### days ( ##.## years)"; T; T / 365
LPRINT CHR$(12);                      'send form feed to eject paper from printer
END

```

**A typical run of the above program.**

SATELLITE ORBITAL DECAY - Model date/time 07-01-1999 @ 15:32:55

Satellite - Test-1

```

Mass = 100.0 kg
Area = 1.0 m^2
Initial height = 300.0 km
F10.7 = 70   Ap = 0

```

TIME	HEIGHT	PERIOD	MEAN MOTION	DECAY
(days)	(km)	(mins)	(rev/day)	(rev/day^2)
0.0	300.0	90.5	15.9139	2.66E-03
10.5	289.9	90.3	15.9463	3.50E-03

19.5	279.9	90.1	15.9823	4.62E-03
26.3	269.9	89.9	16.0182	6.11E-03
31.5	259.9	89.7	16.0546	8.11E-03
35.4	249.9	89.5	16.0908	1.08E-02
38.4	239.8	89.3	16.1279	1.44E-02
40.6	229.8	89.1	16.1642	1.93E-02
42.3	219.6	88.9	16.2018	2.60E-02
43.6	209.1	88.7	16.2406	3.55E-02
44.5	199.3	88.5	16.2768	4.75E-02
45.2	189.2	88.3	16.3146	6.45E-02
45.7	179.5	88.1	16.3507	8.65E-02

Re-entry after 46 days ( 0.13 years)

The columns on the printout should be relatively evident. The first column (TIME) is the elapsed time from when the satellite was at the initially specified height. The HEIGHT is the current height at that elapsed time. The PERIOD is the orbital period of the satellite in minutes. The last two columns are the mean motion in revolutions per day and the decay rate in the mean motion (units are revolutions per day per day). These last two parameters are the standard parameters that appear in the Two Line Element sets that NORAD and others issue for various satellites and that may be found on several web sites. Mean motion is simply calculated by dividing the period (in minutes) into 1440, the number of minutes in a day. Decay is the change in mean motion per day.

There are several parameters in the program that can be easily changed. You may wish to obtain a printout every kilometre, instead of every 10 kilometres. This can be done by setting the value of H1 to 1. You may wish to try a different time increment for the iteration process. At the moment it (dT) is set to one tenth of a day. Experimenting with this will give you a feel for the accuracy inside the iteration loop. The most accurate results are obtained with a small time increment. However, too small a value will make the program slow to execute. The most efficient increment would vary with altitude. For a satellite at 500 km a time increment of a few days would be adequate, but at 200 km, the increment should be reduced to an hour or less. The current value is a compromise to speed and accuracy.

The above simple program should only be viewed as a starting point or a source of ideas to develop your own version which allows for a changing space environment.

## 5. Satellite Lifetime Predictions

The basic scheme to the use of the above program is to find appropriate values for the space environmental parameters (solar 10 cm radio flux and geomagnetic activity index) and estimate or calculate a value for the satellite mass to area ratio. In the absence of any information on the latter quantity it is usual to employ a value of 100 kg per square metre. This is an average value for many satellites. Extreme value do not vary a great deal from this mean, with a low of 50 and a high of 200 covering 90% of relevant spacecraft. Peculiar orbiting objects may however, have values outside this range.

Having substituted the above values in the program and run it, the next step is to compare the output with the actual behaviour of the satellite in which you are interested. This will involve obtaining a list of actual satellite periods, altitudes (semi-major axes) or mean motions for a short period (maybe a few weeks if the satellite is near 500 km altitude, or a few days if the altitude is 300 km). If the program underestimates the actual decay, then you must decrease your mass to area ratio. If the program overestimates the actual decay you must increase the ratio. Eventually you should arrive at a printout which closely approximates the actual behaviour of the satellite. The future predictions of the program, together with the lifetime estimate will then be (assuming the parameters remain constant) the best estimate possible with this model.

Note: When comparing the program printout with mean motion values obtained from two line elements (TLEs), do not worry too much about comparing the decay values from the TLE, particularly if the satellite is a new one. There can initially be quite a discrepancy in the decay values (rev/day/day) as listed in the TLEs.

## 6. Practicalities

There are of course practicalities that need to be considered in undertaking the steps in the previous section. One obvious question is where to obtain values of the space environmental parameters. Other sources are the Space Environment Center in Boulder, Colorado, USA, and the World Data Center-A, also in Boulder. This latter centre specialises in distributing archival space data.

The other problem with the simple program presented here is that it does not allow for the space environment to vary. One solution is to write your own program using the simple model as a guide. Another solution is to use the interactive program available on this Web site, which also gives you automatic access to our predictions of what the space environmental parameters will be in the future.

Another difficulty sometimes encountered in matching model outputs to the real world is that predictions may suddenly cease to be valid (even after working OK for some time). One possibility is that the attitude of the satellite has undergone a substantial change. This can cause a change to the effective cross-sectional area. One reason this might occur is due to a ground control command. It might also be possible that the satellite has run out of energy to keep it stabilised (through use of momentum reaction wheels and/or thrusters) in a desired attitude. This can result in a change of attitude, and even a tumbling. As long as the tumbling period is short compared to its orbital period, it is still possible that the average cross-sectional area it presents to the atmosphere will settle to a new constant value. The biggest changes in cross-sectional area due to attitude changes are likely to occur in the last phases of the satellite's life, just before it is due to re-enter and burn up in the lower atmosphere. Unfortunately, this is normally the time when one would like to make the best prediction!

So far we have only treated orbits that are circular, or very close thereto. Satellites with very elliptical orbits travel to altitudes right outside the sphere of the current atmospheric model, and must be treated quite differently. They are also subject to other perturbations (variations of their orbit) due to the combined gravitational attractions of the Sun and the Moon. However, orbits with only a slight eccentricity can be accommodated by the use of an "effective" height in the model. This effective height is given by the formula:

$$h_e = q + 900 e^{0.6}$$

where  $q$  is the perigee (lowest) height of the orbit (in km) and  $e$  is the orbital eccentricity.

For example, the lifetime of a satellite in an elliptical orbit with a perigee height of 400 km and an eccentricity of 0.01 is the same as the lifetime of a satellite in a circular orbit of height:

$$400 + 900 (0.01)^{0.6}$$

Note that this formula is only a rough approximation with reasonably stringent applicability constraints. It should not be used for orbits with eccentricities in excess of 0.1, and it is really only applicable for conditions of a constant space environment. Thus it might be used for orbits with lifetimes in excess of 10 years (for which we can use an average solar cycle activity), or for orbits that are expected to decay within a few months over which solar activity is expected to be relatively constant.

It is probably fortunate that most satellites nearing the end of their lives will have orbits with very low eccentricities (ie nearly circular). The reason for this is that atmospheric drag acts to circularise orbits. The apogee height is decreased whilst the perigee height is little affected until the orbit becomes close to circular.

## 7. References

- Air Force Geophysics Laboratory, "Handbook of Geophysics and the Space Environment", (1985)  
[This very large blue book is an absolute essential reference for all kinds of information about the space environment - written mainly by scientists responsible for advising the USAF on how this environment affects technological systems that have to operate within and through near space. A very practical reference for information, models and data.]
- Gatland K, "Space Technology", Salamander Books (London, 1981)  
[A quarto size popular level book on space vehicles & space exploration with many tables of data and concluding with a space diary.]
- Hargreaves JK, "The Upper Atmosphere and Solar-Terrestrial Relations", Van Nostrand Reinhold (New York, 1979)  
[A good concise exposition of what the title says. Written at an intermediate level. Contains a short section on satellite drag.]

- King-Hele D, "Observing Earth Satellites", Van-Nostrand Reinhold (New York, 1983)  
[This very readable book at a popular level discusses the observation of satellites from the ground and the various parameters (including atmospheric drag) that affect the orbit.]
- King-Hele D, "Satellite Orbits in an Atmosphere - Theory and Applications", Blackie, Glasgow (1987)
- [This very detailed mathematical text is the "bible" for anyone seriously interested in the affect of the atmosphere on satellite orbits. Analytical expressions are derived with approximations where appropriate for the changes in the orbital elements.]
- Newton I, "Principia"  
[This classic text is not only the basis of our everyday fundamental physical dynamics but it also appears to be the first text to discuss the effect of atmospheric drag on Earth satellite orbits.]
- Planetary and Space Science, Journal published by Pergamon Press.  
[Past issues have contained many articles on detailed aspects of spacecraft interaction with the atmosphere.]
- Tascione T F, "Introduction to the Space Environment", Krieger (Florida, 1994)  
[A good introduction to the space environment by one who has spent many years working in the USAF directly on these issues. A short section on spacecraft operations which includes satellite drag.]
- Tobiska W K, R D Culp & C A Barth, "Predicted Solar Cycle Twenty-Two 10.7 cm Flux and Satellite Orbit Decay", Journal of the Astronautical Sciences, pp419-433, vol 35 (#4), October -December 1987  
[Discusses a simple model for circular orbits. Their atmospheric model is rather more detailed than that used here.]

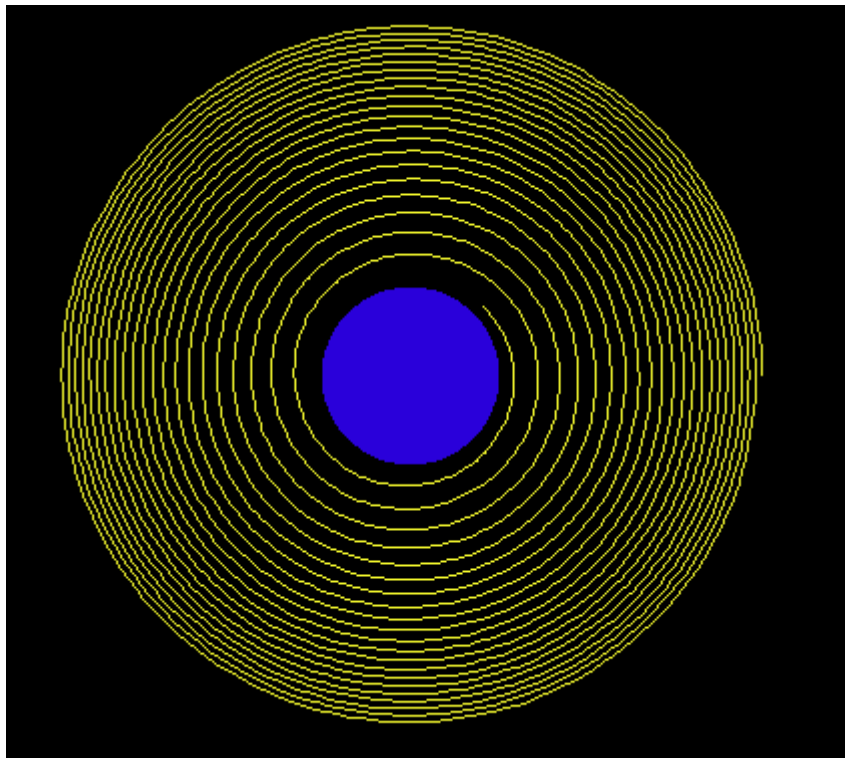


Figure 1:

Diagrammatic view of how a satellite decays in low Earth orbit (LEO). In actual fact many more orbits occur than shown here. Also the radial scale has been drawn very much enlarged to show how the spiral opens out toward the end of the satellite's life, when it undergoes a fiery re-entry into the Earth's lower atmosphere.

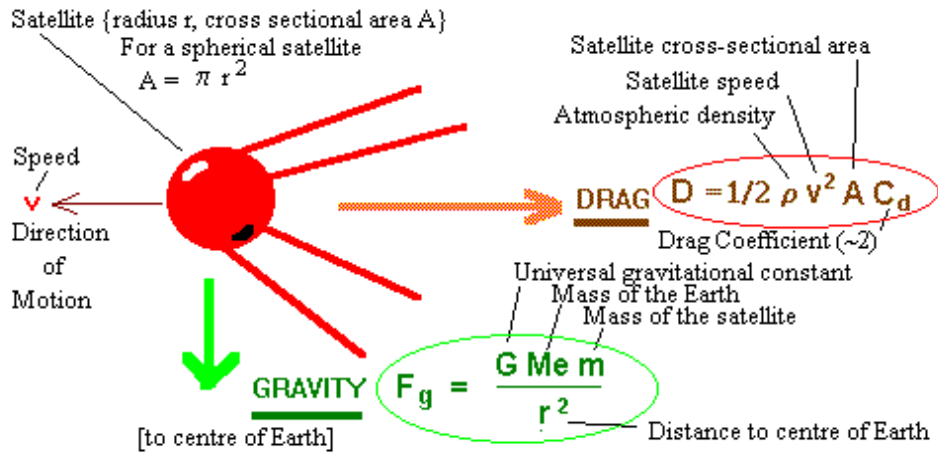


Figure 2:

Forces acting on a satellite in a low circular orbit.

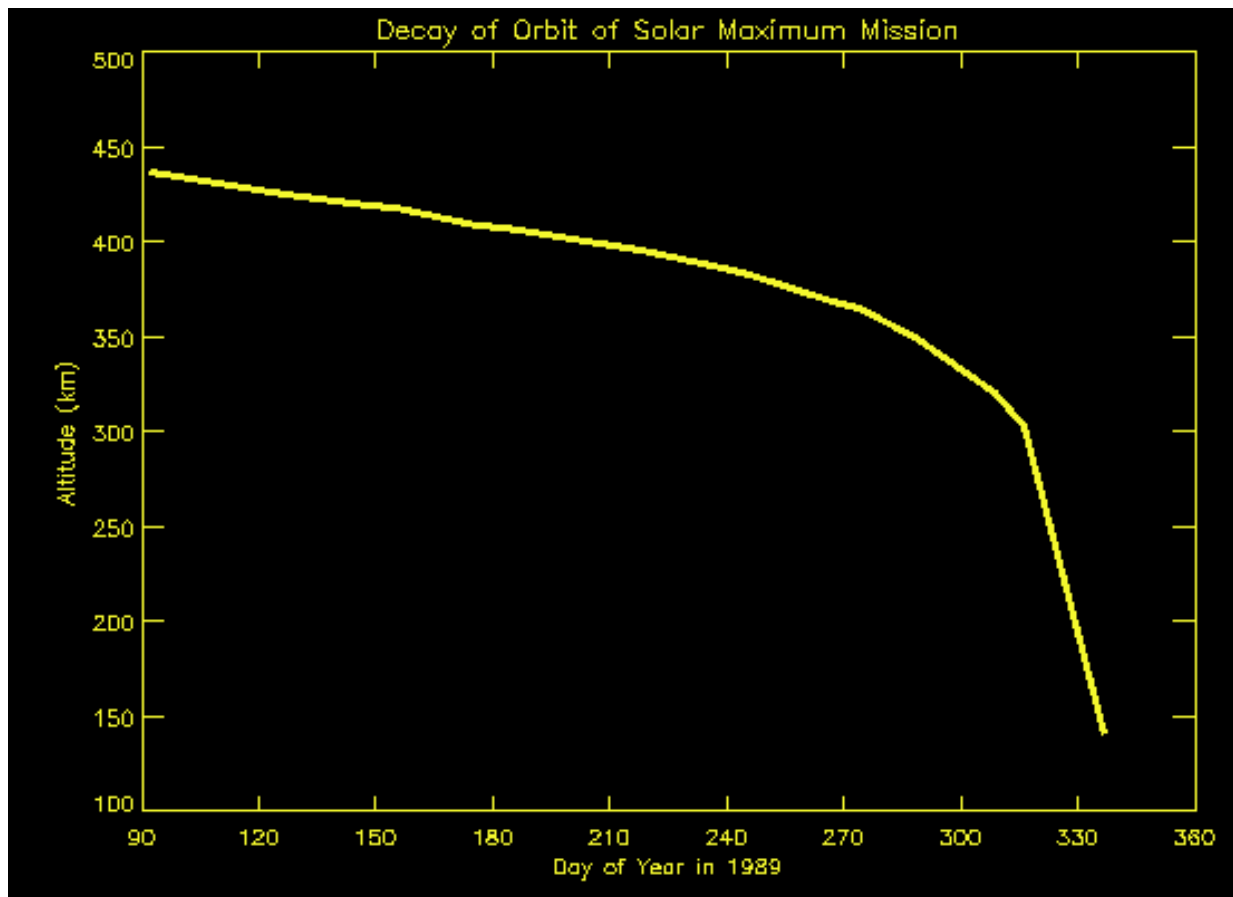


Figure 3:

Actual decay curve for the Solar Maximum Mission satellite which re-entered the Earth's atmosphere at the beginning of December 1989. The satellite was the first spacecraft to be serviced in orbit by a crew from the Space Shuttle. Notice how the satellite decays slowly at higher altitudes, then very rapidly towards the end of its life.



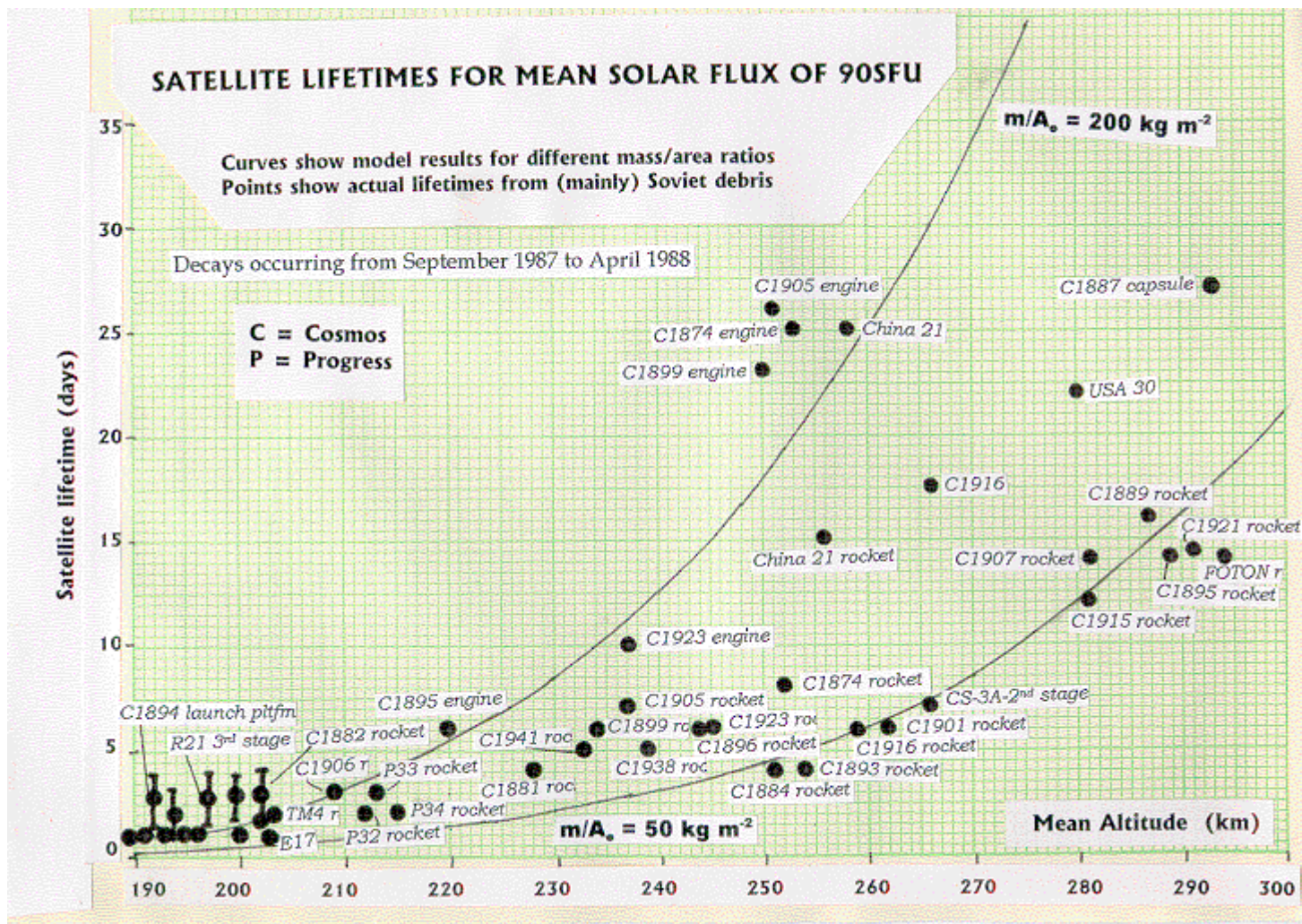


Figure 4:

Lifetimes for various pieces of (mostly Russian) space, as a function of their initial altitude. These decays occurred over a few months when the geomagnetic field was relatively quiet and the solar radio flux had a mean value of 90 SFU. The model curves for  $m/A_e$  ratios of 50 and 200 are also drawn on the graph. Notice that the rocket engines cluster around the  $m/A_e=50$  curve. These objects, devoid of fuel, are relatively "light". The engines on the other hand, are much "heavier".

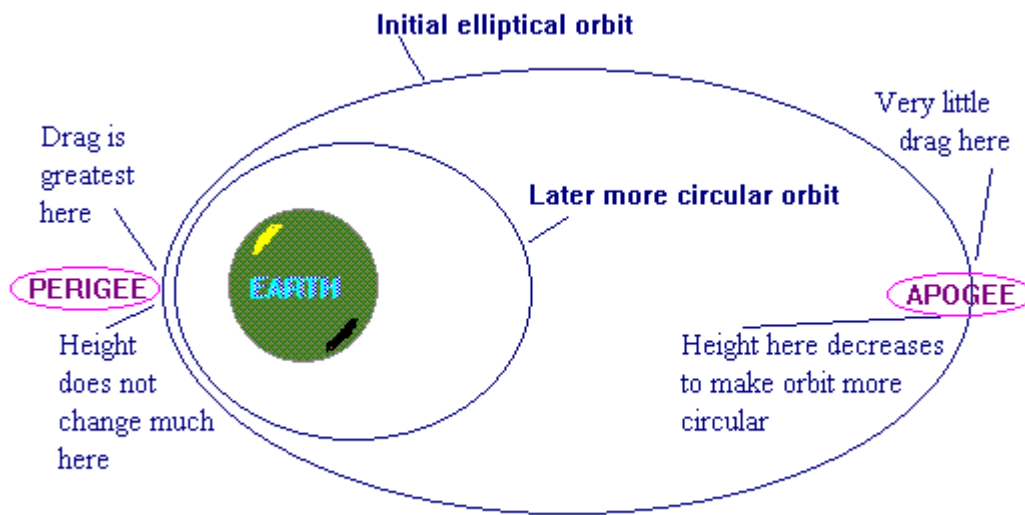


Figure 5:

How an elliptical satellite orbit changes with time. Note that the drag at perigee causes a change in apogee height.

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