Carnegie Observatories Astrophysics Series, Vol. 4:
Origin and Evolution of the Elements, 2004
ed. A. McWilliam and M. Rauch (Pasadena: Carnegie Observatories,
http://www.ociw.edu/ociw/symposia/series/symposium4/proceedings.html)

Fractal Dust in Molecular Clouds

S. DATTA
Department of Applied Mathematics, University of Calcutta,
92 A.P.C. Road, Calcutta 700 009. India

Abstract
The author studies the role of dust in the formation of structure of molecular clouds. Dust has been known to be a catalyst in the process of molecule formation in cold clouds and hence a fractal base structure of dust would mean fractal chains of hydrogen and other molecules. A process of runaway coagulation is highly probable and most probably leads to the formation of the giant molecular clouds (GMCs).

1.1 Introduction

Molecular clouds constitute about 10 to 15 percent of the total mass of our Galaxy (Ferrier 2001) and are self-gravitating, magnetized, turbulent fluids with a hierarchy of structure (Larson 1981; Elmegreen 1999), ranging from Giant Molecular Clouds (GMCs) to cores (Prasad 1995). GMCs are vast complexes with sizes of about 50 pc and from masses about 10^4 to 10^6 $M_\odot$ (Pudritz 2002). The next class consists of normal clouds ranging from 30 pc to a few pc in size and 25 $M_\odot$ to 10^4 $M_\odot$ in mass. Below these are the clumps within clouds, having sizes ranging from 0.4 to a few pc and masses from a few to 25 $M_\odot$. The smallest regions, called the core regions, have sizes ranging from 0.4 pc and less and masses less than 1 $M_\odot$ (Myers 1995). These properties indicate that scale-free or fractal structure (Larson 1981;Prasad 1995; Sitnik 1995; Elmegreen and Falgarone 1996; Hetem & Lepine 1993; Datta 2001b,d) is the base for growth and subsequent evolution.

The physical processes giving rise to a fractal structure are postulated to be a diffusion limited aggregation (DLA) process (Vailati & Giglio 1997), giving rise to single aggregates (Witten & Sander 1981) and then multiple clusters (Witten & Meakin 1983; Meakin 1983; Botet et. al. 1989); turbulence (Elmegreen 1999; Datta 2001) and vorticity (McWilliams 1984; She, Jackson & Orzag 1990) and finally, gravity (Sigalotti & Klapp 1997). In regions of hydrogen density between 10^5 and $10^8/\ cm^3$, temperatures less than or equal to 20K, optical depth greater than 10, the main force driving the grains is turbulence (Ossenkopf 1993) and that above is Brownian motion. The gas dynamics is determined by hydrogen molecules and is not influenced by star formation processes in the neighbourhood. The magnetic field is virial and velocity dispersions are slightly above thermal Jeans limit. Examples where of places where such conditions are found are TMC 1, L134A, L134N, cores in ρ Oph A, L1998 and L1642 among others (Ossenkopf 1993).

The fact that structure formation in molecular clouds is independant of its star-forming properties was first shown by Larson (1981). Larson's laws are given by

$$v = 1.1L^{0.38}, (0.1 \leq L \leq 100) \tag{1.1}$$

$$v = 0.42M^{0.2} \tag{1.2}$$

$$n < H_2 >= 3400L^{-1.1} \tag{1.3}$$

$$1.1L^{0.38} = 0.42M^{0.2} \tag{1.4}$$

where L(parsecs)is the region size given by the maximum projected dimension in the region in which the molecule was observed; v_{rms} (km/s) is the total 3 dimensional rms velocity of all internal motions in each region, calculated by adding the contributions due to large-scale motions across the region, M (in $M_{\odot}$) is the total mass of each region; $< n(H_2) >$ (cm^{-3}) is the average density found in a region of size L. These laws relate mass, size, velocity dispersion and column density in molecular clouds. Further, these relations arise from the two-thirds law of turbulence (Datta 2001), and additionally , particle motion is postulated to be governed by a dynamical non-linear, iterative equation (Datta 2001; 2003) given by

$$v_{t+1} - v_t = {v_t}^2 - v_t + c \tag{1.5}$$

The above equation is analogous to the Navier- Stokes equation in the incompressible case (Datta 2001).

1.2 Mathematical Background

Fractals are self-similar objects with fine structure (B.Mandelbrot 1982; K. Falconer 1997; H.O.Peitgen et. al. 1992). A quantitative measure of a fractal object is it's dimension, the principal definition of which is that given by Kolmogorov (or Minkowski). However, for a practical method, box (or grid) dimension is used, as both definitions are equivalent (Falconer 1997). The box-counting dimension is defined as follows : if X is a non-empty bounded subset of R^n and N_ϵ (X) is the smallest number of sets of diameter at most ϵ which can cover X, then the box-dimension is defined as

$$dimX = \limsup_{\epsilon \to 0} \frac{logN_\epsilon(X)}{log\frac{1}{\epsilon}} \tag{1.6}$$

In the above definition of dimension, the fundamental assumption is that for each ϵ , we measure a set in such a manner that irregularities of size less than ϵ will be ignored. In this paper, the fractal object used to describe molecular cloud geometry is that of the Julia set given by

$$f(z) = z^2 + c \tag{1.7}$$

where z and c are complex quantities and c = - 0.745429 + 0.113008i (Datta 2003). In order to model molecular cloud shapes, we need to consider the gas as a fluid and so we use the equations of fluid motion below (Paterson 1992):

$$\frac{\delta \rho}{\delta t} + \nabla \cdot \vec{v} = 0 \tag{1.8}$$

and in the incompressible case,

$$\nabla \cdot \vec{v} = 0 \tag{1.9}$$

and the Navier-Stokes equation

$$\frac{\delta\vec{v}}{\delta t}+\vec{v}\cdot\nabla\vec{v}=F-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{v} \tag{1.10}$$

where ρ is the density of the fluid, $\vec{v}$ is the fluid velocity at a point inside it at any particular time t, F is the total surface forces on the fluid , p is the fluid pressure at that point, ν is the kinematic viscosity of the fluid. Equation (8) is the equation of continuity, eqn. (9) is that in the incompressible case, and eqn. (10) is the Navier-Stokes eqn. in the incompressible case.

In order to model coagulation processes in molecular clouds, let us consider two clusters, with i and j particles, respectively. Let the i and j cluster collide to form an i+j cluster at the rate $K(i,j) \geq 0$ in a given volume V. The eqn. governing the coagulation process is the Smoluchowski coagulation eqn. given by

$$\frac{d}{\mathrm{d}t}C(j)=\frac{1}{2}\sum_{k=1}^{j-1}K(j-k,k)C(j-k)C(k)-\sum_{k=1}^{\infty}K(j,k)C(j)C(k) \tag{1.11}$$

where j = 1, 2,3 ... and $C(j) \geq 0$ is the expected number of clusters containing particles (Jeon 1999).As no particle is created or destroyed, the total density of particles is conserved. However, it is found to decrease after a finite time if it forms a gel or a cluster of infinite size. Gelation occurs iff

$$K(i,j) \geq \sqrt{ij} \tag{1.12}$$

Additionally, instantaneous gelation occurs if there exists an α such that

$$i^\alpha + j^\alpha \leq K(i,j) \leq (ij)^{\alpha+\epsilon} \tag{1.13}$$

where $\epsilon > 0$ and $\alpha > 2$. A discrete counterpart of eqn. (11) is the Marcus- Lushnikov process (MLP). For the discrete case, gelation occurs iff

$$\lim_{i+j\to 0}\frac{K(i,j)}{ij}=0 \tag{1.14}$$

Forms of K(i,j) where instantaneous gelation occurs are

$$K(i,j)=\frac{2ij^\alpha}{i+j^\alpha-i^\alpha-j^\alpha} \tag{1.15}$$

$$K(i,j)=(ij)^2 \tag{1.16}$$

$$K(i,j) \geq ij\psi(i,j) \tag{1.17}$$

where

$$\psi(j)=\psi(1,j)$$

and

$$\Sigma\frac{1}{j\psi(j)}<\infty \tag{1.18}$$

for all i and j (Jeon 1999).

1.3 Structure Formation

A scale-free or fractal structure of molecular clouds implied by eqn. (1) to eqn. (4) and the fact that the hydrogen molecules forms on dust surfaces (Roser. et. al. 2002) mean that the seed or base must also be fractal. This is supported by extinction calculations of regular fractal graphite clusters of dimension 1.77 and no. of particles less or equal to 50 (Andersen et. al. 2002); the Cosmic Dust Aggregation Experiment (CODAG) by J. Blum et. al. (2002) to study the coagulation of micron-sized dust (figs.3a to c) particles and a study of dust morphology in the central kiloparsec of 20 normal and active galaxies (D.Elmegreen et. al. 2002). A fractal analysis of the Horsehead nebula (B 33) observed from the 2.3m Vainu Bappu Telescope (VBT), Kavalur, India (figure 1) using an automated software ,Benoit 1.3, procured from Trusoft International, St. Petersburg, USA (Seffens 1999) showed that the morphology of B 33 matched that of the Julia set given by eqn (7) (Datta 2003).

As the rate of growth of particles is seen to be enormous (of the order of 10^9 microns/s), we can assume the aggregation process to be instantaneous. Since dust particles are discrete, the process to be used is the ML method. Here, the author takes the case of gelation given by condition (17). The case of $\psi(1, j)$ can be taken to be the scenario where the cluster of j particles (j = 1,2,3,....) attaches to the 'seed' (i = 1). The series given by eqn. (18) has to be convergent and this implies that the structure has a definite cut off in size. The most interesting case is where the number of active sites on a cluster increases faster than it's linear site, i.e. when

$$\lim_{j\to\infty} \frac{j}{K(j)} = 0 \tag{1.19}$$

(Jeon 1999). This results in the interaction of sites in a j cluster is of the order of j^2 or higher. The function given by

$$K(j) = 1 - j^{4/3} \tag{1.20}$$

fits a model with particles having a Maxwellian velocity distribution (van Dongen 1987).

1.4 Discussion

Molecular clouds are the birth place of stars and their thermal and chemical properties determines their position, mass and luminosity and hence the large-scale dynamics of the Galaxy. Attempts made by J.Jeans (1902) and others following him based on the assumption of spherical symmetry, failed to explain observation of filaments, bridges and other non-spherical structures. Such complex structures needed a suitable geometry and that was provided by the geometry of fractals (Mandelbrot 1982; Elmegreen 1999; Datta 2001d). Additionally, studies conducted in the laboratory with both organic and inorganic substances (Matshishita 1989; Maloy et. al. 1985; Botet et. al. 1989; Harrison 1995; Kaiser & Roessler 1997 and others), the building blocks of molecular clouds, showed that growth patterns formed were fractal. Space-based studies (Blum et. al. 2002) have only gone on to confirm terrestrial laboratory results.

Formation of clusters and chains took place at a very fast rate (Maloy et. al. 1985; Blum 2002) and so could be taken to be almost instantaneous. This would be necessary to explain a rate of the order of 10^9 microns/s for growth from a dust particle to GMCs of 50 pc in about 10^7 years. Instantaneous gelation processes for grains follow the ML method and this can result in very dense cloud for the choice of K(i,j) given by eqn. (20).

References

Andresen, A.C.; Sotelo,J.A.; Pustovit, V.N. & Nikalasson, G.A. - 2002 A & A; 386; 296

Blum, J,; Wurm, G.; Poppe,T.; Kempf, S. & Kozasa, T. - 2002 AdSpR; 29; 479 Botet,R.; et. al. - 1989; Fractals Physical Origin & Properties ; ed. L.Pietronero ;Plenum Press; N.Y.; 259

Datta,S. - 2001b ; Bull. of the Cal. Math. Soc.; 93; 451

Datta,S. - 2001d;Automated Data Analysis in Astronomy ;eds. R. Gupta ,H.P.Singh & C. Bailer-Jones; Narosa;New Delhi

Datta,S. - 2003;A & A ; 401; 193

Elmegreen, B.G. - 1999 Ap.J. ; 527; 266

Elmegreen, B.G. & Falgarone, E. - 1996 Ap.J.; 471; 816

Elmegreen, D.M.; Elmegreen, B.G. & Eberwein, K.S. - 2001 AAS; 198; 8.12

Falconer, K. - 1997 ; Fractal Geometry ; Wiley & Sons; Chichester.

Ferriere, K.M. - 2001 ; Rev. of Mod. Physics , 73, 1031

Hetem,A. & Lepine, J.R.D. - 1993 A & A; 270; 451

Jeans, J.H. - 1902 Phil. Trans. A; 312; 1

Jeon, I. - 1999; J. of Statistical Physics; 96; 1049

Jopkii, J.R. - 1999;Interstellar Turbulence ;ed. J. Franco & A. Carraminana; CUP

Kaiser, R.I. & Roessler, K. - 1997 Ap.J. ; 475; 144

Kroupa, P.; - 2002;Science; 295; 82

Larson, R.B. - 1981 ; MNRAS 194, 809

Maloy, A.; et. al. - 1985 ; Phy. Rev. Lett.; 85,2688

Mandelbrot, B.B. - 1990 ; Proceedings of the Royal Society of London, ed. M. Fleishman et. al.

Mandelbrot, B.B. -1982; Fractal Geometry of Nature, Freeman; N.Y.

Matshushita, M. - 1989; The Fractal Approach to Heterogeneous Chemicals; ed. D. Anirved, Wiley, pg. 161

McWilliams, J.C. - 1984 J. of Fluid Mech.; 146; 21

Meakin, P. - 1983 Phy. Rev. Lett.; 51;1119

Myers, P.C. - 1995;Fragmentation of Molecular Clouds and Star Formation ; ed. E. Falgarone, F. Boulanger & G. Douvert; Kluwer Academic , Dordecht, pg.221

Ossenkopf, V. - 1993 A & A; 280; 617

Ott, E. - 1993; Chaos in Dynamical Systems ; CUP

Paterson, A.R. - 1992; A First Course in Fluid Dynamics; CUP.

Peitgen, H.O. ; Jurgens, H. & Saupe, D. - 1992; Fractals in the Classroom ; Springer-Verlag.

Prasad, S.S. - 1995; Fragmentation of Molecular Clouds and Star Formation ; ed. E. Falgarone, F. Boulanger & G. Douvert; Kluwer Academic , Dordecht, pg. 93

Pudritz, R. - 2001; Science;295;82

Roser, J.E.; Manico, G.; Pironello, V. & Vidali, G. - 2002 Ap.J.; 581; 276

Salpeter, E.E. - 1955 Ap.J. ; 121 ; 161

Seffens, W. - 1999; Science ; 285 ; 1228.

She,Z.S.; Jackson,E. & Orzag, S.A. - 1990 Nature ; 344; 226

Sigalotti, L.D.G. & Klapp, J. - 1977 Ap.J. ; 474; 710

Silk, J. - 1995 Ap.J. ; 438; L41

Sitink, T.K. - 1995;Fragmentation of Molecular Clouds and Star Formation ; ed. E. Falgarone, F. Boulanger & G. Douvert; Kluwer Academic , Dordecht

Stutzki, J.; et.al. - 1998 A & A; 336 ; 697.

Thaddeus, P. - 1982 ; Annals of the New York Academy of Sciences ; 395 ; 9

Vailati,A. & Giglio, M. - 1997 Nature ; 390 ; 262

van der Marel; et.al. - 1997 Nature;

van Dongen, P.G. - 1987 J.Phys. A:Math. Gen.; 18; 2779

Wada,K. & Norman,C.A. - 1999;Ap.J.; 516; L13

Weber,F. - 2000; astro-ph 0008376

Witten, T.A. & Meakin, P. - 1983 Phy. Rev. B; 28; 5632 170; 479

Witten, T.A. & Sanders, L.M. - 1981 Phy. Rev. Lett; 47; 1400